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THE PUBLIC SCHOOL ARITHMETIC

FOR GRAMMAR GRADES

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BASED ON McLELLAN AND DEWEY'S

"PSYCHOLOGY OF NUMBER"

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PREFACE

This book has been specially prepared for use in Grammar Grades. Like all the other books of the series, the treatment is based on Dewey and McLellan's "Psychology of Number"; the basal idea of that work is the correlation of number and measurement — *"No number without measurement, no measurement without number."* This implies three factors in the number process, — a Whole to be measured, a Unit of measure, and the How Many of this Unit in the Whole. This basal idea is nothing but an application to quantity of the fundamental principle common to all thinking: —

From undefined whole to defined whole by Analysis, which gives parts (or particulars), and by Synthesis, which gives the defined whole.

The application of this principle in the teaching of Arithmetic makes all the difference between the rational or psychological method and the baldly mechanical methods which too generally prevail. This one idea, the idea of number as measurement, running through all the so-called rules of Arithmetic, secures the unity of the whole subject, and maintains thereby the constantly increasing interest of the learner. The mind naturally works toward unity, and anything that facilitates this normal movement must be essentially attractive, while anything that thwarts it is essentially repellent.

The lack of recognition of this principle largely explains the distaste for Arithmetic which may be found in so many schools. Each rule is practically regarded as a bit of subject-matter which somehow has to be got into the child's mind independently of his previous experiences in arithmetical ideas and processes. When *e.g.* teachers and text-book writers look upon Fractions as having

no connection with "Whole Numbers," as, indeed, not even to be classed as numbers, is it any wonder that the child, when he comes to fractions, is utterly bewildered, separated as he is from his former number experiences by a break which he cannot pass? For the child learns with what he has learned. When any new matter is presented to him, there must be a breaking up into ideas and images of all that part of what "he has learned" which is felt to have a bearing upon the new matter. The selection and adjustment of these ideas is the instrument by which the new is learned; that is, by which it is interpreted and assimilated. The result, in fact, is a remaking of the old — the enlarging, defining, and enriching of the old experiences by means of the new. It is plain, then, that if there is nothing in the old experience connected with the new matter, or if the old experience is only very vaguely connected with it, learning in the true sense of the word cannot take place.

On the other hand, the psychological method, beginning with the vague How Much and How Many of the child's experiences and proceeding by means of constructive exercises in which the need of number is felt, leads to a clear idea of number as a definite So Many of units of measure making up the definite So Much of quantity. There is unity from beginning to end. Each successive lesson grows out of the preceding one and contributes to the development of the number sense. The rules are not isolated groups of ideas and operations having no intelligible connection with one another, but are simply phases in the development of the one idea which the child has been using from the start. The "Images" which come from his first crude notions of number and quantity are the means for the mastery of rational counting; the images which result from adding and subtracting are easily used for the recognition of the new element in multiplication and division — new only in this, that the ratio idea, implicitly present from the first, becomes more explicit.

The same is true of Fractions. Fractions are not something wholly outside the child's experience. He has been unconsciously using the fraction process from the first. He has been breaking

up a whole into parts and putting the parts together again, *e.g.* he has measured a length of eight inches and counted the inch divisions. In this counting he is taking one of the eight, two of the eight, three of the eight, etc. This is the fundamental process of fractions. It is, too, essential to the growth of the right idea of number.

When the method of text-book and teacher is based on a clear conception of the mind's action in dealing with quantity, there will be no need for elaborate controversy as to whether the course in arithmetic should be simplified, or, rather, impoverished, by leaving out certain "Rules" — Least Common Multiple, Percentage, Interest, Ratio and Proportion, etc.

Some of the features of this book may be more specifically stated : —

1. The treatment of the subject being based on the true idea of number — that idea of number which brings the world into subjection to man — the unity of the subject-matter is determined ; the unity of the subject-matter secures unity of interest and the normal action of the mind from beginning to end. This means the highest results with the least waste of power.

2. This book, therefore, makes for thoroughness. Most arithmetics aim at attaining thoroughness by repetition ; but repetition is not thoroughness, nor does it lead to thoroughness ; on the contrary, it may be a hindrance to thoroughness. Endless repetition of forms and symbols, whether by the Grube, the Spiral, or any other method, where there is little or no clear consciousness of the meaning of symbols, or the mental activity symbolized, tends to dulness and mind-wandering instead of alertness and attention. Not repetition of facts and principles, but unity of thought, quickened by unity of interest and working upon facts and principles to organize them into unity, is thoroughness ; or, as Dr. Dewey somewhere says, " One is thorough who pursues a given portion of work *consecutively* and adapts the various parts of it to each other in an effective way so as to make a whole."

3. With this idea of thoroughness constantly in view, all the explanations and exercises are so arranged that there is a contin-

uous transformation of attainments into resources. When a new problem, new thought, or topic is "learned" by means of relevant images derived from past experiences, it is immediately applied in the acquisition of new knowledge. There is thus that *continuity* which alone secures organized knowledge and organized power.

4. In its vast number of new problems founded on and growing out of the commercial, agricultural, industrial, and social interests of the country, this book is without a rival. These problems are so presented that the pupil can hardly help realizing the actual conditions which gave rise to them, or to which they allude; the result is a vital connection between the life of the school and the larger social life. They are therefore practical in the true sense of the word. They make possible the true correlation of Arithmetic, viz., with the activities of the child and of the community in which he lives. The teacher can depend on the accuracy of the figures and statements given in the problems.

5. On account of the careful arrangement of the problems with a view to the gradual and normal growth of the image, the great instrument of the teaching as well as of the learning processes, the book is at once *stimulating and easy to teach*. The book teaches itself. There is, and should be, no break between the subject-matter and method.

6. The miscellaneous exercise at the close of nearly every chapter gives ample opportunity for review, and will prove a great saving of time to the teacher.

7. Abstract problems and principles are constantly and closely associated with concrete examples, that is, with ideas and images derived from the learner's own experience.

8. The ratio idea is used in a sane, practical way throughout the book. No number without measurement and no measurement without ratio.

9. There is a gradual and psychological introduction of "X" at the time when the pupil feels the need of a more powerful and economical instrument of solution than is afforded by arithmetic. The "X" symbolizes, gathers into one the elements which are

more or less distinctly seen to be connected with the problems, and thus facilitates the calling up of the necessary images. This furnishes a real basis and motive for the study of algebra.

10. There is an introduction to concrete geometry in a series of closely related problems.

· This series of "Public School Arithmetics" is the only series founded upon the "Psychology of Number," and from the first book to the last consistently carrying out the fundamental ideas of that work. The many teachers using the series are unanimous in expressing a high opinion of it. It is thought that the words which Dr. Dewey applied to "The Public School Arithmetic" may be applied with emphasis to this "Public School Arithmetic for Grammar Grades" : —

"I am particularly struck with the clearness and conciseness of the method of treatment, the logical order of the selection of topics, and the exclusion of useless and irrelevant matter. The simplification of treatment, due to sticking close to fundamental principles, must recommend the book to teachers and pupils who have been bewildered by the great number of topics treated in the ordinary arithmetics — topics which do not differ at all in their logical or arithmetical basis, but are simply different practical expressions of the same principle."

An answer book will be furnished to classes, free of charge, on the written order of the teacher.

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ARITHMETIC

CHAPTER I

DEFINITIONS AND REVIEW

1. A unit is a quantity used to measure quantity of the same kind.

Thus, 1 mi. is a unit used to measure distance, 1 A. to measure the size of a farm, and 1 doz. eggs to measure the quantity of eggs. If a lady wants to know into how many hair ribbons, each 1 ft. 6 in. long, she can cut a piece of ribbon 2 yd. 1 ft. 6 in. long, she must use 1 ft. 6 in. as a unit with which to measure the length 2 yd. 1 ft. 6 in. Cut a piece of string 2 yd. 1 ft. 6 in. long and divide it into parts each 1 ft. 6 in. long. How many ?

2. As a person measures a quantity he counts the **Number** of units in the quantity. The *number* and the *unit* measure the *quantity*.

Draw a line 7½ in. long and measure it with a unit 2½ in. How many 2½-in. units in 7½ in. ? Measure the same quantity with a unit 1½ in. How many 1½-in. units in 7½ in. ?

3. The number is the **Ratio** of the quantity to the unit.

A line 15 in. long is measured 5 times by the unit 3 in., hence 5 is the ratio of 15 in. to 3 in. What is the ratio of 6 in. to 2 in. ? 12 qt. to 3 qt. ? 1 hr. to 10 min. ? 1 ft. 6 in. to 6 in. ?

Exercise 1

1. Mark two points on the blackboard 6 yd. apart. Cut a string 2 ft. long, and by measuring find how often 2 ft. is contained in 6 yd.

2. Find, without actually measuring, how often 2 ft. is contained in 6 yd. How many widths of carpet 2 ft. wide are needed to carpet a room 6 yd. wide?

3. Draw an oblong 12 in. long and 8 in. wide. Cut out of paper an oblong 4 in. long and 3 in. wide. Using this as a unit of measure, find how often it is contained in the larger oblong.

4. Find, without actually measuring, how many oblongs 4 in. by 3 in. can be cut from an oblong 12 in. by 8 in.

5. A piece of cardboard 18 in. by 12 in. is cut into cards 6 in. long and 4 in. wide. How many are there?

6. Put 2 gal. 1 qt. of water into a pail and measure it with a pint measure. 2 gal. 1 qt. = ? pt.

7. Find, without actually measuring, how many pints are equal to 2 gal. 1 qt.

8. How many pint jars can be filled with 4 gal. 2 qt. of maple syrup?

9. Cut out a piece of cardboard 4 in. long. Measure a distance equal to 8 of these units. Measure this distance again with a foot rule. 8 times 4 in. = ? ft. ? in.

10. Find, without actually measuring, how many feet and inches a string is that can be cut into 8 pieces each 4 in. long.

11. In the following examples what number expresses the measurement of the quantity by the unit?

Quantity	Unit	Quantity	Unit
2 ft. 6 in.	3 in.	3 bu.	12 qt.
11 qt. 2 pt.	6 pt.	$4.50	1 dime
3 wk. 4 da.	5 da.	24 sq. in.	oblong 3 in. × 2 in.
1 hr. 30 min.	18 min.	30¢	3¢ + 2¢

How do you find the number of units in a given quantity?

12. In the preceding example what is the ratio of each quantity to its own unit? What is the ratio of the unit to the quantity?

13. In the following examples find the quantities measured by the numbers and the units:

Number	Unit	Number	Unit
8	2 qt.	6	3 for 10¢
2½	6¢	9	2 pk.
3	35 mi.	40	5 da.
2 doz.	2¢ apiece	6	3 mi. + 4 mi.

14. How do you find the quantity containing a given number of units?

Exercise 2

1. A boy saves $4 a month. In how many months will he save the price of a bicycle worth $32?

2. A man earns $18 a week and spends $15. In how many weeks will he save enough to pay a debt of $24?

3. A family uses 2½ qt. of milk at 6¢ a quart daily. What does their milk cost per day?

What do they pay for milk in a month of 30 da.?

4. A family uses 1 qt. 1 pt. of milk at 6¢ a quart daily. What is their milk bill for the month of April? May?

5. A family uses 2 qt. 1 pt. of milk at 6¢ a quart daily. What do they pay for milk in the month of March, if they get 8 qt. extra?

6. A boy riding on a bicycle gains 3 mi. an hour on a man who is driving. In how many hours after passing him will he be 15 mi. ahead?

7. Two boats travel down a river at the rate of 14 and 9 mi. an hour respectively. If they start together, when will the faster be 20 mi. ahead?

8. Two trains travel, one east and the other west, at the respective rates of 35 and 23 mi. an hour. How far apart will they be in 3 hr.?

9. How do you find the area of an oblong ?

Find the cost of cementing the floor of a cellar 8 yd. long and 6 yd. wide at 15 ¢ a square yard.

10. Find the number of cubic feet in a solid 4 ft. long, 3 ft. wide, and 2 ft. thick. If 1 cu. ft. weighs 8 lb., find the weight of the solid.

11. Find the cost of digging a cellar 6 yd. long, 4 yd. wide, 2 yd. deep at 24 ¢ a cubic yard.

Exercise 3

In the following examples find the sum and prove the answers correct by beginning at the top and adding down :

1. 165 952 674	**2.** 913 567 846	**3.** 319 495 323	**4.** 231 908 259
5. 1829 6513 8661	**6.** 1173 2664 3849	**7.** 2161 7243 5436	**8.** 6963 8196 7749
9. 66981 46938 33430 22109	**10.** 65534 23776 50388 72441	**11.** 36205 18882 68804 15234	**12.** 90195 89974 36632 88353
13. $2581.27 2915.74 187.29 3433.75 2869.28	**14.** $8394.20 7947.38 4698.63 6254.98 6682.57		**15.** $5758.93 2392.34 1332.55 4671.12 6778.86
16. 8667.914 2336.621 1138.325 3657.562 6551.438	**17.** 5639.498 4880.436 2252.203 6767.859 8599.638		**18.** 2542.368 4923.552 1589.865 3656.146 2354.289

Exercise 4

1. An ocean steamer carried 627 cabin and 829 steerage passengers. Find the total number of passengers.

2. A vessel in crossing the Atlantic made daily runs of 496, 443, 470, 457, 481, and 425 mi. Find the length of the trip.

3. In June, 1899, a rural mail-carrier delivered 5089 pieces of mail; in July, 5456; August, 5942; September, 6094; October, 6799. Find the total number of pieces delivered in the five months.

4. In a game of golf a gentleman took the following number of strokes at the different holes: 6, 8, 8, 7, 5, 7, 4, 6, 8. Find his score.

5. Aug. 16, 1899, a Chicago daily paper announced the following contributions to the Porto Rican relief fund: $500, $100, $125, $1, $10, $2, $5, $105. Find the total.

6. The following cash receipts of the Daily News Fresh Air Fund were reported Aug. 5, 1899: $.25, $.10, $9.75, $2.00, $3.45, $1.00, $10.00, $1.50, $1.80, $.50, $2.75, $1.00. Find the total amount.

7. The daily attendance at the Lincoln Park Sanitarium for the week ending Saturday, Aug. 19, 1900, is given below. Find the total attendance for each day of the week.

Monday, Aug. 14. — Sick babies, 157 ; mothers, 157 ; children, 917 ; visitors, 1044.

Tuesday, Aug. 15. — Sick babies, 153 ; mothers, 153 ; children, 696 ; visitors, 1026.

Wednesday, Aug. 16. — Sick babies, 158 ; mothers, 159 ; children, 846 ; visitors, 1210.

Thursday, Aug. 17. — Sick babies, 211 ; mothers, 212 ; children, 1362 ; visitors, 2062.

Friday, Aug. 18. — Sick babies, 145 ; mothers, 145 ; children, 730 ; visitors, 1086.

Saturday, Aug. 19. — Sick babies, 158 ; mothers, 158 ; children, 637 ; visitors, 1107.

8. Find the total number of recruits enlisted for service in the Philippine Islands, Sept. 13, 1899, the record of the different regiments on that date being as follows: 38th, 650; 39th, 800; 40th, 262; 41st, 237; 42d, 484; 43d, 304; 44th, 311; 45th, 448; 46th, 514; and 47th, 592.

9. Four towns lie on a road running north and south. The distance from the first to the second is 6.79 mi., from the second to the third 8.57 mi., and from the third to the fourth 9.84 mi. Find the distance between the first and fourth towns.

10. Aug. 17, 1899, there were inspected in Chicago 132 more cars of oats than of corn. If 281 cars of corn were inspected, find the number of cars of both kinds inspected on that day.

11. A country post-office received 746 pieces of mail matter in August, 144 more in September than in August, and 40 more in October than in September. Find the total number of pieces received during these three months.

12. A real estate agent bought three lots for $375, $490, and $550 respectively. He sold them at a gain of $125 apiece. Find the total selling price.

13. A man paid $3475 for a house and lot. He spent $625 for improvements and $35 in taxes. He then sold at a gain of $450. Find the selling price.

14. Copy neatly the following statement of six weeks' cash receipts; add the amounts vertically and find the sum of the totals:

	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.
1st	$32.25	$24.63	$25.93	$36.27	$23.92	$46.75
2d	41.38	18.79	45.10	24.50	33.37	37.89
3d	29.50	31.83	26.48	39.75	36.76	36.97
4th	27.68	29.65	19.75	18.42	36.61	26.51
5th	38.67	42.77	27.86	31.20	47.53	47.38
6th	28.34	28.64	19.99	26.45	29.75	25.74

15. Copy and add as in the preceding example:

	1st	2d	3d	4th	5th	6th
Mon.	$ 32.25	$ 41.38	$ 29.50	$ 27.68	$ 38.67	$ 28.34
Tues.	24.63	18.79	31.83	29.65	42.77	28.64
Wed.	25.93	45.10	26.48	19.75	27.86	19.99
Thur.	36.27	24.50	39.75	18.42	31.20	26.45
Fri.	23.92	33.37	36.76	36.61	47.53	29.75
Sat.	46.75	37.89	36.97	26.51	47.38	25.74

Why should you get the same total amount for this example as for the previous one?

16. Find the total cost of removing from the business district of the city of Chicago the snow that fell during the week ending Feb. 9, 1901.

Storm No. 1,   Storm No. 2,

Cost of laborers . . . . $8,950 Cost of laborers $3,300

Cost of teams . . . . . 11,710 Cost of teams 4,200

Exercise 5

*Subtract:

1. 833 752	**2.** 527 481	**3.** 496 169	**4.** 943 755
5. 1931 1685	**6.** 6743 2862	**7.** 9236 7698	**8.** 7693 2498
9. 65954 58712	**10.** 47581 29649	**11.** 38145 24628	**12.** 99570 41657
13. 2678.28 1732.50	**14.** 3650.12 1732.43	**15.** 8418.60 1654.75	
16. 1559.834 263.761	**17.** 3976.986 1932.578	**18.** 5436.621 1864.865	

* Use the addition method of subtracting.

| 19. | 2874.931 | 20. | 9025.723 | 21. | 3542.790 |
| | 1458.923 | | 3496.824 | | 1863.254 |

| 22. | 408025.3 | 23. | 53251.04 | 24. | 3868.239 |
| | 372486.7 | | 21740.42 | | 2687.832 |

Exercise 6

1. Aug. 21, 1899, the highest temperature in Chicago was 90° and the lowest 74°. Find the difference.

2. Of 1310 men composing the 30th regiment 706 were from Illinois and the remainder from Michigan. Find how many were from Michigan. How many more were from Illinois than from Michigan?

3. The freight earnings of a certain railroad per mile for 1899 were $4675, and for 1898 $4138. Find the increase.

4. August, 1899, the postal receipts at St. Paul, Minn., amounted to $37,804, an increase of $255 over August, 1898. Find the postal receipts at St. Paul, August, 1898.

5. The cost of police service in a certain town for 1898 was $3759.16 and for 1899 $4263.74. Find the increase.

6. A town board levied for police $3648.73, for a street fund $1682.79, and for a fire and water fund $3763.62. The tax for these purposes in the preceding year was $7854.62. Find the increase.

7. The report of the Chicago Penny Savings Society shows that the deposits for the year ending June 30, 1899, were $43,300, and the withdrawals $31,849.75. Find the balance to the credit of depositors June 30, 1899.

8. The area of the Philippine Islands is 114,326 sq. mi., of Cuba 41,655 sq. mi., and of Porto Rico 3550 sq. mi. How much larger are the Philippine Islands than Cuba and Porto Rico together?

9. June 30, 1900, a school board had $1552.67 on hand. During the next year it received $8795.66 and spent $8973.54. Find the balance on hand June 30, 1901.

10. In 1879 the total length of our postal routes was 79,991 mi., in 1898 it was 174,777 mi. Find the increase.

11. The exports from the United States to the Philippines from Jan. 1 to July 31, 1899, were $386,109, for the same period in 1898 $65,736, and in 1897 $47,754. Find the increase in the amount of exports each year.

12. In 1899 the Chicago, Milwaukee, and St. Paul Railroad owned 7876.84 mi. of track, of which 6142.31 mi. were main line. Find the number of miles in the branch lines.

Exercise 7

* Multiply and prove the answers to the first two, correct by multiplying the multiplier by the multiplicand.

1. 61	96	34	83	35	66
17	35	56	98	75	47
2. 342	190	238	652	385	793
86	19	67	45	98	57
3. $67.40	$72.59	$23.55	$97.64	$84.07
85	67	92	96	45
4. $64.92	$87.96	$53.28	$93.16	$80.57
38	75	36	77	99
5. 601	141	248	884	345
324	188	312	144	432
6. 1491	2983	1504	3844	2998
834	402	386	201	898

* See Multiplication Table, § 39.

7.	62.94	45.61	92.36	39.46	91.64
	32	45	53	68	25

8.	3.229	3.562	6.403	4.899	1.764
	64	35	77	58	86

Exercise 8

1. Find the cost of 32 yd. of carpet at 64 ¢ a yard.

2. A dealer bought 25 horses at an average price of $75, and 48 head of cattle at $28 apiece. Find the total cost.

3. A man drives 5 mi. to the station, rides on the train 4 hr. at 34 mi. an hour, and then drives into the country 6 mi. What is the entire distance?

4. A grocer bought a dozen boxes of soap at $2.25 a box of 60 bars, and sold it for 5 ¢ a bar. Find his gain.

5. A farmer's wife bought 4 lb. of coffee at 32 ¢ a pound, and 16 lb. of sugar at 5½ ¢ a pound, giving in exchange 8 lb. of butter at 18 ¢ a pound. How much money must she pay to settle the bill?

6. How far will a bicyclist travel in 8 da., if he travels 6 hr. a day at 9 mi. an hour?

7. Find the weight of the hogs in a train load consisting of 36 cars, each car containing 54 hogs, the average weight being 195 lb.

8. Find the cost of a 300-ft. breakwater at $38 a foot.

9. In 1898 the State of Indiana employed 237 truant officers at an average salary of $169. Find the total amount paid to truant officers.

10. A grain dealer bought 6275 bu. of wheat in Chicago at 75 ¢ a bushel, and shipped it to New York at a cost of 2 ¢ a bushel. Find his gain if he received 82 ¢ a bushel for it in New York.

11. The city of Stockholm pays $ 300,000 a year for regular teachers in the public schools, and $ 300 a year to each of 304 substitute teachers. Find the total sum paid.

12. It costs $ 30 a carload to remove watermelons from Poseyville, Ind., to Chicago. Watermelons pack 1175 to the car; how much less is this per car than 3 ¢ a melon?

13. Find the total weight of 739 106-lb. sheep, 954 98-lb. sheep, and 268 62-lb. lambs.

14. Find the cost of 90 260-lb. hogs at $ 4.64 per 100 lb.

15. Find the value of a carload of beeves, average weight 1300 lb., at $ 6.40 per 100 lb., there being 19 to a carload.

Exercise 9

In the following examples name the trial divisors, and find the quotients and remainders:

1. 21)714	32)832	41)3239	52)4316
2. 42)7961	63)2961	72)8756	92)7488
3. 19)5344	38)5192	83)4472	69)6365
4. 58)8102	34)2669	57)7608	96)2968
5. 301)8519	225)3447	426)6358	119)8968
6. 108)7400	213)4966	125)3608	524)4853
7. 23)66263	17)15189	54)52341	47)72935
8. 44)50002	95)12855	19)52496	68)35334

Exercise 10

1. A bicycle dealer bought 6 doz. bicycles for $ 2016. Find the cost of each bicycle.

2. A man paid $ 5025 for a farm of 75 A. Find the cost per acre.

3. A horse dealer paid $2520 for 72 horses. What was the average price?

4. If a train travels 34 mi. an hour, how long will it take to travel 816 mi., there being stops amounting to 45 min.?

5. The yield of barley in a certain township was 32,452 bu., the average yield being 28 bu. to the acre. Find the number of acres of barley.

6. A golf course of 18 holes is 5886 yd. around. Find the average distance between two holes.

7. A golf club spends $4000 a year to meet expenses. This is met by the annual dues of its members at $25 apiece. Find the number of members.

8. The total attendance at the Lincoln Park Sanitarium, Chicago, for the 12 weeks ending Sept. 9, 1899, was 125,640. If it was open 6 days a week, find the average daily attendance.

9. A railroad running inland from the sea-coast reaches in 106 mi. an elevation of 15,688 ft. Find the average elevation in feet per mile.

10. The following live stock was received at the Union Stockyards, Chicago, in two days: Cattle, 22,097; hogs, 58,587; sheep, 29,659. Find the number of carloads of each, if cattle average 19 to a car, hogs 55, and sheep 92.

11. 14,912 men enlisted in the 13 regiments for service in the Philippines. This was 1834 more men than were needed; find the number of men to a regiment.

12. A horse dealer sold 32 horses at an average price of $66 each, and used the money to buy 24 more. Find the average price paid for each of the last lot.

13. A wholesale furniture dealer sold 144 beds at $16 apiece, and after using $100 of this money, bought sideboards with the remainder costing $19 each. How many did he buy?

14. A cattle dealer sold a herd of 63 cattle at an average price of $25, and with this money bought 12 sheep at $6.25 each, and 60 cattle. Find the average price of the cattle.

15. 36 carriages of the same kind cost $1332. What will 2 doz. cost at $5 apiece more?

16. An Indiana farmer raised 2294 bu. of corn, averaging 74 bu. to the acre. His neighbor raised 2294 bu., averaging 62 bu. to the acre. The second farmer planted how many acres more than the first?

17. A farmer received $439.56 for the wheat he raised on 27 A., yielding 22 bu. per acre. Find the selling price per bushel.

18. An Ohio farmer sold his wheat and oat crops for 1899 for $894.96. He raised 625 bu. of wheat and 1854 bu. of oats. If the oats sold for 24 ¢ a bushel, find the selling price of the wheat.

CHAPTER II

REVIEW

Exercise 11

1. What is $\frac{1}{4}$ of $\$12$? $\frac{3}{4}$ of $\$12$? $\frac{1}{3}$ of $\$15$? $\frac{2}{3}$ of $\$15$? $\frac{1}{6}$ of $\$24$? $\frac{5}{6}$ of $\$24$? How do you find $\frac{5}{6}$ of a quantity?

2. How do you find $\frac{2}{3}$ of a quantity? $\frac{3}{4}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{4}{9}$?

3. What is $\frac{2}{3}$ of $\$18$? $\frac{3}{4}$ of 16 oz.? $\frac{5}{8}$ of 24 hr.? $\frac{7}{8}$ of 24 sheets of paper? $\frac{4}{9}$ of 36 in.?

4. How many inches in $\frac{3}{4}$ ft.? Minutes in $\frac{2}{3}$ hr.? Quarts in $\frac{5}{8}$ bu.? Quarts in $\frac{3}{4}$ gal.? Inches in $\frac{5}{9}$ yd.?

5. A man earns $\$36$ a week, and spends $\frac{3}{4}$ of it. What are his weekly expenses?

6. A farmer sold $\frac{3}{4}$ of his crop of 624 bu. of oats. How many bushels did he sell? What did they sell for at $30 \cancel{c}$ a bushel?

7. A farmer sold $\frac{3}{4}$ of his crop of 824 bu. of corn at $31 \cancel{c}$ a bushel. What did he receive for it?

8. From 1 yd. of ribbon cut off $\frac{1}{4}$ yd., and what part of a yard is left? Cut off $\frac{3}{4}$ yd., and what part is left?

9. If a man spends $\frac{5}{6}$ of his salary, what part does he save? If a boy sleeps $\frac{2}{5}$ of a day, what part of the day is he awake?

10. A man earns $\$25$ a week, and spends $\frac{4}{5}$ of it. What part of it does he save? How many dollars a week?

11. A lady, who had $\$24$, spent $\frac{3}{4}$ of it for a sofa and the remainder for a chair. Find the cost of the chair. Give two solutions.

12. What is $\frac{1}{3}$ of $\$25.50$? $\frac{2}{3}$ of $\$25.50$? $\frac{3}{4}$ of $\$24.60$? $\frac{5}{6}$ of $\$19.20$? $\frac{3}{7}$ of $\$43.40$?

13. I paid $\$31.50$ for a bicycle for myself, and $\frac{5}{7}$ as much for a boy's bicycle. Find the difference in price.

14. A merchant paid $\$4.32$ for a rug, and sold it for $\frac{3}{4}$ of its cost. Find how much he lost.

Exercise 12

1. Draw a line 1 ft. long. Measure off a part 6 in. long. What part of a foot is 6 in.?

2. Make a drawing to show that 3 in. is $\frac{1}{4}$ ft.; that 4 in. is $\frac{1}{3}$ ft.; that 9 in. is $\frac{3}{4}$ ft.; that 8 in. is $\frac{2}{3}$ ft.; that 10 in. is $\frac{5}{6}$ ft.

3. 6 is what part of 12? What part of 12 is 4? 8? 9? $\frac{3}{4}$ is the fraction $\frac{9}{12}$ reduced to its *lowest terms*.

4. Reduce to their lowest terms: $\frac{3}{12}$, $\frac{4}{12}$, $\frac{8}{12}$, $\frac{10}{12}$, $\frac{9}{12}$, $\frac{6}{8}$, $\frac{8}{10}$, $\frac{6}{15}$, $\frac{12}{16}$.

5. How do you reduce a fraction to its lowest terms?

6. Reduce to their lowest terms: $\frac{6}{9}$, $\frac{15}{18}$, $\frac{18}{24}$, $\frac{10}{16}$, $\frac{15}{20}$, $\frac{20}{16}$, $\frac{14}{21}$, $\frac{24}{32}$, $\frac{30}{20}$, $\frac{20}{25}$.

7. Make a drawing to show that $\frac{1}{2}$ of a quantity is equal to $\frac{2}{4}$ of it.

8. Make drawings to show that:

$\frac{1}{2}=\frac{3}{6}$	$\frac{1}{3}=\frac{3}{12}$	$\frac{1}{3}=\frac{3}{9}$	$\frac{1}{4}=\frac{3}{12}$
$\frac{1}{4}=\frac{4}{16}$	$\frac{1}{5}=\frac{2}{10}$	$\frac{1}{5}=\frac{3}{15}$	$\frac{1}{6}=\frac{3}{18}$

9.

$\frac{1}{2}=\frac{?}{4}$	$\frac{1}{2}=\frac{?}{10}$	$\frac{1}{3}=\frac{?}{9}$	$\frac{1}{3}=\frac{?}{12}$
$\frac{1}{4}=\frac{?}{12}$	$\frac{1}{4}=\frac{?}{20}$	$\frac{1}{5}=\frac{?}{15}$	$\frac{1}{6}=\frac{?}{24}$

10. Make drawings to show that:

$\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$	$\frac{1}{3}+\frac{1}{6}=\frac{1}{2}$	$\frac{1}{2}+\frac{1}{6}=\frac{2}{3}$
$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$	$\frac{1}{10}+\frac{2}{5}=\frac{1}{2}$	$\frac{1}{4}+\frac{1}{3}=\frac{7}{12}$

11. What is the sum of:

$\frac{1}{4}$ and $\frac{1}{2}$?	$\frac{1}{2}$ and $\frac{1}{6}$?	$\frac{1}{3}$ and $\frac{1}{6}$?	$\frac{1}{9}$ and $\frac{1}{3}$?
$\frac{1}{2}$ and $\frac{1}{5}$?	$\frac{1}{3}$ and $\frac{1}{12}$?	$\frac{1}{3}$ and $\frac{1}{4}$?	$\frac{1}{4}$ and $\frac{1}{5}$?
$\frac{1}{12}$ and $\frac{1}{4}$?	$\frac{1}{2}$ and $\frac{1}{3}$?	$\frac{1}{3}$ and $\frac{1}{7}$?	$\frac{1}{6}$ and $\frac{1}{8}$?

12. Make drawings to show that $\frac{2}{3} = \frac{8}{12}$; $\frac{3}{4} = \frac{6}{8}$; $\frac{4}{5} = 1\frac{2}{15}$.

13.

$\frac{2}{3} = \frac{?}{6}$	$\frac{3}{4} = \frac{?}{16}$	$\frac{3}{5} = \frac{?}{30}$	$\frac{3}{8} = \frac{?}{24}$
$\frac{4}{9} = \frac{?}{18}$	$\frac{5}{6} = \frac{?}{18}$	$\frac{4}{9} + \frac{5}{6} = \frac{?}{18}$	
$\frac{2}{3} = \frac{?}{12}$	$\frac{3}{4} = \frac{?}{12}$	$\frac{2}{3} + \frac{3}{4} = \frac{?}{12}$	

14. Make drawings to show that:

(a) $\frac{2}{3} + \frac{3}{4} = 1\frac{5}{12}$ $\qquad\qquad\qquad$ $\frac{4}{9} + \frac{5}{6} = 1\frac{5}{18}$

(b) $\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$ $\qquad\qquad\qquad$ $\frac{5}{6} - \frac{4}{9} = \frac{7}{18}$

15. What is the smallest number in the multiplication tables of 3 and 4? Of 9 and 6? To what do you reduce $\frac{2}{3}$ and $\frac{3}{4}$ before you can add them? $\frac{4}{9}$ and $\frac{5}{6}$?

16. What is the smallest number in the multiplication tables of 2 and 5? 3 and 9? 4 and 3? 4 and 6? 5 and 3? 7 and 3? 4 and 5? 9 and 6? 7 and 2? 8 and 12? 3 and 5?

To what will you reduce each fraction in the following example before adding?

17. Add:

$\frac{1}{2}$ and $\frac{2}{5}$	$\frac{3}{5}$ and $\frac{2}{3}$	$\frac{4}{4}$ and $\frac{2}{3}$	$\frac{2}{3}$ and $\frac{4}{9}$
$1\frac{2}{3}$ and $4\frac{3}{5}$	$\frac{1}{4}$ and $\frac{5}{6}$	$\frac{3}{7}$ and $\frac{2}{3}$	$3\frac{1}{9}$ and $2\frac{5}{6}$
$4\frac{3}{8}$ and $2\frac{5}{12}$	$2\frac{3}{7}$ and $3\frac{1}{2}$	$4\frac{3}{4}$ and $2\frac{1}{5}$	$4\frac{2}{3}$ and $6\frac{3}{5}$

18. How will you find the difference between two fractions?

19. Find the difference between:

$\frac{3}{5}$ and $\frac{1}{2}$	$\frac{2}{3}$ and $\frac{1}{9}$	$\frac{7}{9}$ and $\frac{1}{2}$	$\frac{2}{3}$ and $\frac{4}{9}$
$\frac{5}{9}$ and $\frac{1}{3}$	$\frac{5}{6}$ and $\frac{1}{4}$	$\frac{3}{8}$ and $\frac{2}{3}$	$6\frac{5}{7}$ and $3\frac{3}{8}$
$5\frac{3}{4}$ and $2\frac{3}{7}$	$6\frac{7}{8}$ and $3\frac{5}{6}$	$5\frac{1}{8}$ and $3\frac{3}{4}$	$6\frac{4}{5}$ and $4\frac{2}{3}$

20. William had stamps worth $\$2\frac{1}{2}$ on one penny savings card and worth $\$1\frac{3}{4}$ on another. He cashed both cards, how much money did he get?

Exercise 13

1. A pitcher containing $\frac{5}{8}$ gal. of water is filled by pouring into it $\frac{1}{4}$ gal. more. What part of a gallon does it hold?

2. Wheat, bought at $65\frac{7}{8}\phi$ per bushel, was sold at an advance of $\frac{1}{2}\phi$ per bushel. Find the selling price.

3. Corn, bought at $32\frac{1}{2}\phi$ per bushel, sold for $32\frac{5}{8}\phi$. Find the gain per bushel.

4. Nov. 24, 1899, oats sold on the Chicago market for $25\frac{1}{4}\phi$ per bushel, and the following day the price was $\frac{3}{8}\phi$ lower. Find the price Nov. 25.

5. I bought $10\frac{5}{6}$ T. of coal in the fall, and had $2\frac{1}{4}$ T. left over the next summer. How many tons did I burn? What did it cost at $\$6$ a ton?

6. A kettle weighing $3\frac{3}{4}$ lb. contains $4\frac{5}{6}$ lb. of fruit. Find the total weight.

7. Two bicyclists are travelling along the same road, in the same direction, the first at the rate of $12\frac{7}{8}$ mi. an hour, the second at the rate of $8\frac{1}{2}$ mi. How many miles does the faster gain on the slower in one hour?

8. If turkeys cost $12\frac{1}{2}\phi$ a pound, and chickens $9\frac{1}{4}\phi$, what is the difference between the cost of a 12-lb. turkey and 12 lb. of chickens?

9. A string was cut into two parts, one of which is $2\frac{3}{4}$ ft. long, and the other $3\frac{5}{6}$. How long was it before being cut?

10. $\frac{3}{4}$ yd. of cloth costs 60ϕ.
$\frac{1}{4}$ yd. of cloth costs $?\phi$.
$\frac{4}{4}$ yd. of cloth costs $?\phi$.
1 yd. of cloth costs $?\phi$.

What is your divisor here? Your multiplier?

c

11. $\frac{7}{8}$ of my age is 21 yr.

$\frac{1}{8}$ of my age is ? yr.

My age is ? yr.

What is your divisor here? Your multiplier?

12. What would have been the divisor and multiplier in the previous example if the fraction had been $\frac{7}{9}$? $\frac{5}{8}$? $\frac{4}{9}$? $\frac{2}{3}$? $\frac{4}{3}$? $\frac{9}{8}$? $\frac{7}{3}$? $\frac{5}{16}$? $\frac{14}{5}$?

13. $\frac{5}{6}$ yd. of cloth costs 30 ¢, what is the cost of 1 yd.?

14. $\frac{3}{8}$ of the speed of a railway train is 12 mi. an hour, find the rate at which it travels.

15. A man is $\frac{6}{5}$ as old as his wife. If he is 48 yr. of age, what is his wife's age?

16. A boy said, "When I am $2\frac{3}{4}$ times as old as I am now, I shall be 22 yr. of age." How old is he?

17. A man divided his farm between his two sons, giving the younger $\frac{3}{8}$ of it. If the older son got 90 A., what was the size of the farm?

18. Harriet spent $\frac{4}{9}$ of her money for a chair, and the remainder, which was $15, for a rug. How much had she at first?

19. Make examples like the preceding six questions, using the following price list:

Tea at 60 ¢ a pound.   Ribbon at 36 ¢ a yard.

Spoons at $15 a dozen.   Cloth at 48 ¢ a yard.

Exercise 14

1. Draw two lines, one 10 in., and the other 15 in. long. Divide each line into parts, each 5 in. long. The ratio of 10 in. to 15 in. is $\frac{2}{3}$. The ratio of 15 in. to 10 in. is $\frac{3}{2}$.

2. Make drawings to show that the ratio of 4 to 6 is $\frac{2}{3}$; of 12 to 16 is $\frac{3}{4}$; of 15 to 18 is $\frac{5}{6}$; of 24 to 16 is $\frac{3}{2}$.

3. What is the ratio of 6 to 4 ? 16 to 12 ? 18 to 15 ? 16 to 24 ? 18 to 30 ? 16 to 20 ? 32 to 36 ? 28 to 16 ? 21 to 35 ? 33 to 22 ?

4. Make drawings to show that the ratio of 16 in. to 1 yd. 1 ft. is $\frac{1}{3}$; of 2 ft. to 2 ft. 8 in. is $\frac{3}{4}$.

5. What is the ratio of 8 lb. to 12 lb. ? A 12-lb. turkey costs $1.26. What part of $1.26 will an 8-lb. turkey cost at the same price per pound ? How much ?

6. What is the ratio of 9 to 12 ? Quails cost $1.40 a dozen. What part of $1.40 will 9 quails cost ? How much ?

7. What is the ratio of 18 to 1 doz.? At $5.50 per dozen, what will 18 geese cost ?

8. What is the ratio of 20¢ to 18¢ ? A grocer paid $2.25 for eggs at 18¢ per dozen. What will they sell for at 20¢ per dozen ?

9. A grocer received $1.12 for a jar of butter at 21¢ a pound. What did it cost him at 18¢ a pound ?

10. I paid $4.64 for Early Rose potatoes at 40¢ a bushel. What would the same quantity of Burbanks have cost at 45¢ a bushel ?

11. Draw a rug 18 in. by 36 in. (scale 6 in. to 1 in.). Make a drawing to show that if this costs 84¢, a rug of the same quality, 36 in. by 72 in., should, at the same rate, cost 4 times as much, or $3.36.

12. A Brussels carpet, weighing 25 oz. to the yard, weighs altogether 110 lb. Find the weight of a velvet carpet of the same size, weighing 35 oz. to the yard.

Exercise 15

1. Draw a line .6 of a meter long; .4 m.; .7 m.

2. Make drawings to show lines .5 m. long; .53 m.; .67 m.; .89 m.; .91 m.

3. Make drawings to show lines .254 m. long; .875 m.; .624 m.; .258 m.

4. Make drawings to show lines .25 m. long; .33 m.; .166 m. What part of a meter does each line seem to be?

5. 3.26 lb. is read three and twenty-six hundredths pounds; 5.486 lb. is read five and four hundred eighty-six thousandths pounds.

6. Read the following, supplying different units:

6.42	2.358	60.006	329.017
5.84	14.405	412.15	425.7
3.16	70.4	72.6	19.934
3.06	70.04	459.245	9.009
8.2	159.72	300.001	226.043
24.09	7.415	721.01	724.981
16.74	8.009	455.554	999.999

7. In the expression 5.429 T. the *place value* of each figure is given thus: 5 is five tons, 4 is four tenths of a ton, 2 is two hundredths of a ton, and 9 is nine thousandths of a ton.

8. Give the place value of each figure in the following: 2.5 mi., 9.34 ft., 7.258 yd., 8.04 lb., 6.104 T., 64.291 A., 36.09, 100.102, 6.056, 249.003.

Add:

9. 33.725	65.876		274.796
64.059	6.932		989.307
49.374	87.04		891.2
62.498	29.985		56.238

Subtract:

10. 3.945	6.412	46.279	62.070
1.897	3.547	38.564	34.638

11. 5.29	4.7	8.	846.739
3.645	3.125	5.273	312.888

12. Find the number of acres of land in a farm divided into four fields containing, respectively, 14.25 A., 25.875 A., 23.75 A., 16.125 A.

13. From a piece of cloth containing 20.125 yd. were cut 12.875 yd. How much was left in the piece?

14. From a farm containing 160 A. the owner sold 62.875 A. How much did he still own?

15. A man bought a house and lot with .375 of his money, and invested the remainder in business. What part of his money was invested in business?

Exercise 16

Multiply:

1. 2	.2	.3	.32	.24	.56
3	3	3	3	3	3

2. .213	.457	2.47	5.814	21.049
4	2	6	7	5

3. .7	6	.23	36	145
6	.7	36	.23	.48

4. 9.362	8.364	54.31	932	5.768
53	29	645	.28	129

5. What will 15 loads of coal weigh, the average weight being 2.375 T.?

6. A grocer sold 16 bbl. of apples a day for one week at a gain of $.625 a barrel. Find his gain on a week's sales.

7. Find the cost of 26 thousand feet of lumber at $34.625 per thousand.

8. A merchant had $4875 in the bank and withdrew .36 of it to pay for goods. Find the amount still remaining in the bank.

9. A drover, after selling part of a herd of 840 cattle, had .55 of it left. How many did he sell?

10. Reduce to quarts : .5 pk., .375 pk., .875 pk., .23 pk., .424 pk.

11. Find the cost of .625 pk. of beans at 6¢ a quart.

12. Reduce to quarts : .375 bu., .75 bu., .87 bu., .694 bu., .825 bu.

13. Find the cost of .875 bu. of clover seed at 22.5¢ a quart.

14. How many yards in 1 mi. ? How many yards in .5 mi. ? .25 mi. ? .125 mi. ? .35 mi. ? .675 mi. ? .725 mi. ?

15. How many steps will a boy take in walking .725 mi., if he takes 1.5 steps to a yard ?

16. How many days in .146 of 300 da. ? If there are 300 working days in one year, how much does a man earn in .245 yr. at $1.50 a day ?

17. How many feet in 1 mi. ? In .125 mi. ? What is the cost of laying a concrete sidewalk .125 mi. long at $.625 a foot ?

18. How many quarts in .375 bu. ? What is the cost of .375 bu. of flaxseed at 4.75¢ a quart ?

Exercise 17

1. Divide 6.969 by 3; 4.864 by 2; 9.356 by 4.

2. Find the value of :

4.864 ÷ 4	9.465 ÷ 5	19.35 ÷ 2
39.588 ÷ 6	8.0 ÷ 5	231. ÷ 4
27.291 ÷ 3	37. ÷ 4	2150.42 ÷ 4
.994 ÷ 7	2.520 ÷ 8	721.44 ÷ 3
2.097 ÷ 9	54.76 ÷ 8	59.2 ÷ 8

In the first two divisions what is the place value of each figure in the quotient ?

3. A vessel travels 54.5 knots in 4 hr., what is the rate per hour ?

4. 1 gal. contains 231 cu. in., how many cubic inches are there in 1 qt. liquid measure ?

5. 1 pk. contains 537.6 cu. in., how many cubic inches in 1 qt. dry measure ?

6. A quart of oats occupies how many more cubic inches than a quart of water?

7. Find the value of:

$178.92 \div 21$	$362.79 \div 87$
$212.16 \div 68$	$132.634 \div 94$
$70.578 \div 27$	$876.6 \div 18$
$9.798 \div 46$	$4.15 \div 25$
$13.088 \div 32$	$18.468 \div 76$

8. A farmer got 18 loads of hay from one field. If the total amount was 15.57 T., find the average weight of one load.

9. A farmer sold 36 bu. of oats for $8.46. Find the price per bushel.

10. Show by measuring that 12 m. divided by 4 m. gives the same quotient as 1.2 m. divided by .4 m. Why is this so?

11.

18 in. $\div$ 3 in. $= ?$	1.8 in. $\div$ .3 in. $= ?$
24 yd. $\div$ 4 yd. $= ?$	2.4 yd. $\div$ .4 yd. $= ?$

$1.5 \div .3 = ?$	$9.6 \div .8 = ?$	$4.8 \div .6 = ?$
$4.9 \div .7 = ?$	$7.2 \div .9 = ?$	$3.5 \div .5 = ?$

12. Divide 4.548 by .6.

```
.6 | 4.548
 6 | 45.48
     7.58
```
Move the decimal point one place to the right in both divisor and dividend, and divide 45.48 by 6, as in the example. Why can you do this?

13. Divide 157.324 by .74.

```
.74)157.324(
74)15732.4(212.6
   148
   ‾‾‾
    93
    74
    ‾‾
   192
   148
   ‾‾‾
    444
    444
```

Move the decimal point two places to the right in both divisor and dividend before dividing. Why can you do this?

14. Find the value of :

$7.932 \div .6$	$10.101 \div 3.9$	$16 \div .8$
$4.536 \div .12$	$149.85 \div .37$	$36 \div .09$
$43.05 \div .15$	$.936 \div .15$	$6 \div .75$
$12.186 \div .18$	$5963.6 \div 6.8$	$36 \div .45$
$1.75 \div .05$	$75.250 \div 86$	$150 \div .25$
$3.78 \div 6$	$59.328 \div .48$	$27 \div .09$

15. A man divided a field containing 4 A. into lots, each containing .25 A. Find the number of lots.

16. Reduce to yards: 12 ft., 63 ft., 6.3 ft., 25.2 ft., 8.58 ft., .741 ft.

17. Reduce to feet: 48 in., 624 in., 50.4 in., 3.936 in., 3.876 in., 9.168 in.

Exercise 18

1. Draw a line 8 in. long, and divide it into fourths. Count the number of units in the line. How many ?

2. A quantity considered in percentage is measured by how many units ?

3. What is $\frac{1}{2}$ of 100 ? $\frac{1}{4}$ of 100 ? $\frac{3}{4}$ of 100 ? $\frac{4}{4}$ of 100 ?

4. $\frac{1}{2}$ of a quantity equals what per cent of it ? $\frac{1}{4}$? $\frac{3}{4}$? $\frac{4}{4}$?

5. What part of a quantity is 25% of it ? 50% ? 75% ? 100% ?

6. Find 50% of $\$48$; 25% of $\$36$; 75% of $\$36$; 100% of $\$15$.

7. A farmer gave his son 25% of his farm of 640 A. What was the size of his son's farm ?

8. A grocer paid $28 \cancel{\text{c}}$ a lb. for coffee, and sold it so as to gain 25% ; find the gain. Find the selling price.

9. My furnace burns 12 T. of coal during the winter and my neighbor's 75% as much. How much coal does he burn ?

10. A merchant paid $32 \cancel{c}$ a yard for cloth and sold it at a gain of $8 \cancel{c}$ a yard. What part of the cost did he gain? What per cent?

11. 25% of a man's age is 12 yr., how old is he?

12. What is $\frac{1}{3}$ of 100? $\frac{2}{3}$ of 100? $\frac{1}{3}$ of a quantity is what per cent of it? $\frac{2}{3}$? $\frac{1}{4}$? $\frac{3}{4}$? $\frac{3}{3}$?

13. A man built a house which cost $\$2250$ on a lot that cost $66\frac{2}{3}\%$ of that sum. Find the cost of the lot. Find the cost of the house and lot together.

14. A grocer bought peaches at $90 \cancel{c}$ a bushel, and sold them at a gain of $33\frac{1}{3}\%$. Find the selling price per bushel. Per peck.

15. What part of $\$36$ is $\$12$? A merchant bought rugs at $\$36$ each, and sold them at an advance of $\$12$. Find his gain per cent.

16. A speculator gained $\$400$ on a lot by selling it at a gain of $66\frac{2}{3}\%$. Find what he paid for it.

Exercise 19

1. What is $\frac{1}{5}$ of 100? $\frac{2}{5}$ of 100? $\frac{3}{5}$ of 100? $\frac{4}{5}$ of 100? $\frac{5}{5}$ of 100?

2. Commit to memory: $20\% = \frac{1}{5}$, $40\% = \frac{2}{5}$, $60\% = \frac{3}{5}$, $80\% = \frac{4}{5}$. What is meant by saying that $20\% = \frac{1}{5}$?

3. What is $\frac{1}{8}$ of 100? $\frac{3}{8}$ of 100? $\frac{5}{8}$ of 100? $\frac{7}{8}$ of 100? $\frac{1}{6}$ of 100?

4. Commit to memory: $\frac{1}{8} = 12\frac{1}{2}\%$, $\frac{3}{8} = 37\frac{1}{2}\%$, $\frac{5}{8} = 62\frac{1}{2}\%$, $\frac{7}{8} = 87\frac{1}{2}\%$, $\frac{1}{6} = 16\frac{2}{3}\%$.

5. How many minutes in 20% of 1 hr.? Quarts in $12\frac{1}{2}\%$ of 1 bu.? Ounces in $62\frac{1}{2}\%$ of 1 lb. avoir.? Hours in $37\frac{1}{2}\%$ of 1 da.? Square inches in $16\frac{2}{3}\%$ of 1 sq. ft.?

6. $\frac{1}{12}$ of $100\% = ?$ $\frac{1}{16}$ of $100\% = ?$ $\frac{1}{15}$ of $100\% = ?$ Memorize: $\frac{1}{12} = 8\frac{1}{3}\%$, $\frac{1}{16} = 6\frac{1}{4}\%$, $\frac{1}{15} = 6\frac{2}{3}\%$.

7. What part is 3 in. of 1 yd. ? What per cent ? What per cent is $5 of $20 ?

8. A man paid $640 for a lot, and sold it at a gain of $6\frac{1}{4}\%$. Find the gain and the selling price.

9. A man sold a cow for $\frac{2}{3}$ of the cost price. Find his loss per cent.

10. What per cent is gained by selling goods at $\frac{3}{2}$ of the cost? At $\frac{4}{3}$ of the cost ? At $\frac{6}{5}$ of the cost ?

11. Cloth which cost 30 ¢ a yard was sold at a loss of $16\frac{2}{3}\%$. Find the selling price.

12. A merchant paid 60 ¢ a yard for cloth, and sold it at 75 ¢ a yard. Find his gain. Find his gain per cent.

13. What per cent is gained if cloth costing 40 ¢ a yard is sold for 48 ¢ a yard ? 50 ¢ ? 44 ¢ ? 60 ¢ ? 45 ¢ ? 55 ¢ ?

14. What per cent is lost if cloth costing 48 ¢ a yard is sold for 36 ¢ a yard ? 40 ¢ ? 42 ¢ ? 30 ¢ ?

Miscellaneous Exercise 20

1. The number of cars of grain received in Chicago, Sept. 8, 1899, was as follows: winter wheat, 37; spring wheat, 54; corn, 690; oats, 323; rye, 9; barley, 40. Find the total number.

2. During the year 1899 the different railroads entering Chicago elevated, respectively, 2 mi., 2 mi., 21 mi., 12 mi., 12 mi., 1 mi., 11 mi., 8 mi., 8 mi., and 4 mi., of track. Find the total number of miles of track elevated in 1899.

3. Find the amount of the following bill:

> 12 yd. cotton at $6\frac{1}{2}$ ¢ a yard;
> 8 yd. lining at 11 ¢ a yard;
> 6 yd. dimity at 22 ¢ a yard;
> 1 doz. towels at 25 ¢ each.

4. A speculator bought 28 A. of land at $ 54 an acre, and 35 A. at $ 65 an acre. If he sold it all at $ 63 an acre, find his gain.

5. If 25 cars of spring wheat contain 51,000 bu., find the average number of bushels to the car.

6. If 116 cars of corn contain 278,400 bu., find the average number of bushels to the car.

7. In the city of Chicago in 1899 there were 243.57 mi. of sidewalk built and 67.88 mi. repaired. Find the total number of miles built and repaired.

8. Make a drawing to show that $\frac{2}{3}$ ft. $+ \frac{5}{6}$ ft. $= 1\frac{1}{2}$ ft.

9. Two boys picked the fruit off a cherry tree. One picked $\frac{1}{2}$ bu. and the other $\frac{5}{6}$ bu. How much fruit did the tree bear?

10. What part is $ 2 of $ 5? What per cent? What per cent is $ 1 of $ 3? $ 2 of $ 3? $ 3 of $ 4?

11. A drover bought sheep at $ 5 each, and sold them at an advance of $ 1. Find his gain per cent.

12. A fruit dealer bought oranges at 24 ¢ a dozen, and sold them for 3 ¢ each. Find his gain per cent.

CHAPTER III

NUMERATION AND NOTATION

4. **Numeration** is counting, or the expression of number in words.

The ordinary system of numeration is the *Decimal System*, so called because it is based on the number ten.

5. The names of the first group of numbers in regular succession are: one, two, three, four, five, six, seven, eight, nine.

Other number names are: ten, hundred, thousand, million, billion, trillion, etc.

6. The number one applied to any unit denotes a quantity which consists of a single unit of the kind named.

The number next following nine is ten, which applied to any unit denotes a quantity consisting of nine such units and one unit more.

The number hundred applied to any unit denotes a quantity which consists of ten ten-units.

The number thousand applied to any unit denotes a quantity which consists of ten hundred-units.

The number million applied to any unit denotes a quantity which consists of a thousand thousand-units.

7. The number tenth applied to any unit denotes that quantity of which ten make up the unit.

The number hundredth applied to any unit denotes that quantity of which one hundred make up the unit.

The number thousandth applied to any unit denotes that quantity of which one thousand make up the unit.

8. Instead of speaking of " the number denoted by " 5, 75, or 375, we may for brevity speak of the number 5, 75, or 375.

9. Notation is the art of expressing numbers by means of certain number symbols called numerals or figures.

10. The *Arabic Numerals*, styled also *Figures*, are

$$0, \quad 1, \quad 2, \quad 3, \quad 4, \quad 5, \quad 6, \quad 7, \quad 8, \quad 9,$$

denoting naught, one, two, three, four, five, six, seven, eight, nine respectively. The first of these is called *naught*, cipher, or zero; the remaining nine are called *digits*. By means of these numerals and a dot called the *decimal point*, we can write down any number expressed decimally. The method of doing so may be described as follows:

A figure immediately to the left of the decimal point denotes so many single units.

A figure immediately to the left of the single-units figure denotes so many tens of the units, while a figure immediately to the right of the single-units figure denotes so many tenths of the unit.

Figures to the left of the tens-figure, taking them in order from right to left, denote so many hundreds of the unit, so many thousands of the unit, so many ten-thousands of the unit, etc.

Figures to the right of the tenths-figure, taking them in order from left to right, denote so many hundredths of the unit, so many thousandths of the unit, so many ten-thousandths of the unit, etc.

The function of the decimal point is to mark the place of the standard unit when the quantity is measured.

11. Consider the following quantities :

$$
\begin{array}{ll}
9 & \text{yd.} \\
59 & \text{``} \\
259 & \text{``} \\
3259 & \text{``} \\
43259 & \text{``} \\
843259 & \text{``} \\
59.7 & \text{``} \\
59.76 & \text{``} \\
59.761 & \text{``} \\
59.7613 & \text{``}
\end{array}
$$

In the above quantities

9 denotes 9 of the unit one yard.

5	"	5 " " "	ten yards.
2	"	2 " " "	one hundred yards.
3	"	3 " " "	one thousand yards.
4	"	4 " " "	ten thousand yards.
8	"	8 " " "	one hundred thousand yards.
7	"	7 " " "	one tenth of a yard.
6	"	6 " " "	one one-hundredth of a yard.
1	"	1 " " "	one one-thousandth of a yard.
3	"	3 " " "	one ten-thousandth of a yard.

12. The number 8 always denotes 8 of the unit; 86 denotes 8 of the ten-unit or 80 of the unit and 6 of the unit, and is read *eighty-six* of the unit.

865 denotes 8 of the *hundred*-unit and 6 of the *ten*-unit and 5 of the unit; *i.e.* 800, 60, and 5 of the unit, and is read *eight hundred sixty-five* of the unit.

Thus, the numbers 8, 86, 865 always denote eight, eighty-six, eight hundred sixty-five respectively, the position of the figures in each case giving the *unit*. For example :

In the number 865,865,865 each 865 is read eight hundred sixty-five, the difference being in the unit only; 865 of the million-unit, 865 of the thousand-unit, and 865 of the one-unit.

This number is read eight hundred sixty-five million eight hundred sixty-five thousand eight hundred sixty-five.

Exercise 21

1. Express in words the numbers given in Exercises 28, 30, 38, 39, 41.

Read the following statements:

2. During the year 1899 about 6,349,662,320 pieces of mail matter were posted in the United States. Of this number 6,312,-732 were sent to the dead letter office.

3. To show the increase in the paper-bag business it is said that the sales for the United States were in 1871 about 391,000,000 a year, and in 1897 the sales were 3,943,000,000. The sale for the year 1899 was about 5,000,000,000.

4. During the ten months ending Oct. 31, 1899, the exports from the United States were valued at $ 1,029,242,286, and beat the record by $ 41,344,597.

5. For the year ending Oct. 31, 1899, the exports were $ 1,296,-890,945, an increase of more than $ 70,000,000 over the previous year.

Exercise 22

Express in words:

1. .7 ton; .64 ton; .643 ton.   **3.** 9.403 hr.; 29.04 min.; .09 sec.

2. 4.92 lb.; 8.09 lb.; 2.734 lb.   **4.** 7.456 A.; 6.7985 A.

5. 8452.69 sq. mi.; 21.4394 A.

6. Express in words the numbers given in Exercises 111 and 112.

Exercise 23

Write in figures:

1. Three hundred forty-nine; eight thousand four hundred sixty-nine; nine thousand five hundred seventy.

2. Twenty-nine thousand one hundred thirty-four; fifty thousand eight hundred seventy-six; seventy-eight thousand three hundred.

3. Nine hundred fifty-two thousand seven hundred forty; six hundred forty-nine thousand nine hundred five; nine hundred thousand eight hundred sixty-four.

4. One hundred sixty-eight thousand six hundred eighteen; three hundred twelve thousand seven hundred forty-two; four hundred sixty-one thousand eight hundred twenty-one.

5. Seven hundred fourteen thousand thirty; three hundred thousand two hundred four; one hundred thousand fifty.

6. Eight hundred ninety thousand one hundred twenty-three; two hundred four thousand six hundred seventy-eight; nine hundred one thousand two hundred thirty.

7. Sixteen thousand sixty; seven hundred two thousand nine hundred five; one hundred seventy-five thousand two hundred fifteen.

8. Ninety thousand twenty-three; five hundred twenty thousand sixty-four; three hundred twenty-four thousand four hundred forty-four.

Exercise 24

Express in figures:

1. Five tenths; eight tenths; one tenth; six and four tenths; twenty-five and nine tenths; twenty and two tenths.

2. Forty-five hundredths; sixty-seven hundredths; twenty-seven hundredths; seven hundredths; nine hundredths; two hundredths; six and thirty-four hundredths.

3. Six and three tenths; sixty-three hundredths; ninety-five hundredths; nine and five tenths; two and four tenths; two and four hundredths; eight tenths; eight hundredths.

4. Six hundred twenty-five thousandths; five hundred eight thousandths; two hundred forty-seven and eight hundred six thousandths; nine and fifteen thousandths; six thousandths.

5. Five tenths; two and sixty-seven hundredths; nine hundredths; fourteen thousandths; sixteen thousandths.

6. Three and twelve thousandths; three thousand four hundred fifty-three; six thousand eight hundred seventy-five ten-thousandths; two and one hundred ninety-nine ten-thousandths.

7. Nine hundred six thousandths; nine hundred six ten-thousandths; twenty and forty-five thousandths; seventeen and seven ten-thousandths; three hundred three and nine ten-thousandths.

THE ROMAN NOTATION

13. The Arabic Notation is the one in general use. It was introduced into Europe by the Arabs. The system of notation which was used among the Romans is now used only to denote the chapters and sections of books, etc.

14. The following letters are used to denote numbers, and their values are written below:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

15. The numbers 6, 8, 15, 20 are represented thus:

<div align="center">

VI VIII XV XX

</div>

Hence if a character in the Roman Notation be followed by another of equal or less value, the number denoted by the expression is equal to the *sum* of the simple values.

D

16. The numbers 4, 9, 40, and 90 are represented by IV, IX, XL, XC.

Hence if a character in the Roman Notation is followed by one of greater value than itself, the number denoted by the expression is the *difference* of their simple values.

17. Express 1896 in Roman numerals.

$$1896 = 1000, 800, 90, \text{ and } 6$$
$$1000 = M$$
$$800 = DCCC$$
$$90 = XC$$
$$6 = VI$$
$$\therefore 1896 = MDCCCXCVI$$

Hence to write any number in Roman numerals, separate the number into its different parts, and write down the parts in order, beginning at the left.

Exercise 25

Write in Roman numerals:

1. 14, 25, 54, 89, 99.

2. 178, 304, 871, 982, 999.

3. 1204, 1590, 1756, 1876, 1895.

Write in figures:

4. XLVI, LXXIX, XCIV, LXXXIII.

5. XCIX, CXXXIX, CLX.

6. DLIV, MDCII, MDCCCXIX, MXC.

CHAPTER IV

ADDITION

18. Let the length of a room be measured by the parts, 2 ft., 3 ft., 4 ft., and 5 ft. Here the common unit of measure, 1 ft., has been repeated 2, 3, 4, and 5 times to measure the parts.

The number of units in all is the sum found primarily by counting 2, 3, 4, and 5, or 14 units of 1 ft. Hence the length of the room which is now definitely measured is 14 ft.

Addition may, therefore, be considered as the operation of finding the quantity, which, as a whole, is made up of two or more given quantities as its parts. Each of these quantities must have the same measuring unit. Not only is it impossible to add 5 ft. to 4 min., it is impossible to add 5 ft. to 4 in.; *i.e.* to express without change of unit the whole quantity by a number of either feet or inches.

The parts added are called **Addends**.

The **Sum** is the quantity obtained by adding the quantities expressed in terms of a common unit.

19. The Sign of Addition is +, and is read *plus;* thus 6 + 8 is read 6 plus 8.

The Sign of Equality is =, and is read *equals* or *equal;* thus 4 + 5 = 9 is read 4 plus 5 equals 9.

20. I bought 3 farms of 50 A. each, 6 farms of 50 A., and 4 farms of 50 A. How much did I buy altogether?

Here we are required to find the whole quantity measured by the sum of 3, 6, and 4 farms of 50 A.

∴ the whole quantity = 13 farms of 50 A.

Exercise 26

1. What quantity is measured by the parts 2 in., 5 in., and 4 in.?

2. 3 ten-dollar bills + 4 ten-dollar bills + 6 ten-dollar bills = ? ten-dollar bills. How many dollars?

3. How many five-cent pieces are equal to 4 five-cent pieces, 9 five-cent pieces, and 5 five-cent pieces? How many cents?

4. What is the quantity denoted by the sum 6, 7, and 5 times the measuring unit?

5. I paid out in one day 6 ten-dollar bills, 8 ten-dollar bills, and 5 ten-dollar bills. How much did I spend all together?

6. If I sell two lots, one for 8 units of value, and the other for 6 units, what do I get for both, the unit of value being $100?

7. A fruit dealer who arranges his apples in piles of 6 for 5¢, sells 1 pile to each of a company of 4 persons, and 3 piles to another customer. How much does he sell all together? How many apples?

8. A speculator bought 5 farms of 100 A. for $5000, 6 farms of 100 A. for $7000, and 3 farms of 100 A. for $4000. How much land did he buy? What did it cost?

9. A fruit dealer sells his apples at the rate of 3 for 5 cents. He sells 5 cents' worth to each of 8 customers. How many apples did he sell?

10. 1 gal. 3 qt. = ? qt. 3 gal. 1 qt. = ? qt. What is the cost of 2 gal. 3 qt. of milk at 6¢ a quart?

11. 1 pk. 6 qt. = ? qt. 4 pk. 2 qt. = ? qt. What is the cost of 1 pk. 4 qt. of berries at 7¢ a quart?

12. 1 yd. 2 ft. = ? ft. 3 yd. 2 ft. = ? ft. Find the weight of 2 yd. 2 ft. of carpet if 1 ft. weighs 7 oz.

13. How many working days in 1 wk. 5 da.? What will a man earn in 1 wk. 4 da. at $1.50 a day?

14. 1 bu. 8 qt. = ? qt.   1 bu. 12 qt. = ? qt.   I bought 1 bu. 6 qt. of cherries at 5¢ a quart   Find the cost.

15. 1 doz. + 3 = ?   2 doz. + 6 = ?   What did I pay for 1 doz. and 6 eggs at 2¢ apiece?

21. Drill on the following addition combinations to secure accuracy and rapidity:

1	1	1	2		1	2		1	2	3		1	2	3		1	2	3	4
1	2	3	2		4	3		5	4	3		6	5	4		7	6	5	4

1	2	3	4		1	2	3	4	5		2	3	4	5		3	4	5	6
8	7	6	5		9	8	7	6	5		9	8	7	6		9	8	7	6

4	5	6		5	6	7		6	7		7	8		8		9
9	8	7		9	8	7		9	8		9	8		9		9

Enlarge each combination thus:

8	8	8		8	18	28		18	38	68
9	19	29, etc.		9	9	9, etc.		19	29	39, etc.

	80	80		80	180	280
	90	190, etc.		90	90	90, etc.

Give such problems as the following, requiring instantaneous answers:

How many square feet in 1 sq. yd. 8 sq. ft.?   1 sq. yd. 6 sq. ft.? etc.

How many quarts in 1 pk. 4 qt.? etc.

When 7 is the number added, base the problem on days, thus:

How many days in 1 wk. 4 da.?

When the number is 6, on minutes, thus:

How many seconds in 1 min. 30 sec.? and so on.

Exercise 27

Count by :

1. 2's from 0 to 100; 1 to 101.

2. 3's from 0 to 102; 1 to 103: 2 to 104.

3. 4's from 0 to 100; 1 to 101; 2 to 102; 3 to 104.

4. 5's from 0 to 100; 1 to 101; 2 to 102; 3 to 103; 4 to 104.

5. 6's from 0 to 102; 1 to 103; 2 to 104; 3 to 105; 4 to 106; 5 to 107.

6. 7's from 0 to 105; 1 to 106; 2 to 107; 3 to 108; 4 to 109.

7. 8's from 0 to 104; 1 to 105; 2 to 106; 3 to 107; 4 to 108.

8. 9's from 0 to 99; 1 to 100; 2 to 101; 3 to 102; 4 to 103.

22. A person paid $38 for a cow, $146 for a horse, and $255 for a carriage.   Find the cost of all.

$ 38
146
255
————
$439

In this problem we are required to find the cost which is the whole measured by the parts $38, $146, and $255.

This may, for convenience, be broken up into the sum of 5, 6, and 8 units of $1, 5, 4, and 3 units of $10, and 2 and 1 units of $100.   The sum of 5, 6, and 8 units of $1 = 19 units of $1 = 1 unit of $10 and 9 of the $1 unit.

Add the 1 unit of $10 in with the tens' column.

The sum of 1, 5, 4, and 3 units of $10 = 13 units of $10, or 1 unit of $100, and 3 units of $10.

Add the 1 unit of $100 in with the hundreds' column.

The sum of 1, 2, and 1 unit of $100 = 4 units of $100.

Hence the cost = 4 units of $100, 3 units of $10, and 9 units of $1 = $439.

23. Find the sum of the following numbers, using the unit employed in stating your age :

234
983
567
842
————
2626

∴ the sum = 2626 yr., since 1 yr. is the unit of age.

Add thus, beginning at the units' column : 9, 12, 16 ; write down 6 and carry 1 to the tens' column ; 5, 11, 19, 22 ; write down 2 under the tens' column and carry 2 to the hundreds' column ; 10, 15, 24, 26 ; write down 26, putting the 6 under the hundreds' column.

To prove the answer correct, add downward.   If the same answer is obtained, the result is likely to be correct.

Exercise 28

Find the sum of the following quantities and explain your work clearly. Prove each answer correct by beginning at the top and adding down.

1. $ 441	2. 341 ct.	3. 532	4. 687
234	225 "	233	956
518	343 "	154	888

5. 543 min.	6. 635 hr.	7. 247	8. 976
666 "	87 "	859	485
752 "	256 "	23	329·
231 "	742 "	271	786

9. 2576	10. 2598	11. 4397	12. 8649
3491	6776	8999	6262
7743	4259	5637	8497
8988	7362	8249	9773

13. 64251	14. 89435	15. 850439	16. 262933
3789	62789	973642	998757
45278	576	845867	639364
99	43	939894	286753
6472	416	768795	486325
2573	7235	649879	744638

17. 634879	18. 453798	19. 296843
273548	667788	796276
506644	549763	947362
979721	438925	288433
342994	648888	334455
434696	999999	667854

20. How many days in 1 leap year?

Find the number of days in 1 leap year and 257 da.

21. How many square inches in 1 sq. ft.?

Find the number of square inches in 1 sq. ft. 96 sq. in.

Find the number of square feet in 1 sq. yd. 6 sq. ft.

22. How many yards in 1 mi. ?

How many steps will a man take in walking 1 mi. 468 yd., if he goes 1 yd. in each step ?

23. How many feet in 1 mi ?

How many feet apart are two places the distance between which is 1 mi. 3480 ft. ?

Add :

24.		25.		26.		27.	
	84		99		893		733
	93		77		254		842
	89		86		767		951
	75		25		899		258
	91		43		654		365
	27		88		473		874
	30		76		129		935
	45		52		895		273

28.		29.		30.		31.	
	542		9834		7594		5846
	879		729		821		7593
	666		8345		2357		3819
	257		728		8463		5578
	389		3403		1525		2904
	983		17		7469		8392
	365		295		2856		9576
	874		8943		8888		2882

24. 1 mi. contains 5280 ft. ; find, by adding, the number of feet in 4 mi.

Here we are required to find the sum of four equal addends, each of which is 5280 ft. ; thus :

$$\begin{array}{r} 5280 \text{ ft.} \\ 5280 \text{ ft.} \\ 5280 \text{ ft.} \\ \underline{5280 \text{ ft.}} \\ 21120 \text{ ft.} \end{array}$$

Multiply 5280 ft. by 4 and prove this answer correct.

Exercise 29

1. 1 bu. contains 32 qt., find, by adding, the number of quarts in 4 bu.   Multiply 32 qt. by 4 and prove your answer correct.

2. 1 mi. contains 1760 yd., find, by adding, the number of yards in 3 mi.   Multiply and prove your answer correct.

3. One square foot contains 144 sq. in.   Find, by adding, the number of square inches in a rug containing 6 sq. ft.   Multiply and prove your answer correct.

4. How many hours in one day ?   Find, by adding, the number of hours in one week.   Multiply and prove your answer correct.

5. One square mile contains 640 A.   Find, by adding, the number of acres in 6 sq. mi.   Multiply and prove your answer correct.

6. One gallon contains 231 cu. in.   Show, by adding, that a 5-gallon can contains 1155 cu. in.   Show this by multiplication.

7. Find the number of days in 6 yr. of 365 da. each and 2 leap years of 366 da. each.

8. If a boy attends school 189 da. each year, find, by adding, the number of days he will be in school in 8 yr.   Multiply and prove your answer correct.

Exercise 30

In each of the following questions state in each case which is the whole quantity to be measured and what are the parts measuring it.   .

1. A spent the following sums of money : $425, $342, $673, and $897.   How much did he spend all together ?

2. An encyclopædia consists of three volumes.   In the first there are 693 pages, in the second 745, and in the third 892.   Find the number of pages in the encyclopædia.

3. Using the table in § 150, find the number of days in the first six months of the year.   In the last six months.   In a year.

4. Find the number of days in the three spring months. In the three summer months. In the three fall months. In the three winter months.

5. Find the total area of these lakes :

Lake Erie,   area   7,750 sq. mi.;
Lake Ontario,   area   6,950 sq. mi.; .
Lake Michigan, area 22,000 sq. mi.;
Lake Superior, area 31,500 sq. mi.

6. A merchant bought 150 yd. of cloth for $ 232, 254 yd. for $ 175, 1875 yd. for $ 2395, and 640 yd. for $ 1966. Find the number of yards bought and the total cost.

7. How many years are there between the establishment of the Republic of Rome in 509 B.C. and the Declaration of Independence in 1776 A.D. ?

8. Find the sum of three hundred seventy-six thousand fifty-four; one hundred ninety-seven thousand two hundred fifty-one; four hundred fifty-seven thousand six hundred forty-nine.

9. The population of Maine is 694,466; New Hampshire, 411,588; Vermont, 343,641; Massachusetts, 2,805,346; Rhode Island, 428,556; and Connecticut, 908,355. Find the population of the New England states.

10. According to the census of 1900 the population of the six largest cities of the United States is: New York, 3,437,202; Chicago, 1,698,575; Philadelphia, 1,293,697; St. Louis, 575,238; Boston, 560,892; and Baltimore, 508,957. Find the total population.

11. Find the number of times a clock strikes from a quarter · of nine A.M. until a quarter of nine P.M.

12. A, B, and C engaged in trade; A put in $ 3475, B $ 4593, and C as much as the other two together. How much money was put into the business ?

13. A man, dying, willed to his widow, $ 6875; to his son, $ 4294, and to his daughter, $ 3875. What was his estate worth ?

14. According to the census of 1900 the population of the following states is: Wisconsin, 2,069,042; Illinois, 4,821,550; Indiana, 2,516,462; Ohio, 4,157,545; Michigan, 2,420,982. Find their total population.

15. The area of the basin of the Colorado River is 250,000 sq. mi.; Columbia, 250,000; Mackenzie River, 440,000; Missouri-Mississippi, 1,250,000; Nelson, 355,000; Rio Grande, 180,000; St. Lawrence, 350,000. Find the total area of these river basins.

16. What is the area of the New England states; that of Maine being in square miles 33,040, of New Hampshire 9305, of Vermont 9565, of Massachusetts 8315, of Rhode Island 1256, of Connecticut 4990 ?

17. A father left his eldest son $ 24,000 more than he left his second son, and the second son $ 7560 more than the third; to the third he left $ 60,480. What was the second son's portion ? What was the portion of the oldest son ?

18. Aug. 17, 1899, there were inspected in the city of Chicago 66 carloads of wheat, 281 of corn, 413 of oats, 3 of rye, and 30 of barley. Find the total number of carloads of grain inspected.

19. For the week ending Aug. 17, 1899, there was received in Chicago the following number of live stock. Find the total number of each kind:

	CATTLE	CALVES	HOGS	SHEEP
Thursday, Aug. 10	10,122	374	23,007	14,565
Friday, Aug. 11	3,342	147	16,113	9,017
Saturday, Aug. 12	100	10	8,614	2,171
Monday, Aug. 14	19,095	293	20,466	28,960
Tuesday, Aug. 15	5,960	1,488	12,496	18,860
Wednesday, Aug. 16	23,869	565	23,697	28,134
Thursday, Aug. 17	10,500	300	25,000	15,000

20. The imports to the United States from Cuba, Porto Rico, and the Philippines, from Jan. 1 to July 31, 1899, were:

1899	CUBA	PORTO RICO	PHILIPPINES
January	$994,220	$63,481	$348,019
February	2,307,940	124,618	277,003
March	3,398,723	349,785	147,452
April	4,419,712	782,172	937,164
May	4,762,970	647,179	622,101
June	3,614,904	814,803	61,882
July	2,632,845	448,267	880,515

Find the total in each case.

21. The exports from the United States to Cuba, Porto Rico, and the Philippines, from Jan. 1 to July 31, 1899, were:

1899	CUBA	PORTO RICO	PHILIPPINES
January	$1,980,982	$224,150	$15,382
February	1,671,846	267,619	19,529
March	2,503,110	375,529	43,180
April	1,723,062	316,669	112,267
May	2,124,679	305,564	63,905
June	2,123,935	361,423	67,775
July	1,989,379	448,267	64,408

Find the total in each case.

22. Dec. 1, 1899, there were 905 officers and 30,578 men of the regular army of the United States in the Philippines, and 594 officers and 15,388 men of the volunteer force. Find the total number of officers and of men in the Philippines Dec. 1, 1899.

23. Dec. 1, 1899, the regular army of the United States was distributed as follows:

	OFFICERS	ENLISTED MEN
In Cuba	334	10,796
In Porto Rico	87	2,855
On the continent of North America . .	910	17,317
In Hawaii	12	453
In the Philippine Islands	905	30,578

Find the number of officers and of enlisted men in the army.

24. In five months of the year 1899, a rural mail-carrier handled the following mail matter:

	PIECES DELIVERED	PIECES COLLECTED
June	5089	2086
July	5456	2233
August	5942	746
September	6094	890
October	6799·	930

Find the total number of pieces delivered and collected each month.

25. Add vertically and horizontally the following statement of eight weeks' cash receipts:

	MON.	TUES.	WED.	THUR.	FRI.	SAT.	TOTAL
1st	$3862.93	$1391.76	$6760.68	$1098.91	$1696.65	$43.68	
2d	396.74	6168.37	864.39	964.26	167.69	1864.86	
3d	1768.63	467.89	2035.68	3165.03	691.83	785.97	
4th	3976.98	76.05	364.76	93.68	1948.39	1759.46	
5th	263.76	1035.84	36.10	386.41	3.45	1396.71	
6th	1559.83	1932.57	1268.15	8.37	279.72	67.85	
7th	62.24	318.62	134.36	1763.29	1468.29	543.66	
8th	194.87	3.85	7643.82	685.38	765.42	39.67	
Total							

26. Add vertically and horizontally the following statement:

							TOTAL
	$1169.84	$3650.12	$189.10	$97.22	$26.55	$851.02	
	909.58	866.78	914.19	239.49	297.02	312.60	
	575.72	742.49	1654.70	196.17	859.69	1477.42	
	2678.28	1180.66	119.25	8418.60	2223.42	568.35	
	312.83	1638.24	2016.72	1542.24	5300.20	116.02	
	1052.47	342.65	108.00	349.95	136.97	1214.03	
	339.11	687.23	215.17	1020.00	1124.50	1732.25	
	1732.50	514.02	557.60	600.00	475.00	138.50	
	1237.50	3839.25	777.60	136.70	4656.65	1097.47	
	113.56	1291.98	112.50	1850.14	738.75	1204.74	
	3661.00	973.03	311.20	636.99	243.44	142.91	
	1139.67	670.22	1201.64	7357.51	252.47	694.62	
Total							

25. Find the sum of: 2.46, 23.973, 15.025, 643.319, and .468.

2.46
23.973
15.025
643.319
.468
―――――
685.245

Since we can add numbers of the same unit, we write the addends so that units will be under units, tenths under tenths, and so on. This is easily done by placing the decimal points directly below each other. Then, beginning at the right, we add the figures as if they were integers, and place the decimal point in the sum between the units' and tenths' column.

Exercise 31

Add:

1. 3.456	2. 27.43	3. 76.425
4.593	18.314	39.639
7.245	5.687	28.764
9.864	34.986	21.385
3.124	22.425	13.026

Write in columns and add:

4. $4.396 + 7.295 + 6.478 + 5.765.$

5. $.432 + .987 + .593 + .666.$

6. $84.63 + 46.892 + 24.7 + 95.657.$

7. $\$24.375 + \$95.875 + \$16.125 + \$19.50.$

8. Find the capacity of four bins, the first of which contains 66.384 bu., the second 89.645 bu., the third 27.437 bu., and the fourth 75.938 bu.

9. What is the area of a farm which is divided into three fields containing, respectively, 25.936 A., 14.56 A., and 24.504 A.?

10. Four towns, A, B, C, D, lie on a road running directly east and west. The distance from A to B is 5.693 mi., from B to C 8.421 mi., from C to D 12.768 mi. Find the distance from A to D.

11. In the city of Chicago in 1899 there were 243.57 mi. of sidewalk built and 67.88 mi. repaired. Find the total number built and repaired.

Miscellaneous Exercise 32

1. A boy worked 20 problems in arithmetic and got wrong answers to $\frac{1}{10}$ of them. How many did he miss? How many did he work correctly?

2. Caryl had a spelling lesson of 48 words, and spelled correctly $\frac{7}{8}$ of them. How many did she spell correctly?

3. During the month of June a commercial traveller was away from home $\frac{4}{5}$ of the time. How many days was he at home?

4. What is the ratio of 8 lb. to 16 lb.? Of the cost of 12 lb. of sugar to that of 15 lb.? Of 15 lb. to 12 lb.? Of the weight of 36 yd. of carpet to 27 yd. of the same kind?

5. A pole 6 ft. high casts a shadow 8 ft. long. By what must 8 ft. be multiplied to find the length of the shadow cast by a pole 9 ft. long? How long is it?

6. The carpet on a certain room weighs 35 oz. to the yard, and its entire weight is 105 lb. What would have been its weight if it had been made out of carpet weighing 45 oz. to the yard?

7. What is the ratio of 75 lb. to 90 lb.? A carpet weighs 30 oz. to the yard, and its entire weight is 90 lb. Another carpet of the same size weighs 75 lb. Find how many ounces 1 yd. of the second carpet weighs.

8. Find the cost of 2 yd. of cloth at 32 ¢ a yard. Find the cost of 3 doz. oranges at $.25 a dozen.

9. Find the selling price of 4 bu. of wheat at 64 ¢ a bushel. Find the selling price of 5 bu. of corn at $.38 a bushel.

10. Find the cost of 12 yd. of oil cloth at $.21 a yard. Of 18 yd. at $.25 a yard.

11. What is 20% of $30? 12½% of $24? 37½% of $72? 33⅓% of $69? 6¼% of $32? 62½% of $48? 11⅑% of $54? 16⅔% of $54? 66⅔% of $21? 8¼% of $36? 75% of $84? 6⅔% of $45? What is meant by saying that 6⅔% = $\frac{1}{15}$?

12. A grocer bought tea at 75 ¢ a pound, and sold it at a gain of 33⅓%. Find his gain. Find the selling price per pound.

13. How do you find the gain when you are given the cost and the gain per cent? How do you find the selling price?

14. In the following examples, find the gain and also the selling price:

Cost	Gain Per Cent	Gain	Selling Price
40 ¢	12½ %		
39 ¢	33⅓ %		
$ .65	20 %		
$2.60	25 %		
$2.10	6⅔ %		
$ .28	14⅞ %		
$ .48	16⅔ %		
$ .16	37½ %		
$ .55	9 1/11 %		
$2.19	66⅔ %		
$1.00	75 %		
$1.32	8⅓ %		
$ .63	22⅔ %		
$ .08	100 %		

15. A grocer bought berries at 8 ¢ a box and sold them at a gain of 25%. Find the selling price. What did he receive for a crate containing 24 boxes ?

16. A grocer bought berries at 6 ¢ a box and sold them at a gain of 33⅓%. What did he receive for a crate of 16 boxes ?

17. In the year 1899 the United States sold to the British Colonies the following articles :

Cotton	$2,994,674
Corn	7,501,508
Wheat	6,159,136
Flour	9,961,230
Provisions	16,886,946
Refined petroleum	4,211,709
Live cattle	701,947
Tobacco	1,251,407

Find the total value.

18. Add :

32.49	144.938	92.778
63.824	992.807	89.006
765.653	25.069	729.054
4.159	76.238	308.298
36.768	4.718	123.864

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CHAPTER V

SUBTRACTION

26. A man who earned $14 a week, spends $5 a week for his board. How much has he left?

We are here given the whole quantity, or $14, and one part, or $5, and we are required to find the other part.

The question may be viewed in two ways: How much must be added to $5 to make $14? Or how much must be taken from $14 to leave $5? The answer in both cases is known from addition. $5 and $9 are two quantities making $14. Therefore, if one of them, $5, is given, the other must be $9. Or, in other words, $9 is the *difference* between $14 and $5. It is this view of difference that gives the name *Subtraction*.

27. Subtraction may therefore be defined as the operation of finding the part of a given quantity that remains when a given part has been taken from the quantity.

The given quantity is called the **Minuend**, and the given part the **Subtrahend**, while the part that remains is called the **Difference** or **Remainder**.

28. The Sign of Subtraction, − , is called *minus*. Thus 8 − 6 is read 8 minus 6, and signifies that 6 is to be subtracted from 8.

Exercise 33

Read the following questions, filling in the blanks:

1. 6 and 7 are —,   8 and 9 are —,   4 and 8 are —.
2. 4 and 6 are —,   4 and — are 10,   9 and — are 15.

3. 2 and — are 11,   3 and — are 8,   6 and — are 14.

4. 22 and — are 25,   4 and — are 36,   9 and — are 27.

5. 8 and — are 29,   5 and — are 16,   6 and — are 48.

Subtract (Note: Let the process be *not* 6 from 9 leaves 3, but 6 *and* 3 are 9):

6. $\frac{9}{6}$; $\frac{19}{6}$; $\frac{29}{6}$; $\frac{39}{6}$; $\frac{49}{6}$; $\frac{69}{6}$; $\frac{99}{6}$.

7. $\frac{90}{60}$; $\frac{190}{60}$; $\frac{8}{5}$; $\frac{28}{5}$; $\frac{80}{50}$; $\frac{380}{50}$.

8. $\frac{12}{7}$; $\frac{120}{70}$; $\frac{32}{7}$; $\frac{320}{70}$; $\frac{14}{8}$; $\frac{54}{8}$.

What numbers added respectively to 9, 7, 6, 8, 5, and 4, make
9. 12?   **10.** 15?   **11.** 17?   **12.** 14?   **13.** 18?   **14.** 16?

29. Drill, as in § 21 in addition, on the fundamental subtractions, connecting with corresponding additions, until accuracy and rapidity are secured; thus:

$\frac{9}{8}$; $\frac{19}{8}$; $\frac{29}{8}$; $\frac{37}{8}$; $\frac{49}{8}$; $\frac{59}{8}$; and so on.

$\frac{90}{80}$; $\frac{190}{80}$; $\frac{210}{80}$; $\frac{390}{80}$; $\frac{490}{80}$; and so on.

$\frac{17}{8}$; $\frac{27}{8}$; $\frac{36}{8}$; $\frac{47}{8}$; $\frac{57}{8}$; and so on.

30. A man who owned 18 farms of 50 A., sold 7 of them. How much had he left?

Because $7 + 11 = 18$, it is evident that he had left 11 farms of 50 A. each.

Exercise 34

1. 9 ft. + ? = 16 ft.   9 yd. + ? = 16 yd.   12 yd. − 4 yd. = ?

2. How many dimes must be added to 6 dimes to get 14 dimes? What is the difference between 14 dimes and 6 dimes?

How much less is 6 five-dollar bills than 14 five-dollar bills?
How many dollars?

3. A fruit dealer arranges his oranges into 12 piles of 4
oranges each. He sells 8 piles. How many has he left? How
many oranges?

4. A fruit dealer sold 10 piles of 3 apples each. How many
had he left if he had at first 15 piles of 3 apples? How many
apples?

5. I owe a debt of 12 ten-dollar bills and have 5 ten-dollar
bills in my pocket. If I pay this toward the debt, how much
do I still owe? How many dollars?

6. What must be added to 15 units to get 20 units? Taken
from 20 units to get 15 units? To get 5 units? If I sell my
horse for 20 units of $5 each, find the selling price.

7. What is the difference between a quantity denoted by 14
times the measuring unit and one denoted by 8 times the meas-
uring unit?

8. A person who has $50 pays a debt of $30. How much
money has he left? If this is in ten-dollar bills, how many? If
in five-dollar bills, how many?

9. 3 gal. = ? qt.   5 gal. = ? qt.   8 gal. = ? qt.   From a can
containing 2 gal. of milk a milkman poured 3 qt. How many
quarts were left in the can?

10. 8 qt. = ? pt.   12 qt. = ? pt.   27 qt. = ? pt.   A lady bought
16 qt. of fruit in pint jars. How many jars were left after 13
had been used?

11. How many quarts in 1 bu.? In 2 bu.? A bushel basket
lacked 5 qt. of being full. How many quarts did it contain?

12. A piece 3 ft. 4 in. long is cut from a rope 8 ft. 6 in. long.
Find the length of the other piece.

13. In paying for a suit of clothes that cost $16 I gave the
clerk 2 ten-dollar bills. What change should I receive back?

14. 1 sq. ft. = ? sq. in. A piece of cloth 8 in. long and 6 in. wide is cut from a square foot of cloth. How many square inches in the remainder ?

15. 2 ft. = ? in. 4 ft. = ? in. 5 ft. = ? in. A man takes a step 3 ft. long and his son a step 8 in. shorter. How many inches does his son step ?

16. Out of 6 doz. eggs half a dozen were found to be broken. How many were whole ?

31. The following method of subtraction, which is nearly always adopted in making change, is almost universally employed by professional computers and is considered by many teachers the best way to perform subtraction.

It is superior in accuracy and rapidity to the method of the next paragraph.

It is based on the principle that the sum of the subtrahend and remainder is equal to the minuend.

From 875 take 451.

$$\begin{array}{r} 875 \\ 451 \\ \hline 424 \\ \hline \end{array}$$

Thus: 1 and **4** are 5 ; 5 and **2** are 7 ; 4 and **4** are 8. In this operation let the pupil fancy that he is doing addition with the sum at the top, and as he works set down the figures, 4, 2, and 4.

32. A merchant bought 965 yd. of silk and sold 723 yd. How much had he left ?

Here we are required to find the unknown part. This is the difference between the measured whole, or 965 yd., and the given part, 723 yd.

965 yd. = the measured whole.
723 yd. = the measured part.
242 yd. = the difference, which is now definitely known.

EXPLANATION. — As in addition, we write *units* under *units*, *tens* under *tens*, and *hundreds* under *hundreds*. Beginning with the units, we say 3 units from 5 units leaves 2 units, which we write below the line in the units' column. Then 2 tens from 6 tens leaves 4 tens. Place *this* in the tens' column.

Lastly, 7 hundreds from 9 hundreds leaves 2 hundreds, which we write in the hundreds' column.

This difference, 242 yd., is the other part, which is now definitely measured.

Exercise 35

Subtract, and prove your answer correct in each case:

1. 946	2. 785	3. 659	4. 897
324	323	236	683

5. 8498	6. 9999	7. 8395	8. 7948
2361	7265	4073	5216

9. 7684	10. 8697	11. 2578	12. 3796
6450	1082	1506	1542

Exercise 36

In the following questions, name (1) the unknown part, (2) the whole quantity, (3) the given part:

1. A merchant sold 246 yd. from a piece of cloth 258 yd. in length. How many yards had he remaining?

2. A person deposited in a bank $8495, but shortly after drew out $1035. How much had he left in the bank? If he should draw out this sum in ten-dollar bills, how many would he get?

3. On Tuesday a merchant deposited in a bank $3475, on Wednesday $4690. If he withdrew $1010 on Thursday, how much did he still have on deposit?

4. What is the difference between 1 yr. and 213 da.?

5. A bankrupt has debts amounting to $8496; his assets are $3015. How much more does he owe than he can pay?

6. A man left property to the value of $36,875 to his two children. The son received $14,250; what was the daughter's share?

7. At an election the successful candidate received 953 votes, and the unsuccessful candidate 613 votes. Find the majority of the former.

33. Computers' Method.

From 94,275 take 67,492.

```
94275
67492
26783
```

Thus: 2 and **3** are 5 ; 9 and **8** are 17 ;
carry 1 to 4 as in addition, making it 5 ;
5 and **7** are 12 ; carry 1 to 7, making it
8 ; 8 and **6** are 14 ; carry 1 to 6, making
it 7 ; 7 and **2** are 9.

The numbers 3, 8, 7, 6, and 2 are written down in order to give the remainder.

34. Find the difference between 642 and 375.

```
642
375
267
```

As we cannot take 5 units from 2 units, we take 1 *ten* from the 4 *tens*, and adding this 1 ten, which equals ten units, to the 2 units, we have 12 units. Then 5 units from 12 units leaves 7 units, which we write under the units' column. Now as we took 1 ten from 4 tens, we have left only 3 tens ; we borrow 1 hundred from the 6 hundreds, and considering the 1 hundred as 10 *tens*, we add it to the 3 tens, making 13 tens ; then 7 tens from 13 tens leaves 6 tens, which we write under the tens' column.

Now as we took 1 hundred from 6 hundreds, we have left only 5 hundreds ; hence we subtract 3 hundreds from 5 hundreds, leaving only 2 hundreds, which we write in the hundreds' column.

The remainder, or difference, is thus 2 hundreds, 6 tens, and 7 units, or 267.

Exercise 37

* In the following questions subtract and prove the correctness of your results by adding the two parts :

1.	653	**2.**	307	**3.**	642
	269		268		375
4.	921	**5.**	255	**6.**	907
	87		99		859
7.	3849	**8.**	9345	**9.**	7007
	2567		8367		6609
10.	8000	**11.**	9041	**12.**	7968
	5348		7385		2693

* Use computers' method of subtraction.

13. 43970
26784

14. 50062
37891

15. 12009
11376

16. 34060
29143

17. 986403
728547

18. 620703
444444

19. 850439
473642

20. 759826
378934

Exercise 38

* Subtract:

1. 57261
38877

2. 40359
9998

3. 10000
1021

4. 89437
15790

5. 67182
30293

6. 81349
47538

7. 654375
412884

8. 986392
826957

9. 303233
192001

10. 233826
204739

11. 310865
270326

12. 605487
584598

13. 164326
48476

14. 982623
897674

15. 1000101
707707

Exercise 39

* Subtract:

1. 755903
699004

2. 640021
400569

3. 716287
662763

4. 100794
81685

5. 143812
109758

6. 948735
473596

7. 4731246
4342760

8. 9487352
5999999

9. 1737682
739908

* Use computers' method of subtraction.

| 10. | 3801572 | 11. | 5745861 | 12. | 5048650 |
| | 2003789 | | 2837154 | | 4243091 |

| 13. | 1217191 | 14. | 4100293 | 15. | 2047000 |
| | 1038182 | | 1925867 | | 1054888 |

| 16. | 5468305 | 17. | 7086543 | 18. | 1671498 |
| | 1490673 | | 2889454 | | 536819 |

| 19. | 8235460 | 20. | 2679953 | 21. | 1521815 |
| | 3530089 | | 1346397 | | 1432568 |

Exercise 40

1. 1 bu. = ? qt.

Subtract 32 qt. from 128 qt., and from the remainder, and so on, until no remainder is left. How many times did you subtract? How many bushels in 128 qt.?

2. Divide 128 qt. by 32 qt., and thus show that your answer to example 1 is correct.

3. 1 lb. = ? oz.

As in example 1, subtract 16 oz. from 80 oz. until no remainder is left. How many pounds in 80 oz.? Prove your answer correct by division.

4. 1 da. = ? hr.

As in example 1, subtract 24 hr. from 168 hr. until no remainder is left. How many days in 168 hr.? Prove your answer correct by division.

5. A township contains 36 sq. mi.

As in example 1, subtract 36 sq. mi. from 216 sq. mi. until no remainder is left. Into how many townships can a section of country containing 216 sq. mi. be divided? Prove your answer correct by division.

6. 1 sq. ft. = 144 sq. in.

As in example 1, subtract 144 sq. in. from 1152 sq. in. until no remainder is left. How many square feet in 1152 sq. in.? Prove your answer correct by division.

7. 1 mi. = 1760 yd.

As in example 1, subtract 1760 yd. from 8800 yd. until no remainder is left. How many miles in 8800 yd.? Prove by division.

8. 1 mi. = 5280 ft.

As in example 1, subtract 5280 ft. from 31,680 ft. until no remainder is left. How many miles in 31,680 ft.? Prove by division.

Exercise 41

Solve the following questions and prove your answers correct:

1. Subtract $819 from $918, explaining the process.

2. I paid $3500 for a house and lot and sold it for $4275. Find my gain.

3. A speculator sold cattle at a loss of $3145 and some horses at a gain of $2578. How much did he lose on both transactions?

4. A merchant exchanges a stock of goods worth $6725, and a house worth $3120, with a farmer for a farm valued at $5900, the farmer paying the balance in money. What sum must the merchant receive?

5. A lends B $9780; B repays A by giving him bank stock to the amount of $1946, a farm worth $6385, and the balance in cash. How much cash did B pay A?

6. A is worth $6215, B is worth $876 less than A, and C is worth as much as A and B together, lacking $2343. How much are B and C worth, respectively? How much are all three worth?

7. How much larger is Lake Erie than Lake Ontario? Lake Superior than Lake Michigan? The total areas of the three smaller lakes than Lake Superior? (For the areas of these lakes see Exercise 30, question 5.)

8. During August, 1899, the postal receipts at Chicago amounted to $495,093, an increase of $37,714 over the receipts for the same month of 1898. The receipts at Detroit in August last were $53,238, a decrease of $239 as compared with August,

1898; at Minneapolis, $53,201, an increase of $1756. Find the postal receipts in each of these cities for August, 1898.

9. What is the difference between 9 and 5? What quantity is equal to this difference if the unit of measure is 3 in.? 8 in.? $5? $10?

10. What is the difference between 643 and 579 when the unit of value is $1? $10? $100? $1000?

11. How much greater are 253 units of $1000 than 1864 units of $100?

12. A man bought a house and lot for $8450. He spent $1379 in improvements and $212 for insurance. He then sold the house and lot for $12,000; did he gain or lose, and how much?

13. A collector received $1300 from five men; from the first he received $367, from the second $194 less than from the first, from the third $36 more than from the second, from the fourth as much as from the second and third together. How much did he collect from each man?

14. From the difference between 784 and 8305, take the difference between 17,012 and 21,410.

15. Two men start from the same point and travel in the same direction. The first travels 84 mi. in one day and the second 69 mi. How far were they apart at the end of the first day? If they had travelled in opposite directions, how far would they have been apart?

16. The population of Texas in 1900 was 3,048,710, and of Illinois, 4,821,550. How much greater was the population of Illinois in 1900 than that of Texas?

17. The area of Texas is 265,780 sq. mi., of England, 50,800 sq. mi., and of Germany, 208,700 sq. mi. How much larger is Texas than the united area of England and Germany?

18. The population of New York State in 1880 was 5,082,871; in 1890 it was 5,997,853; in 1900 it was 7,268,012. What was

the increase in population from 1880 to 1890? From 1890 to 1900? How much greater was the latter increase than the former?

19. The population of Illinois in 1900 was 4,821,550; of Iowa, 2,231,853. Show by subtraction that the population of Illinois in 1900 was more than twice that of Iowa.

20. A Frenchman came to the United States June 20, 1844, when he was 23 years old. How old was he June 20, 1899?

21. Find the increase or decrease in the earnings of the following railroads for the first week of August, 1899, over the first week of August, 1898:

	1899	1898
Pittsburg & Western:		
First week August	$64,934	$65,115
Norfolk & Western:		
First week August	270,086	215,392
Ohio River:		
First week August	28,689	22,780
Rio Grande Western:		
First week August	60,800	44,600

22. The area of winter wheat sown in Iowa in the fall of 1898 was 154,177 A. On account of winter-killing only 27,427 A. were harvested. Find the number of acres of wheat destroyed by the cold.

23. 154,243 A. were planted in potatoes in Iowa in 1899, and 155,131 A. in 1898. Find the decrease in 1899.

24. The United States government paid $30,393,209.53 for carrying the mails in 1888, and $52,294,383.23 in 1898. Find the increase.

25. The imports into the United States from Cuba, Porto Rico, and the Philippines for the seven months ending July 31, 1898 and 1899, are given below. Find the increase in each case for the year 1899.

Imports					Cuba	Porto Rico	Philippines
1899 .	.	.	.	.	$ 19,976,956	$ 3,379,944	$ 3,274,134
1898 .	.	.	.	.	12,474,770	2,253,800	2,283,775

26. As in the previous example, find the increase in exports from the United States to these islands.

Exports					Cuba	Porto Rico	Philippines
1899 .	.	.	.	.	$ 14,116,993	$ 2,299,221	$ 386,109
1898 .	.	.	.	.	4,485,937	569,110	65,736

35. From 25.3846 take 18.6397.

25.3846
18.6397
——————
6.7449

We write units under units, tenths under tenths, and so on. Beginning at the right, we subtract as if the figures were integers, and place the decimal point in the difference between the units' and the tenths' column.

Do this problem by the computers' method.

Exercise 42

Find the difference :

1. 26.437	**2.** 94.568	**3.** 102.4951
15.254	29.783	58.2876

From :

4. 75.093 take 34.267. **5.** 6.4297 take 3.5824.

6. 41.7453 take 27.937. **7.** 3.1111 take 1.4682.

8. 3.1416 take .9885.

9. A car containing 24.875 T. of coal was divided between A and B. A received 11.375 T. What did B get?

10. Show by subtracting 6 times that a field containing 52.584 A. can be divided into 6 fields, each containing 8.764 A.

11. The highest rate of taxation paid in Cook County, Illinois, for the year 1899 was $10.68 on $100 property, and the lowest $1.96. Find the difference.

12. The taxes on $100 property for the year 1899, for the different towns in the city of Chicago, were as follows:

North Chicago $6.635
South Chicago 6.34
West Chicago 7.232
Hyde Park 6.35
Lake 6.426
Lake View 6.376
Jefferson 6.105

Find the difference between the tax on $100 property in West Chicago and in the other towns of Chicago.

Miscellaneous Exercise 43

1. Draw a line. Mark off on this line 6 parts, each $\frac{1}{3}$ ft. long. How many feet long is the line? 6 times $\frac{1}{3}$ ft. = ? $\frac{1}{3}$ ft. multiplied by 6 = ? $\frac{1}{3}$ of 6 ft. = ?

2.

$\frac{1}{3}$ of 12 ft. = ?	$\frac{3}{4}$ of $20 = ?	$30 \times \frac{5}{6} = ?$
$12 \times \frac{1}{3}$ ft. = ?	$20 \times \$\frac{3}{4} = ?$	$240 \times \frac{3}{4} = ?$
$\frac{2}{3}$ of 6 lb. = ?	$\$\frac{3}{4} \times 20 = ?$	$155 \times \frac{3}{5} = ?$
$6 \times \frac{2}{3}$ lb. = ?	$56 \times \frac{5}{8}$ yd. = ?	$147 \times \frac{3}{7} = ?$
$\frac{2}{3}$ lb. $\times 6 = ?$	$40 \times \frac{5}{8}$ hr. = ?	$365 \times \frac{2}{5} = ?$

3. Find the weight of 12 packages of tea each containing $\frac{3}{4}$ lb. Of 16 yd. of carpet at $\frac{7}{8}$ lb. a yard.

4. Find the weight of 6 spoons at $\frac{1}{2}$ oz. each; 8 at $\frac{3}{4}$ oz. each; $\frac{3}{4}$ doz. at $\frac{2}{3}$ oz. each; $1\frac{1}{2}$ doz. at $\frac{5}{6}$ oz. each.

5. Find the weight of 48 yd. of carpet at $1\frac{5}{16}$ lb. a yard.

6. Find the cost of:

12 bu. potatoes at $\$\frac{3}{4}$ a bushel.
6 crates berries at $\$\frac{2}{3}$ a crate.
10 bu. apples at $\$\frac{4}{5}$ a bushel.
6 lb. coffee at $\$\frac{1}{3}$ a pound.

7. By what number do you multiply to reduce gallons to quarts?

Reduce to quarts:

1 gal. 2 qt.	6 gal. 2 qt.	26 gal. 3 qt.
3 gal. 1 qt.	12 gal. 1 qt.	35 gal. 2 qt.

8. I bought 3 gal. 2 qt. of milk at 6¢ a quart, and gave a dollar bill in payment. What change should I get back?

9. By what number do you multiply to reduce yards to feet?

Reduce to feet:

6 yd. 2 ft.	12 yd. 1 ft.	84 yd. 1 ft.
8 yd. 1 ft.	18 yd. 2 ft.	72 yd. 2 ft.
9 yd. 2 ft.	24 yd. 2 ft.	65 yd. 1 ft.

10. Find the weight of an iron rod 8 yd. 2 ft. long if 1 ft. weighs ½ lb.

11. What part of $8 is $2? What per cent? What part of 24 lb. is 8 lb.? What per cent? What per cent of $18 is $3? Of $12 is $8? Of $36 is $27? Of $15 is $9?

12. A grocer bought coffee at 30¢ a pound, and sold it at a gain of 6¢ a pound. The gain is what part of the cost? What per cent of the cost?

13. Find the gain per cent:

Cost	Gain	Cost	Gain
$5	$1	75¢	25¢
$15	$5	42¢	6¢
$30	$5	40¢	25¢
$24	$3	64¢	4¢
$75	$50	90¢	6¢
$48	$18	36¢	27¢
$36	$4	27¢	18¢
$36	$3	72¢	12¢

14. A merchant bought carpet at 64¢ a yard, and sold it for 88¢ a yard. Find his gain on each yard. Find his gain per cent.

15. A merchant bought handkerchiefs at 25 ¢ each, and sold them for 35 ¢. Find his gain per cent.

16. Find the gain per cent:

Cost	Selling Price	Cost	Selling Price
$ 18	$ 24	$ 72	$ 120
$ 44	$ 66	$ 54	$ 60
$ 60	$ 96	$ 32	$ 52
$ 56	$ 77	$ 27	$ 45
$ 63	$ 91	$ 66	$ 72
$ 64	$ 68	$ 48	$ 54

17. The following is the statement of the total cost and the net returns of a peach orchard in Maryland for a period of ten years:

Net returns for peach sales	$ 46,361.07
Land	$ 2100.00
Trees	135.00
Planting, etc.	60.78
Machinery	170.07
Cultivation	1320.00
Fertilizers	769.00
Taxes	312.06
Interest	2520.90
Incidentals	11.88

Find the total cost and the profits.

18. For the year ending June 30, 1899, the exports from the United States to Great Britain and her colonies were:

To the United Kingdom	$ 511,816,475
British North America	89,573,609
British West Indies	8,751,817
British Guiana	1,749,545
British East Indies	4,341,936
British Asia	7,732,525
British Australia	19,777,129
British Africa	15,155,610

Find their total value.

19. The total exports from the United States to all countries for the same year as in the preceding example were $1,201,931,222. Find the value of the exports to countries other than Great Britain and her colonies.

20. How much less was this than the value of the exports to Great Britain and her colonies?

CHAPTER VI

MULTIPLICATION

36. (1) Beginning with $2, add by $2, till you reach $26. What are the $2 called? Addends. What the result? Sum.

(2) In getting this *sum* have you definitely thought of *how many* $2 there are? No. Do you know from the sum *how many* there are? No.

(3) If you add $2 to $2, etc., till you reach the *sum*, $182, do you know *how many* twos there are? No.

(4) How do you look upon the sum $26 (say) and the $2? The $26 is simply the sum of an unknown number of $2.

(5) Now count the number of $2. There are 13. Did you think of this 13 in the addition process? No.

(6) Now consider this 13 in relation to the *addend* $2, and the *sum* $26, what *new* idea is introduced? The *idea* of *how many times* $2 is repeated to make $26?

(7) Then what is the *number* which measures $26? 13. What is the unit of measure? $2. From what you know of number, say what *ratio* 13 is? The ratio of $26 to $2. We say at once (without adding) that 13 times $2 is $26.

(8) In this do we depend at all on addition? *Yes.* We first find the *sum*, and connect this in memory with the *number of times* the addend is repeated.

(9) But is it then correct to say that the processes $2 + $2 + $2 ⋯ = $26, is *identical* with the process $13 × $2 = $26? *No;* for *13* represents the "*new* idea" referred to, and $2 has become a *definite unit of measure*, which with 13 denotes the quantity $26. The *addend* has become a *factor*, and the *sum* a *product*.

37. Find the cost of 9 yd. of cloth at $5 a yard.

(1) Here we think of $5 as a *derived unit* measuring the value of 1 yd. Hence the cost of 9 yd. is equal to $9 \times \$5$, or to $45.

(2) Thus 45, the number of primary units in the total cost, is called the *product* of the number of primary units in the derived unit $5, which is 5, by the number of units, viz. 9, in the given quantity of cloth.

(3) In the above example the total cost was given by 9 units of $5 each, and after *multiplication* by 45 units of $1 each. Thus multiplication does not change the total cost (*i.e.* the measured quantity); it changes only the *number* which measures it (in this case from 9 to 45) by changing the unit of measure, $5, to the primary unit, $1.

The numbers to be multiplied together, viz. 9 and 5, are called *factors* of the product, *i.e.* of the number that measures the quantity.

38. Multiplication is the operation of finding the number of primary units in a quantity expressed by a given number of derived units, or, more briefly,

Multiplication is the operation of finding the product of two numbers.

The **Multiplicand** is the derived unit of measure.

The **Multiplier** denotes how many times this unit of measure is to be repeated, *i.e.* it denotes the *ratio* of the measured quantity to the unit of measure.

39. $8 \times \$6$ is read 8 times $6.

$\$6 \times 8$ is read $6 multiplied by 8.

$\times$ is the Sign of Multiplication.

(1) Require the multiplication table to be memorized in regular order; also, so that it can be given by the pupil in irregular order, thus:

$9 \times 4 = 36,$ $9 \times 6 = 54,$ $9 \times 10 = 90,$ etc.

(2) Drill, requiring instantaneous oral and written answers to such questions as: What is 6×7? 9×8? 8×9?

(3) Drill, requiring instantaneous answers: What is

$6 \times 7 + 4$? $9 \times 8 + 3$? $8 \times 9 + 7$?

MULTIPLICATION TABLE

TWICE	THREE TIMES	FOUR TIMES	FIVE TIMES	SIX TIMES	SEVEN TIMES
1 is 2	1 is 3	1 is 4	1 is 5	1 is 6	1 is 7
2 " 4	2 " 6	2 " 8	2 " 10	2 " 12	2 " 14
3 " 6	3 " 9	3 " 12	3 " 15	3 " 18	3 " 21
4 " 8	4 " 12	4 " 16	4 " 20	4 " 24	4 " 28
5 " 10	5 " 15	5 " 20	5 " 25	5 " 30	5 " 35
6 " 12	6 " 18	6 " 24	6 " 30	6 " 36	6 " 42
7 " 14	7 " 21	7 " 28	7 " 35	7 " 42	7 " 49
8 " 16	8 " 24	8 " 32	8 " 40	8 " 48	8 " 56
9 " 18	9 " 27	9 " 36	9 " 45	9 " 54	9 " 63
10 " 20	10 " 30	10 " 40	10 " 50	10 " 60	10 " 70
11 " 22	11 " 33	11 " 44	11 " 55	11 " 66	11 " 77
12 " 24	12 " 36	12 " 48	12 " 60	12 " 72	12 " 84

| EIGHT TIMES	NINE TIMES	TEN TIMES	ELEVEN TIMES	TWELVE TIMES
1 is 8	1 is 9	1 is 10	1 is 11	1 is 12
2 " 16	2 " 18	2 " 20	2 " 22	2 " 24
3 " 24	3 " 27	3 " 30	3 " 33	3 " 36
4 " 32	4 " 36	4 " 40	4 " 44	4 " 48
5 " 40	5 " 45	5 " 50	5 " 55	5 " 60
6 " 48	6 " 54	6 " 60	6 " 66	6 " 72
7 " 56	7 " 63	7 " 70	7 " 77	7 " 84
8 " 64	8 " 72	8 " 80	8 " 88	8 " 96
9 " 72	9 " 81	9 " 90	9 " 99	9 " 108
10 " 80	10 " 90	10 " 100	10 " 110	10 " 120
11 " 88	11 " 99	11 " 110	11 " 121	11 " 132
12 " 96	12 " 108	12 " 120	12 " 132	12 " 144

40. In the preceding diagram we have 36 dots, signifying 36 units of any kind, arranged in 4 rows of 9 dots each, and at the same time 9 rows of 4 dots each. Hence we think of 36 as equal to 4 times 9 or 9 times 4. Arrange the dots to show that 36 is equal to 3×12 or 12×3, and also to 2×18 or 18×2.

4 and 9 are called *factors* of 36, and 36 is called the *product* of 4 and 9.

This illustrates the **law of commutation**, a law of great importance in Arithmetic.

Thus we think of 24 as equal to 2×12 or 12×2, 3×8 or 8×3, 4×6 or 6×4.

$$\bullet \;\bullet \;\bullet \;\bullet \;\bullet \;\bullet$$
$$\bullet \;\bullet \;\bullet \;\bullet \;\bullet \;\bullet$$
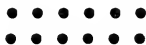
41. If in the above arrangement we think of each dot as representing $1, then the diagram shows that $12 \div 2 = 6$.

What other measurement is shown by the *same* arrangement?

Exercise 44

1. Arrange dots, representing any units, to show that

$$18 = 3 \times 6, \text{ or } 6 \times 3 = 2 \times 9, \text{ or } 9 \times 2.$$

2. Give the factors of 45 (9×5 or 5×9), 66, 56, 72, 96, 63, 90, 54, 99, 84, 132, 108.

3. Give the factors of 9, 16, 25, 36, 49, 64, 81, 100, 121, 144.

4. Arrange 30 dots to show how often 30 is measured by 10. By 3.

5. How often is 72 measured by 9? 8? 6? 12? 4? 18? 3? 24? 2? 36?

6. If one factor of 96 is 12, what is the other? If one factor is 8, what is the other?

7. What will 5 yd. of cloth cost at $4 a yard? What will 4 yd. cost a $5 a yard?

8. If 9 men can do a piece of work in 6 da., how long will it take 1 man to do it? If 6 men can do a piece of work in 9 da., how long will it take 1 man to do it?
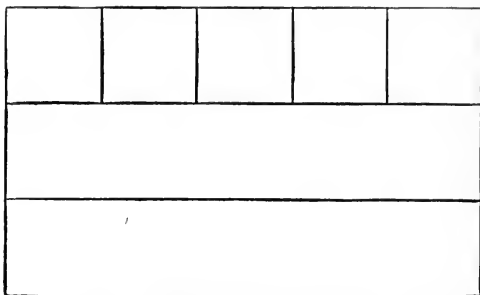
Scale: ½ in. = 1 in.

42. (1) Find the area of an oblong 5 in. long and 3 in. wide.

Let the oblong be divided into 3 strips by lines 1 in. apart, as in the figure.

The area of 1 strip = 5 sq. in.

∴ the area of the oblong = 3 × 5 sq. in.

= 15 sq. in.

Prove the answer correct by dividing the oblong into square inches, and counting.

Make a drawing to show that the area is equal to 5 × 3 sq. ft.

Thus we may think of the area as either 5 × 3 or 3 × 5 sq. in.
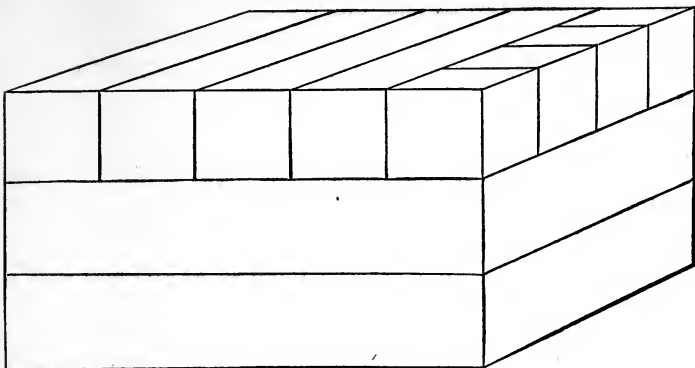
(2) Reduce 9 pk. 6 qt. to quarts.

9 pk. = 9 × 8 qt. = 72 qt.

∴ 9 pk. 6 qt. = 78 qt.

Here the problem is to add 6 qt. to 9 units of 8 qt. each.

NOTE. — By the law of commutation we may think of 9 pk. as 9 × 8 qt., or as 8 × 9 qt. Thus in reducing pecks to quarts we may use 8 as a constant multiplier.

What constant multiplier will reduce yards to feet? Feet to inches? Gallons to quarts? Quarts to pints? Weeks to days?

Scale: $\frac{1}{2}$ in. = 1 in.

(3) Find the volume of a rectangular solid 5 in. long, 4 in. wide, and 3 in. thick.

Let the solid be divided into 3 slices by horizontal planes 1 in. apart.

Let the upper slice be divided into 5 rows by vertical planes 1 in. apart.

Let the right-hand row be divided into 4 cu. in. by vertical planes 1 in. apart.

The volume of 1 row = 4 cu. in.

The volume of 5 rows or 1 slice = 5 × 4 cu. in.

The volume of 3 slices or the solid = 3 × 5 × 4 cu. in.

= 60 cu. in.

Exercise 45

1. Make a drawing to show that an oblong 6 in. long and 4 in. wide contains 24 sq. in.

2. Find the area of the following oblongs: 6 in. by 8 in.; 7 in. by 9 in.; 8 in. by 11 in.; 9 in. by 12 in.

3. How do you find the number of units of area in an oblong?

4. Find the area of the floor of each of the rooms whose dimensions are 6 yd., 5 yd.; 8 yd., 8 yd.; 12 yd., 10 yd.; 12 yd., 12 yd.

5. A garden bed 5 ft. long and 3 ft. wide is surrounded by a walk 1 ft. wide. Make a drawing to show the entire area of the bed and walk. What is this area? (Scale 1 in. to 1 ft.)

6. Find the number of square inches in each of the stamped linen squares whose sides are, respectively, 6, 8, 10, and 12 in.

7. Find the number of square feet in a square yard; of square inches in a square foot.

8. A rug occupies the centre of a square room, and is everywhere 2 ft. from the wall. If the room is 14 ft. long, find the size of the rug.

9. How long is a township? How wide? What is its area?

10. Make a drawing to show the volume of a rectangular solid 2 in. by 3 in. by 4 in. What is it? How do you find the volume of a rectangular solid?

11. Find the volume of each of the rectangular solids whose dimensions are: 2 in., 3 in., 5 in.; 4 in., 6 in., 9 in.; 4 in., 5 in., 7 in.; 6 in., 6 in., 6 in.; 2 ft., 2 ft., 2 ft.; 3 units of length, 4 units, 5 units.

12. Find the number of cubic feet in a cubic yard. Of cubic inches in a cubic foot.

13. Make a drawing to show the perimeter of an oblong 3 in. by 5 in. What is it? How do you find the perimeter of an oblong?

14. Find the perimeter of rooms whose dimensions are: 6 yd., 8 yd.; 7 yd., 9 yd.; 8 yd., 11 yd.; 11 ft., 12 ft.; 12 ft., 15 ft.

15. Find the number of yards of braid needed to carpet a rug 4 yd. long and 3 yd. wide. What does it cost at 8 ¢ a yard?

16. How long will it take a bicyclist to ride around a square township at 8 mi. an hour?

Exercise 46

1. Reduce to lower denominations:

(1) 5 yd. 2 ft.; 7 yd. 1 ft.; 8 yd. 2 ft.; 9 yd. 1 ft.; 11 yd. 1 ft.; 12 yd. 2 ft.

(2) 5 ft. 4 in.; 8 ft. 10 in.; 7 ft. 8 in.; 11 ft. 3 in.; 9 ft. 7 in.; 12 ft. 6 in.

2. Find the cost of making a sidewalk 10 yd. 2 ft. long at 25¢ a foot.

3. Find the cost of 5 curtains, each 3 yd. 1 ft. long, at 6¢ a foot.

Reduce to lower denominations:

4. 3 sq. yd. 5 sq. ft.; 5 sq. yd. 7 sq. ft.; 12 sq. yd. 6 sq. ft.; 8 sq. yd. 8 sq. ft.; 9 sq. yd. 2 sq. ft.; 11 sq. yd. 10 sq. ft.

5. 6 qt. 1 pt.; 8 qt. 1 pt.; 11 qt. 1 pt.; 7 pk. 6 qt.; 9 pk. 4 qt.; 9 bu. 3 pk.; 8 bu. 2 pk.; 11 bu. 3 pk.; 7 gal. 2 qt.; 9 gal. 3 qt.

6. Find the cost of 3 gal. 2 qt. of milk at 6¢ a quart.

7. A grocer sold 6 bu. 3 pk. of potatoes at 18¢ a peck. Find the selling price.

Reduce to lower denominations:

8. 7 wk. 4 da.; 9 wk. 2 da.; 11 wk. 6 da.; 8 wk. 1 da.; 12 wk. 5 da.; 5 hr. 40 min.; 8 hr. 9 min.; 9 hr. 22 min.; 12 hr. 45 min.; 5 da. 4 hr.; 8 da. 2 hr.

9. How many hours are there in one week?

43. What is the cost of 6 town lots at $894 a lot?

Here we think of the whole cost as 6 units of $894 each.

EXPLANATION. — The unit $894 may be considered as made up of 4 units of one dollar, 9 units of ten dollars, and 8 units of one hundred dollars.

$894

6

$5364

6 × 4 units of one dollar = 24 units of one dollar = 2 units of ten dollars + 4 units of one dollar.

6 × 9 units of ten dollars = 54 units of ten dollars.

54 units of ten dollars + 2 units of ten dollars = 56 units of ten dollars = 5 units of one hundred dollars + 6 units of ten dollars.

6 × 8 units of one hundred dollars = 48 units of one hundred dollars.

48 units of one hundred dollars + 5 units of one hundred dollars = 53 units of one hundred dollars = 5 units of one thousand dollars + 3 units of one hundred dollars.

5 units of one thousand dollars + 3 units of one hundred dollars + 6 units of ten dollars + 4 units of one dollar = $5364.

Exercise 47

Multiply separately:

1. 231 by 2, 4, 6, 8, 10, and 12.
2. 690 by 3, 5, 7, 9, and 11.
3. 897 by 4, 6, 8, 10, and 12.
4. 2463 by 3, 5, 7, 9, and 11.
5. 5781 by 2, 4, 8, 10, and 12.
6. 9654 by 3, 5, 7, 9, and 11.
7. 8267 by 2, 4, 6, 8, 10, and 12.
8. 5280 by 3, 5, 7, 9, and 12.
9. 1728 by 2, 4, 6, 8, 10, and 12.
10. 4840 by 3, 5, 7, 9, and 11.
11. 63,360 by 2, 4, 6, 8, 10, and 12.
12. 24,793 by 3, 6, 7, 9, and 11.
13. 98,654 by 2, 4, 6, 8, 10, and 12.
14. 89,743 by 3, 5, 7, 9, and 11.
15. 64,789 by 2, 4, 5, 8, 10, and 12.

Exercise 48

1. How do you find the perimeter of a room? The area of an oblong?

2. Find the perimeter of a room 18 ft. long and 12 ft. wide. What is the area of an oblong whose length is equal to the perimeter of this room and whose width is 9 ft.? (Draw this oblong on the blackboard, scale = 1 in. to 1 ft.)

3. The length of a room is 15 ft. and the width 10 ft. Find its perimeter. Find the area of an oblong whose length is equal to the perimeter of this room and whose width is 8 ft.

4. What is the perimeter of a room whose dimensions are 15 ft. and 10 ft.? What is the area of the four walls of a room whose dimensions are 15 ft. and 10 ft., and height 8 ft.?

5. (a) Find the perimeters of rooms whose dimensions are:

Length	Width	Height
16 ft.	14 ft.	8 ft.
17 ft.	15 ft.	8 ft.
20 ft.	18 ft.	9 ft.
22 ft.	20 ft.	12 ft.

(b) Find the areas of the four walls of these rooms.

6. A cord of wood is 8 ft. long, 4 ft. wide, 4 ft. high. How many cubic feet does it contain?

7. Find the number of cubic inches in a cubic foot.

8. A gallon of water will exactly fill a rectangular box 11 in. long, 7 in. wide, and 3 in. high. Find the number of cubic inches in a gallon.

9. There are 1760 yd. in 1 mi. Find the number of yards in 2 mi.; 5 mi.; 8 mi.; 9 mi.; 12 mi.

10. There are 5280 ft. in 1 mi. Find the number of feet in 3 mi.; 6 mi.; 7 mi.; 9 mi.; 11 mi.

11. Reduce to square inches: 5 sq. ft.; 8 sq. ft.; 10 sq. ft.; 12 sq. ft.

12. Reduce to square feet: 3547 sq. yd.; 8426 sq. yd.; 9819 sq. yd.

13. One square mile contains 640 A. Find how many acres there are in 6 sq. mi.; 8 sq. mi.; 10 sq. mi.; 12 sq. mi.

14. Reduce to cubic inches: 4 cu. ft.; 5 cu. ft.; 7 cu. ft.; 9 cu. ft.; 11 cu. ft.

15. How do you reduce weeks to days? gallons to quarts?

16. Reduce to days: 453 wk.; 769 wk.; 827 wk.; 852 wk.

17. Reduce to quarts: 765 gal.; 917 gal.; 763 gal.; 789 gal.

18. Reduce to quarts: 735 pk.; 892 pk.; 679 pk.; 728 pk.

19. Reduce to days: 3 yr.; 5 yr.; 6 yr.; 8 yr.; 11 yr.; 12 yr. (1 yr. = 365 da.)

44. What is the cost of 76 town lots at \$894 a lot?

$$
\begin{array}{r}
\$894 \\
76 \\
\hline
5364 \\
6258 \\
\hline
\$67944
\end{array}
$$

The explanation is similar to that given in § 43. Since, when 7 is used as a multiplier, the 4 units of one dollar are multiplied by 7 *tens*, the product is the same as that found by multiplying 4 units of *ten* dollars by 7. This is 28 units of ten dollars, and is equal to 2 units of one hundred dollars and 8 units of ten dollars. Hence the 8 is written under the 6 in the tens' column, and the 2 is carried to be added in the hundreds' column, and so on.

To prove the answer correct, multiply 76 by 894; thus:

$$
\begin{array}{r}
76 \\
894 \\
\hline
304 \\
684 \\
608 \\
\hline
67944
\end{array}
$$

∴ the answer is correct.

Exercise 49

Multiply and prove your answers correct:

1. 423 by 36.	11. 8647 by 365.
2. 479 by 32.	12. 7245 by 168.
3. 295 by 16.	13. 8939 by 224.
4. 798 by 24.	14. 6558 by 144.
5. 581 by 52.	15. 9275 by 231.
6. 649 by 27.	16. 9475 by 1760.
7. 959 by 24.	17. 8213 by 5280.
8. 764 by 31.	18. 4781 by 1728.
9. 953 by 56.	19. 5893 by 2240.
10. 825 by 48.	20. 6439 by 1728.

Exercise 50

Multiply:

1. 8245 by 684.	6. 8746 by 675.
2. 7639 by 797.	7. 9687 by 897.
3. 5927 by 395.	8. 4786 by 478.
4. 4399 by 927.	9. 9467 by 769.
5. 8999 by 868.	10. 8769 by 567.

45. (1) A drover bought 36 horses at $145 a head, and 96 cows at $28 a head. Which cost the more, and how much?

Here the problem is to find the difference between 36 units of $145 each and 96 units of $28 each. Multiply $145 by 36 and $28 by 96, and find the difference.

(2) A's barn cost $175, his house 16 times as much, and his farm cost as much as both. What was the cost of all?

Here the problem is to find the cost of 1 + 16 + 17, or 34 units of $175 each. Multiply $175 by 34 and find the cost of all.

Exercise 51

1. A dry goods firm had 28 clerks, and paid an average salary of $45 a month. Find the total amount of their monthly salaries.

2. A real estate dealer bought 8 lots, and sold them at a gain of $250 on each lot. Find his total gain.

3. A dealer bought 150 head of cattle and 47 mules. He made a profit of $13 a head on the former and $17 each on the latter. Find his total gain.

4. A vessel took 3 da. to make a trip. If the average rate was 12 mi. an hour, find the length of the trip.

5. A boy left home on his bicycle, and rode 5 hr. at the rate of 8 mi. an hour. He returned home at the rate of 6 mi. an hour. How far was he from home 4 hr. after starting back? Draw a line and mark off the distances.

6. A train started from St. Louis and travelled 12 hr. at the rate of 35 mi. an hour. The next day it returned at the rate of 32 mi. an hour. Find its distance from St. Louis 3 hr. after starting back.

7. A farmer raised a crop of 58 bu. of corn an acre from 48 A. Find the number of bushels of corn. What is its value at 38 ¢ a bushel?

8. A farmer raised a crop of 37 bu. of oats an acre from 65 A. Find the value of the oats at 26 ¢ a bushel.

9. Make out a bill for the following goods:

23 yd. cotton @ 11¢; 13 yd. gingham @ 23¢;
25 yd. flannel @ 37¢; 18 yd. tweed @ $1.50;
12 yd. serge @ $1.75; 6 yd. broadcloth @ $4.50.

10. A produce merchant exchanged 48 bu. of oats at 39¢ per bushel and 13 bbl. of apples at $3.85 a barrel for 200 lb. of butter at 37¢ a pound. How much should he pay to settle the account?

11. A grain dealer buys 4795 bu. of wheat in Chicago at 63¢ a bushel, and ships it to New York at a cost of 3¢ a bushel. Find his gain if he sells it in New York for 71¢ a bushel.

12. A man bought 51 horses at $97 each, and sold them at $136 each. How much did he gain?

13. Find the amount of the following bill:

63 brooms, at 16¢ each;
13 yd. print, at 11¢ per yard;
17 lb. tea, at 35¢ per pound;
4 doz. oranges, at 4¢ each;
287 lb. sugar, at 5¢ per pound;
84 eggs, at 13¢ per dozen.

14. A fruit dealer sold apples at the rate of 6 for 5¢. What was the price per dozen? If he sold oranges at the rate of 3 for 5¢, what was the price per dozen?

15. A fruit dealer paid 8¢ a dozen for bananas, and sold them at the rate of 4 for 5¢. Find his gain on a bunch containing 5 doz.

16. Bought oranges at the rate of 18¢ a dozen, and sold them at the rate of 6 oranges for 15¢. How much did I gain on 11 boxes, each containing 20 doz.?

17. If in the previous example two boxes were spoiled, what was the gain?

18. How far will a bicyclist ride in 12 da., if he rides 6 hr. a day at 8 mi. an hour?

19. Two vessels start from the same point and travel, the one down a river at the rate of 15 mi. an hour, the other up the river at the rate of 9 mi. an hour. How far will they be apart in 8 hr. ?

20. If the first vessel travelled up the river at the rate of 12 mi. an hour, how far apart would they be in 8 hr. ?

21. A speculator bought 45 A. of land at $ 65 an acre, and 63 A. at $ 78 an acre. If he sold the whole at $ 75 an acre, how much did he gain or lose ?

22. In 1898 there were shipped from the melon district of Indiana 1156 carloads of melons. If the average cost of shipping was $ 29.75 a car, find the total cost.

23. The wholesale price of a dressed beef weighing 800 lb. is given in the following table:

	Pounds	Per Pound		Pounds	Per Pound
Forequarters :			Hindquarters :		
Roast . . .	76	16½ cents	Round . . .	180	9 cents
Plate . . .	90	3½ cents	Loin . . .	140	16½ cents
Shank . . .	24	3 cents	Suet . . .	24	4 cents
Chuck . . .	210	6½ cents	Flank . . .	24	3 cents
			Shank . . .	32	2½ cents

Find the cost of the forequarters and of the hindquarters.

24. In the previous example what is the entire cost of the beef, and how much less is this than 9 ¢ a pound ?

25. The grain receipts of the city of Chicago, Aug. 24, 1899, were 80 cars of wheat, 365 of corn, and 480 of oats. If a car of wheat contains 400 bu., of corn 400 bu., and of oats 680 bu., find the number of bushels of each kind of grain received.

26. A farmer bought 80 A. of land at $ 25 an acre, and drained it at a cost of $ 475, thus increasing its value to $ 45 an acre. Find the increase in the value of the farm above the cost of drainage. .

27. A section of land in the state of Missouri, containing 108,200 A., was drained at a cost of $700,000, thus increasing its value from $5 to $40 an acre. Find the total increase in value above the cost of drainage.

46. Multiply .948 by 6.

.948
  6
─────
5.688

948 thousandths multiplied by 6 equals 5688 thousandths or 5.688.

Exercise 52

Multiply:

1. .5 by 9.	**6.** .842 by 9.	**11.** 4.79 by 32.
2. .8 by 3.	**7.** .1416 by 25.	**12.** 3.295 by 16.
3. .26 by 3.	**8.** .988 by 76.	**13.** .7568 by 144.
4. .39 by 8.	**9.** 3.54 by 12.	**14.** 8.754 by 172.
5. .624 by 3.	**10.** .543 by 36.	

15. Show by measuring a plate that its circumference is more than three times its diameter.

16. Draw a circle 4 in. in diameter. *The circumference of a circle is* 3.1416 *times the diameter.* Multiply 3.1416 by 4 and find the length of the circumference. Measure the circumference and see if your answer appears to be correct.

17. By what do you multiply 3.1416 to find the circumference of a circle? Find the circumference of a circle whose diameter is 3 in.

18. Find the circumferences of circles whose diameters are 6 in., 8 in., 10 in., 7 ft., 9 yd., 2 mi.

19. Find the circumference of a circular flower bed whose diameter is 4 ft.

20. A circular lake is 5 mi. in diameter. How many miles does a boy go in skating around the lake?

21. Find the volume of a rectangular piece of wood 2 ft. by 3 ft. by 4 ft. If this weighs 36.125 lb. per cubic foot, find its weight.

22. Find the weight of a rectangular solid of oak, 3 ft. by 2 ft. by 1 ft., weighing 47.375 lb. per cubic foot.

23. A cubic foot contains 7.48 gal. of water. Find how many gallons of water will fill a tin box containing 8 cu. ft.?

24. A cubic foot contains 7.48 gal. of water. Find how many gallons can be poured into a tin-lined box, whose interior dimensions are 3 by 4 by 6 ft.

25. A drover bought 12 sheep at $5.375 per head, 36 at $4.625, and 212 at $4.125. Find the total cost.

26. Multiply each of the following numbers by 10: .43; .576; 4.23; .017; 89.4263.

27. In multiplying by 10 how many places to the right do you move the decimal point?

28. Write down the product of each of the following numbers multiplied by 10: 7.4; 8.946; 5.32; .008; 62.9347.

29. Multiply each of the following by 100: .435; 8.027; 9.12; 46.5928.

30. In multiplying by 100 how many places to the right do you move the decimal point?

31. Write down the product of each of the following numbers multiplied by 100: 4.95; 84.793; 9.714; 52.1967.

Miscellaneous Exercise 52 (a)

1. Show by a drawing that $1\frac{1}{2}$ ft. $= \frac{3}{2}$ ft. Show by a drawing that $1\frac{2}{3}$ ft. $= \frac{5}{3}$ ft. Show by a drawing that $2\frac{3}{4}$ ft. $= \frac{11}{4}$ ft.

2. $1\frac{3}{4}$ ft. $= \frac{?}{4}$ ft. $\qquad$ $2\frac{1}{4}$ lb. $= ?$ lb. $\qquad$ $3\frac{1}{5} = ?$

$\quad$ $2\frac{2}{3}$ ft. $= \frac{?}{3}$ ft. $\qquad$ $4\frac{1}{4}$ lb. $= ?$ lb. $\qquad$ $4\frac{2}{5} = ?$

$\quad$ $3\frac{1}{5}$ yd. $= \frac{?}{5}$ yd. $\qquad$ $2\frac{1}{8}$ lb. $= ?$ lb. $\qquad$ $6\frac{3}{5} = ?$

3. $20 \times \frac{8}{5} = ?$ $\qquad$ $12 \times \frac{5}{3} = ?$ $\qquad$ $14 \times 2\frac{2}{7} = ?$

$\quad$ $24 \times \frac{5}{3} = ?$ $\qquad$ $12 \times 1\frac{2}{3} = ?$ $\qquad$ $40 \times 1\frac{5}{8} = ?$

$\quad$ $16 \times \frac{9}{8} = ?$ $\qquad$ $18 \times 1\frac{5}{6} = ?$ $\qquad$ $45 \times 2\frac{7}{9} = ?$

$\quad$ $42 \times \frac{7}{6} = ?$ $\qquad$ $36 \times 3\frac{1}{8} = ?$ $\qquad$ $15 \times 3\frac{2}{5} = ?$

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4. What is the cost of 10 lb. of sugar at $5\frac{1}{2}$ ¢ a pound?   Of 8 yd. of silk at $\$2\frac{1}{2}$ a yard?

5. What is the weight of 16 yd. of carpet at $2\frac{1}{4}$ lb. a yard? Of 8 yd. at $1\frac{5}{8}$ lb. a yard?   Of 8 sq. yd. of oil-cloth at $1\frac{3}{4}$ lb. per square yard?   Of 24 sq. yd. of floor oil-cloth at $2\frac{3}{4}$ lb. per square yard?

6. Find the cost of:

> 40 yd. carpet lining at $2\frac{1}{2}$ ¢ a yard.
> 12 yd. binding at $\frac{3}{5}$ ¢ a yard.
> 100 fish hooks at $\frac{1}{5}$ ¢ each.
> 3 doz. penholders at $2\frac{3}{4}$ ¢ each.

7. Reduce to pecks:

8 bu.; 6 bu. 3 pk.; 9 bu. 2 pk.; 12 bu. 1 pk.; 25 bu. 3 pk.

8. Find the selling price of 6 bu. 2 pk. of apples at 25 ¢ a peck.   Of 8 bu. 3 pk. of potatoes at 18 ¢ a peck.

9. How many quarts in one bushel?   Reduce to quarts:

3 bu. 6 qt.; 12 bu. 18 qt.; 27 bu. 8 qt.; 45 bu. 9 qt.; 33 bu. 6 qt.

10. Find the selling price of 3 bu. 8 qt. of beans at 8 ¢ a quart.

11. A merchant bought cloth at 25 ¢ a yard and sold it at a loss of 10 ¢ a yard.   The loss is what part of the cost?   What per cent of the cost?

12. Find the loss per cent:

Cost	Loss	Cost	Loss
$ 3	$ 1	75 ¢	5 ¢
$ 40	$ 16	56 ¢	21 ¢
$ 32	$ 4	27 ¢	18 ¢
$ 63	$ 42	84 ¢	12 ¢

13. A dealer paid $36 apiece for bicycles and sold them for $33.   How much did he lose on each bicycle?   What per cent of the cost?

14. Find the loss per cent:

Cost	Selling Price	Cost	Selling Price
$ 25	$ 20	50 ¢	30 ¢
$ 25	$ 15	99 ¢	90 ¢
$ 3	$ 2.25	45 ¢	42 ¢
$ 6	$ 3.75	39 ¢	26 ¢
$ 9	$ 7.29	16 ¢	4 ¢

15. Great Britain's "grocery bill" with the United States for 1899 was:

Corn	$27,512,398	Canned beef	2,066,308
Wheat	55,367,397	Salted beef	1,080,351
Flour	41,335,609	Tallow	1,538,114
Fresh beef	23,456,488	Butter	1,705,190
Live cattle	28,213,572	Cheese	2,063,409
Bacon	30,312,477	Petroleum	8,563,518
Hams	16,366,864	Tobacco	7,808,850
Lard	12,310,730	Horses	3,024,952
Pickled pork	3,119,067	Sheep	702,347
Fresh pork	2,686,191		

Find the total.

16. In addition to the above Great Britain purchased in the United States cotton to the value of $99,709,352; find the total.

17. In 1889 the total number of vessels passing through the Sault Ste. Marie canal was 5579. The total freight 7,516,022 T. In 1899 the total number of vessels was 20,055, the total freight 25,255,810 T. Find the increase in each case.

18. The average price of cotton was $2\frac{1}{4}$ ¢ a pound more in 1899 than in 1898. Find the increase in the price of a bale of 500 lb.

19. Find the increase in the value of a cotton crop of 625 bales due to an advance of $1\frac{3}{4}$ ¢ a pound.

20. Multiply:

.643	72.95	3.624	6.438
48	29	88	75

CHAPTER VII

DIVISION

47. What will 4 oranges cost at 5¢ apiece? If 4 oranges cost 20¢, what is the cost of each? What must 5¢ be multiplied by to get 20¢?

At $5 a yard, how much will 9 yd. of cloth cost? What must $3 be multiplied by to get $27? At $3 a yard, how many yards can be bought for $27?

48. Of what product are 5 and 6 the factors? (30.) If 5 is one factor of 30, what is the other? Of what product are 12 and 4 the factors? If 4 is one factor of 48, what is the other? If 6 is one factor of 42, what is the other? If 9 is one factor of 63, what is the other? 4 is one factor of each of the following numbers: what are the other factors? 24, 36, 20, 28, and 16.

49. In **Multiplication** we are given two factors and we are required to find their product.

In **Division**, on the other hand, we are given the product, and also one of the factors, and we are required to find the other factor.

Thus: Find how many yards of cloth at $5 a yard can be bought for $45?

In this problem, 45, the number measuring the cost of the cloth, is the product of two factors. One of these is 5, the *given* number, which measures the value of the unit, and the other is 9, which is the number of yards.

If 9 yd. of cloth cost $45, what will 1 yd. cost?

As before, 45 is the product of two factors. The given factor is 9, and the required factor 5. Therefore 1 yd. costs $5.

50. Division is the operation of finding either of two factors, when their product and the other factor are given.

The factor found is called the **Quotient.** It shows how often the Divisor is contained in the Dividend.

The given factor is called the **Divisor.**

The given product of the Quotient and Divisor is called the **Dividend.**

When the Divisor is not contained an exact number of times, the excess is called the **Remainder.** See § 57.

51. The sign of Division is ÷; thus $8 ÷ $2 = 4 is read $8 divided by $2 is equal to 4.

$8 ÷ 2 = $4, is read $8 divided by 2 is equal to $4.

9 ÷ 3 may also be written $\frac{9}{3}$, where 9 is the dividend and 3 the divisor.

52. When the divisor does not exceed 12, the operation can be performed mentally, and the process is called Short Division.

When all the different steps of the division are written, the process is called Long Division.

53. SUGGESTIONS TO THE TEACHER. — Give questions similar to the following, in order to secure facility in interpreting results and accuracy and rapidity in using the multiplication table.

Thus: (1) A product is 72; one factor is 8. Find the other.

(2) What is $36 ÷ 4? $72 ÷ $9? $72 ÷ 9? 132 ÷ 11? Associate simple practical questions with these numbers.

(3) Extend the table thus: Divide 210 by 7 ; 3500 by 5 ; 450 by 90 ; 840 by 12.

(4) Give the quotient and remainder when 86 is divided by 7 ; 93 by 12 ; 43 by 6.

(5) Reduce to the next higher denomination : 32 qt.; 40 ¢; 96 in.; 45 da.; 450 min.

(6) The unit of area is 9 sq. rd. What number expresses the ratio of the area of a field containing 270 sq. rd. to the unit of area ?

54. If 1 T. of coal costs $ 6, how many tons will $ 4764 buy ?

Here the product is 4764 ; one factor is 6, and we are required to find the other factor, which is the number of tons.

$$6)\overline{4764}$$
$$\overline{794}$$

6 divides 47 of the hundreds' unit 7 times in the hundreds' place, with a remainder 5 of the hundreds' unit ; 5 of the hundreds' unit and 6 of the tens' unit equal 56 of the tens' unit.

6 divides 56 of the tens' unit 9 times in the tens' place, with a remainder 2 of the tens' unit. 2 of the tens' unit and 4 of the one-unit, equal 24, which divided by 6 equals 4.

∴ the number of tons = 794.

Exercise 53

1. Divide each of the following numbers by 3: 5187 ; 7864 ; 1783 ; 96,231 ; 52,867 ; 84,829.

2. Divide by 4: 6552 ; 2496 ; 9897 ; 79,284 ; 70,837 ; 66,894.

3. Divide by 5: 9565 ; 3127 ; 2704 ; 33,375 ; 80,346 ; 45,404.

4. Divide by 6: 1698 ; 5934 ; 3353 ; 66,554 ; 46,893 ; 39,577.

5. Divide by 7: 5964 ; 8828 ; 2495 ; 99,573 ; 87,049 ; 98,425.

6. Reduce to weeks: 273 da.; 365 da.; 4365 da.

7. Divide by 8: 2564 ; 3683 ; 4992 ; 46,264 ; 84,364 ; 60,578.

8. Divide by 9: 3141; 6283; 6562; 15,708; 52,664; 82,315.

9. Divide by 10: 3140; 6408; 2989; 43,825; 61,413; 84,375.

10. Divide by 11: 1760; 5280; 4379; 30,005; 52,275; 65,341.

11. Divide by 12: 2736; 5592; 1875; 28,060; 96,725; 10,008.

12. Divide 54,716 by each of these numbers: 3, 6, 9, 12.

13. Divide 63,360 by each of these numbers: 5, 7, 11, 12.

14. Divide 86,468 by each of these numbers: 4, 8, 10, 12.

15. Divide 75,918 by each of these numbers: 6, 8, 9, 11.

16. How many quarts in 2 gal.? A city milk dealer sold 448 qt. of milk. How many 2 gal. cans are required to hold all the milk?

17. A dealer sold a quantity of coal for $ 6 a ton, and received for it $ 7950. How many tons did he sell?

18. A merchant sold cloth at $ 3 a yard, and received for it $ 1344. Find the number of yards.

19. A rod 540 in. long has a piece 8 in. long cut off from it, then another piece of the same length, then another, and so on. How often may this be done, and what is the length of the piece remaining at last?

20. $ 15,108 was paid for sheep at $ 6 apiece. Find the number of sheep.

21. What number must be added to 91 to make it exactly divisible by 8?

22. The expense of carpeting a room was $ 45; but if the breadth had been 3 ft. less than it was, the expense would have been $ 36. Find the breadth of the room.

23. The expense of carpeting a room was $75; but if the breadth had been 6 ft. more than it was, the expense would have been $ 105. Find the breadth of the room.

24. Find the number of strips of carpet each 3 ft. wide required to carpet a room 15 ft. wide; 21 ft. wide; 27 ft. wide; 16 ft. wide; 6 yd. wide; 8 yd. wide.

55. (1) Find the length of an oblong which contains 96 sq. in. and is 8 in. wide.
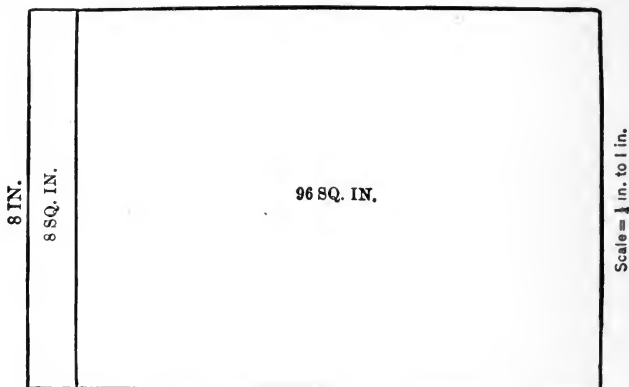
Cut off from the oblong a strip 1 ft. wide. This strip contains 8 sq. in.

Here we are given the whole quantity, or 96 sq. in., and the measuring unit, 8 sq. in. The number 12 gives the number of primary units of 1 in. contained in the length. Therefore the length is 12 in.

The area of 1 strip 1 in. wide = 8 sq. in.
The number of strips 1 in. wide = 96 sq. in. ÷ 8 sq. in. = 12.
∴ the length = 12 in.

Hence to find the measure of the length divide the measure of the area (96) by the measure of the width (8). Thus 96 ÷ 8 = 12.

(2) Find the number of yards of carpet required to carpet a room 32 ft. long and 26 ft. wide, the carpet running lengthwise, if each strip is 2 ft. wide.

The number of strips of carpet = 26 ft. ÷ 2 ft. = 13.
The length of the carpet = 13 × 32 ft. = 416 ft.
= 138 yd. 2 ft.
∴ 138 yd. 2 ft. of carpet are needed.

Make a diagram for this question on the scale of $\frac{1}{4}$ in. to 1 ft.

(3) Reduce 6498 da. to weeks.

In this question the time is expressed in terms of the unit, 1 da., and we are required to express it in terms of the unit, 1 wk. or 7 da. Dividing 6498 da. by 7 da., the result is 928 wk. 2 da.

Exercise 54

In the following exercise prove the correctness of your answers:

1. Reduce to quarts: 36 pt.; 78 pt.; 96 pt.; 65 pt.; and 257 pt.

2. Find the number of strips of carpet 2 ft. wide, required to carpet a room 16 ft. wide; 20 ft. wide; 24 ft.; 32 ft.

3. Reduce to yards: 384 ft.; 456 ft.; 723 ft.; 897 ft.; 5280 ft.

4. Find the number of strips of carpet 3 ft. wide required to carpet a room 15 ft. wide. If the room is 6 yd. long, how many yards are needed to carpet the room?

5. Find the number of strips of carpet 2 ft. wide required to carpet a room 16 ft. wide. If the room is 8 yd. long, how many yards are needed to carpet the room?

6. Make a drawing (scale 1 in. to 1 ft.) to show how many yards of carpet, 3 ft. wide, are needed to carpet a room 12 ft. wide and 15 ft. long. How many?

7. Reduce to gallons: 576 qt.; 893 qt.; 798 qt.; 962 qt.

8. Divide by 5: 475; 827; 593; 890; 646.

9. Divide by 6: 252; 435; 728; 846; 777.

10. Reduce to weeks: 245 da.; 365 da.; 678 da.; 899 da.; 987 da.

11. Reduce to pecks: 32 qt.; 892 qt.; 958 qt.; 2456 qt.; 9472 qt.

12. Reduce to square yards: 756 sq. ft.; 894 sq. ft.; 3478 sq. ft.; 9864 sq. ft.

13. Reduce to dimes: 620¢; 840¢; 729¢; 843¢; 5246¢; 8795¢.

14. Divide by 11: 451; 628; 847; 956; 8297; 7887.

15. How many dozen are there in 840 units? 957 units? 1459 units? 4596 units?

16. Reduce to feet: 459 in.; 897 in.; 2641 in.; 63,360 in.

17. Find the number of square inches in a square whose side is 5 in. Find the side of a square containing 64 sq. in.

18. Find the lengths of the sides of squares whose areas are 4, 25, 81, 36, 121, 49, 9, 100, 64, and 144 sq. in. respectively.

19. Find the length of each of these oblongs:

Area	Width	Area	Width
12 sq. in.	3 in.	45 sq. ft.	5 ft.
24 sq. in.	4 in.	240 sq. yd.	12 yd.
72 sq. in.	8 in.	144 sq. mi.	9 mi.

20. Find the length of a room 12 ft. wide, which contains 192 sq. ft.

Exercise 55

Find the quotient and remainder and prove your results correct:

1. $36 \div 2$; $48 \div 3$; $72 \div 4$; $65 \div 5$; $84 \div 6$.

2. $56 \div 7$; $96 \div 8$; $81 \div 9$; $80 \div 10$; $77 \div 11$; $48 \div 12$.

3. $249 \div 3$; $842 \div 6$; $941 \div 8$; $654 \div 8$.

4. $137 \div 2$; $439 \div 5$; $849 \div 7$; $999 \div 12$.

5. 6)743   11)682   9)847   10)895

6. 5)679   4)673   8)976   12)899

7. 7)2457   10)6430   9)1978   12)4994

8. 6)3975   2)2557   11)7381   3)4565

9. 6)4544   12)8256   5)1935   4)3191

10. 11)8149   8)7919   9)4676   7)6769

11. 4)2319   5)9847   6)1764   12)9543

12. 11)8682   8)7798   9)7992   12)63360

56. (1) A father dying left an estate valued at $48,832 to be divided equally among his wife, his two sons, and his four daughters. What was the share of each?

In this problem we are required to find the share of each, which is the unit of measure. In order to find this, we are given the value of the estate, which is the whole quantity, and one factor, which is the number of shares; viz. $1 + 2 + 4$, or 7. Therefore, dividing the whole quantity by 7, we find the share of each to be $6976.

(2) The area of the four walls of a room whose dimensions are 8 yd. and 6 yd. is 112 sq. yd. Find the height of the room.

We are here required to find the height of the room. In order to find it we are given the area. We are also given the dimensions with which we can find the perimeter of the room.

Thus we may think of the area of the walls as an oblong whose sides are the perimeter (28 yd.) and height, and area 112 sq. yd. Hence the measure of the height $= 112 \div 28 = 4$.

$$\therefore \text{ the height} = 4 \text{ yd.}$$

57. (1)

```
              245
         31)7598
              62
             ───
             139
             124
             ───
             158
             155
             ───
               3 Remainder
```

31 divides 75 of the hundreds' unit two hundred times. Put 2 as the first term in the quotient.

Multiply 31 by 2, and subtract the product 62 from 75. The remainder is 13 of the hundreds' unit. Annex the 9 tens of the dividend, making 139 tens. 31 divides 139 tens 4 tens times. Put 4 as the second term in the quotient. Multiply 31 by 4, and subtract the product 124 from 139. The remainder is 15 of the tens' unit. Annex the 8 units, making 158 units. 31 divides 158 units 5 times. Put 5 as the third term in the quotient. Multiply 31 by 5, and subtract the product 155 from 158. The remainder is 3.

What is the quotient on dividing 7598 by 31? What does it show?

(2) To prove that the answer in the last example is correct:

$$
\begin{array}{r}
245 \text{ Quotient} \\
31 \text{ Divisor} \\
\hline
245 \\
735 \\
\hline
7595 \text{ Product} \\
3 \text{ Remainder} \\
\hline
7598 \text{ Dividend}
\end{array}
$$

∴ the answer is correct.   Or thus, by division,

$$
\begin{array}{r}
31 \\
245)\overline{7598} \\
735 \\
\hline
248 \\
245 \\
\hline
3
\end{array}
$$

∴ 245 quotient and 3 remainder is the correct answer.

58. Divide 39,726 by 87.

$$
\begin{array}{r}
456 \\
87)\overline{39726} \\
348 \\
\hline
492 \\
435 \\
\hline
576 \\
522 \\
\hline
54
\end{array}
$$

In this division name the unit to which each remainder and each partial dividend belongs.

59. *Trial divisor and trial dividend.*

The work of finding the quotients can be much simplified by using the trial divisor and trial dividend.

Thus in § 57, as 31 is nearer 30 than 40, the trial divisor is 3.   Dividing 3 into the trial dividends 7, 13, and 15, the quotients are 2, 4, and 5.

In § 58, as 87 is nearer 90 than 80, the trial divisor is 9.   Dividing 9 into the trial dividends 39, 49, and 57, the quotients are 4, 5, and 6.

In general, if the divisor is 61, 62, 63, 615, 627, or 634, the trial divisor is **6.**
If the divisor is 67, 68, 69, 675, 689, or 697, the trial divisor is **7.**

If the divisor is 64, 65, or 66, the use of the trial divisor is less certain ; but the rule is to use 6 as the trial divisor for 64, 7 for 66, and either 6 or 7 for 65.

Name the trial divisors in the next exercise.

Exercise 56

Find the quotients and remainders of the following, and prove the answers to the odd numbers correct by multiplying, and the even numbers correct by dividing:

1. $712 \div 31$.	14. $7948 \div 29$.	27. $75,643 \div 97$.
2. $2341 \div 51$.	15. $8543 \div 49$.	28. $23,877 \div 24$.
3. $6287 \div 71$.	16. $9765 \div 69$.	29. $38,753 \div 34$.
4. $2195 \div 70$.	17. $8720 \div 89$.	30. $63,056 \div 64$.
5. $5894 \div 91$.	18. $8888 \div 38$.	31. $74,111 \div 25$.
6. $2068 \div 22$.	19. $9894 \div 18$.	32. $96,433 \div 75$.
7. $3572 \div 42$.	20. $9320 \div 58$.	33. $56,159 \div 95$.
8. $1576 \div 62$.	21. $16,324 \div 78$.	34. $27,766 \div 36$.
9. $8189 \div 82$.	22. $30,086 \div 98$.	35. $56,139 \div 56$.
10. $6285 \div 23$.	23. $18,874 \div 27$.	36. $78,045 \div 76$.
11. $7549 \div 53$.	24. $21,803 \div 37$.	37. $204.80 \div 32$.
12. $8476 \div 63$.	25. $36,989 \div 67$.	38. $185.76 \div 24$.
13. $9989 \div 93$.	26. $52,298 \div 87$.	39. $27.144 \div 36$.

Exercise 57

1. An Illinois farmer raised 2784 bu. of corn from 48 A. Find the number of bushels per acre.

2. A farmer raised 962 bu. of oats. If the average yield per acre was 37 bu., find the number of acres.

3. A farmer received $199.80 for his crop of wheat at 74¢ a bushel. Find the number of bushels. If he sowed 15 A. with wheat, find the average yield per acre.

4. A farmer received $158.08 for his crop of oats at 38 ¢ a bushel. If he sowed 16 A. with oats, find the average yield per acre.

5. How many pecks in 2 bu. ? 2 bu. 3 pk. ? 3 bu. 1 pk. ? 6 bu. 2 pk. ? How many pecks in 8 bu. ? 48 bu. ? 69 bu. ? 125 bu. ? 336 bu. ?

6. A shipper bought 225 bu. of apples and packed them in barrels containing 2 bu. 2 pk. Find how many barrels he used.

7. A man bought a house and lot for $4500 and paid $3750 in cash. If he paid the balance in monthly instalments of $25 each, in how many months would he pay off the debt ?

8. A miller put up 3675 lb. of flour in 49-lb. sacks and sold them at $1.10 each. Find the total selling price.

9. A grocer bought 360 lb. of whitefish in 15-lb. pails at $1.55 a pail and sold it for 16 ¢ a pound. Find his gain.

10. A grocer bought 864 lb. of tapioca in cases containing 36 one-pound packages at $3.30 a case and sold it at 12 ¢ a pound. Find his gain.

Exercise 58

1. 10,377 ÷ 13.	**9.** 37,847 ÷ 86.	**17.** 854,300 ÷ 49.
2. 29,452 ÷ 14.	**10.** 84,374 ÷ 45.	**18.** 537,047 ÷ 36.
3. 99,624 ÷ 15.	**11.** 22,158 ÷ 23.	**19.** 624,839 ÷ 75.
4. 87,643 ÷ 16.	**12.** 84,999 ÷ 69.	**20.** 802,666 ÷ 33.
5. 63,277 ÷ 17.	**13.** 15,273 ÷ 34.	**21.** 263,204 ÷ 54.
6. 64,935 ÷ 18.	**14.** 42,965 ÷ 88.	**22.** 467,989 ÷ 68.
7. 99,658 ÷ 19.	**15.** 335,296 ÷ 47.	**23.** 467,989 ÷ 67.
8. 29,943 ÷ 99.	**16.** 582,934 ÷ 56.	**24.** 633,600 ÷ 76.

25. 604,826 ÷ 29. **27.** 494,358 ÷ 65.

26. 253,789 ÷ 96. **28.** 832,016 ÷ 79.

In the following exercise, before dividing, make a careful guess as to what the quotient will be.

Exercise 59

1. $395,267 \div 105$.
2. $300,498 \div 207$.
3. $227,876 \div 121$.
4. $407,253 \div 309$.
5. $839,428 \div 224$.
6. $719,888 \div 421$.
7. $584,287 \div 593$.
8. $495,638 \div 784$.
9. $597,445 \div 656$.
10. $386,777 \div 921$.
11. $811,394 \div 675$.

12. $\$367,989 \div 476$.
13. $578,243$ cu. in. $\div 231$ cu. in.
14. $578,243$ rd. $\div 320$ rd.
15. $987,655$ cu. ft. $\div 128$ cu. ft.
16. $\$128,821 \div 360$.
17. $599,647 \div 176$.
18. $313,947$ da. $\div 365$ da.
19. $444,555 \div 366$.
20. $574,381$ A. $\div 640$ A.
21. $987,432$ sq. in. $\div 144$ sq. in.
22. $358,049 \div 528$.

Exercise 60

1. 1 sq. ft. = ? sq. in. How many square feet in 864 sq. in.? 1728 sq. in.? 3456 sq. in.? 5616 sq. in.?

2. 1 sq. in. = 640 A. How many square miles in a section of land containing 1920 A.? 4480 A.? 16,000 A.? 23,680 A.?

3. A section of land containing 16,640 A. is drained. Find its area in square miles.

4. 1 gal. contains 231 cu. in. Find the number of gallons in 1848 cu. in.; 3003 cu. in.; 8085 cu. in.

5. How many gallons will a tank hold that contains 45,276 cu. in.?

6. 1 cord contains 128 cu. in. Find the number of cords in 1152 cu. in.; 9344 cu. in.; 35,328 cu. in.

7. How many square inches in an oblong 4 in. long and 3 wide? 8 in. by 6 in.? 12 ft. by 9 ft.?

How many square miles in a section of land 12 mi. long and 6 wide? Into how many parts each containing 8 sq. mi. can it be divided?

8. A section of land in the form of an oblong is 27 mi. long and 12 mi. wide. Into how many townships each containing 36 sq. mi. can it be divided?

9. The freight rates from Chicago to Hartford, Conn., are $ .55 per 100 lb. Find the number of sacks of flour, each weighing 100 lb., that can be sent for $ 36.30. Find their weight in pounds.

10. The freight rates from Chicago to Sioux City, Iowa, are $.48 per 100 lb. Find the number of pounds in a shipment on which the freight charges were $ 35.52.

Exercise 61

1. 493,287 yd. ÷ 1760 yd.   **8.** 819,634 ÷ 4972.

2. 298,456 ft. ÷ 5280 ft.   **9.** 819,634 ÷ 3264.

3. 140,008 cu. in. ÷ 1728 cu. in.   **10.** 205,639 ÷ 7459.

4. 680,442 cu. in. ÷ 2150 cu. in.   **11.** 726,998 ÷ 9543.

5. 998,209 lb. ÷ 2240 lb.   **12.** 337,877 ÷ 9961.

6. 857,864 gr. ÷ 5760 gr.   **13.** 698,206 ÷ 8456.

7. 398,125 gr. ÷ 7000 gr.   **14.** 729,453 ÷ 5879.

Exercise 62

1. Find the wages due a workman who has worked 423 hr. at $ 1.50 a day, of 9 hr. each.

2. A man sold two city lots for $ 1875 and $ 2125 respectively, and used the money to buy a farm of 80 A. Find the cost per acre.

3. The temperatures in Chicago, March 2, 1900, are given below:

8 A.M.	 24	2 P.M.	 29
10 A.M.	 26	4 P.M.	 30
12 M.	 27	6 P.M.	 30
	8 P.M. 30		

Find the average temperature.

4. March 2, 1900, there were received in Chicago 27,200 bu. of wheat. At 340 bu. to the car, how many cars of wheat were received? At 40 cars to a train, how many train-loads would be necessary to bring in this grain?

5. At 680 bu. to a car and 42 cars to a train, how many trains are necessary to move a crop of 6,854,400 bu.?

6. The corn crop of Kansas for 1899 was 360,000,000 bu. At 400 bu. to a car and 40 cars to a train, how many trains were necessary to move this crop?

7. The area of Rhode Island is 1053 sq. mi., and in 1900 the population was 428,556. How many inhabitants were there to the square mile in 1900?

8. The area of Massachusetts is 8040 sq. mi., and in 1900 the population was 2,805,346. How many inhabitants to the square mile?

9. Turn to your geography and find the area and population of several states. Divide and find the average population per square mile. Why is this average so much greater in some states than in others?

10. For the 35 weeks ending Feb. 26, 1900, there were received in the city of St. Louis 7,945,000 bu. of wheat. Find the average number of bushels per week.

11. For the 35 weeks ending Feb. 26, 1900, there were received in Toledo 10,654,000 bu. of wheat; in Detroit, 2,359,000 bu.; in Kansas City, 13,517,000 bu. Find the average number of bushels per week received in each city.

12. How many years from the beginning of the year 1493 to the close of the year 1600?

From the beginning of 1493 to the close of 1600, $501,640,000 worth of gold was found in the world. Find the average amount each year.

13. State how to divide one number by another by long division. State how to solve each of the preceding questions in this exercise.

H

60. Divide 77.968 by 8.

8)77.968   8 is contained in 77 units 9 times, with remainder 5 units.
  9.746   8 is contained in 59 tenths 7 tenths times, with remainder
          3 tenths. 8 is contained in 36 hundredths 4 hundredths times,
with remainder 4 hundredths. 8 is contained in 48 thousandths 6 thousandths
times with no remainder.

Hence the operation of dividing a dividend containing a decimal is similar
to that of dividing when the dividend does not contain a decimal. Care
must be taken to insert the decimal point in the quotient, as in the example,
immediately after the units' figure is used in the dividend.

Exercise 63

Divide:

1. 12.6 by 6.   **4.** 239.76 by 37.   **7.** 195.2544 by 473.

2. 7.56 by 7.   **5.** 596.36 by 17.   **8.** 192.4947 by 171.

3. 16.38 by 13.   **6.** 889.92 by 72.

9. Divide 389.904 A. of land equally among 16 persons.

10. 43 bu. of wheat cost $ 37.625, find the cost of 1 bu.

11. 25 bu. of corn cost $ 8.625, find the cost of 1 bu.

12. 94 bu. of oats cost $ 22.09, find the cost of 1 bu.

13. Divide the following numbers by 10:

        47.39   543.21   62   7.64

14. State how you move the decimal point to find the quotient
on dividing a number by 10.

15. Write down the quotients obtained on dividing the follow-
ing numbers by 10:

        64.52   3.9   742.63   95.614

16. State how to find the quotient without actual division when
a decimal is divided by 10.

17. Divide by 100:

        792.6   8943.62   54.15   89467.1

18. State how to find the quotient without actual division when
a decimal is divided by 100. By 1000.

19. Write down the quotients obtained on dividing the following numbers, (a) by 10, (b) by 100 :

24.3   2163.4   59.44   743.92   84   175   242

20. Write down the quotients obtained on dividing the following numbers, (a) by 1000, (b) by 100 :

831.5   4926.4   2235   3852.1   4829   384

Miscellaneous Exercise 64

1. Divide a line 12 in. long into parts each 2 in. long. How many parts ?   12 in. ÷ 2 in. = ?

2. Divide a line 1 ft. long into parts each $\frac{1}{4}$ ft. long. How many parts ?   1 ft. ÷ $\frac{1}{4}$ ft. = ?

3. Make drawings to show the value of 1 ft. ÷ $\frac{1}{2}$ ft. ; 1 ft. ÷ $\frac{1}{3}$ ft. ; 1 ft. ÷ $\frac{1}{6}$ ft.

4. Into how many lots, each containing $\frac{1}{4}$ A., can you divide 1 A. ?

5. A society divided 1 T. of coal among several families, giving to each $\frac{1}{6}$ T.   How many families were there ?

6. Into how many parts, each $\frac{1}{4}$ ft., can you divide a line 1 ft. long ? 3 ft. long ? 5 ft. long ?

5 ft. ÷ $\frac{1}{4}$ ft. = ?	4 gal. ÷ $\frac{1}{4}$ gal. = ?	9 pk. ÷ $\frac{1}{2}$ pk. = ?
8 ft. ÷ $\frac{1}{3}$ ft. = ?	6 bu. ÷ $\frac{1}{8}$ bu. = ?	11 lb. ÷ $\frac{1}{6}$ lb. = ?
7 yd. ÷ $\frac{1}{5}$ yd. = ?	12 da. ÷ $\frac{1}{3}$ da. = ?	12 mi. ÷ $\frac{1}{4}$ mi. = ?

7. A lady has 6 gal. of fruit in cans, each containing $\frac{1}{8}$ gal. Find the number of cans.

8. Find the number of cans in each case: 6 lb. beef in $\frac{1}{2}$ lb. cans ; 8 lb. extract of beef in $\frac{1}{4}$ lb. cans ; 12 lb. extract in $\frac{1}{8}$ lb. cans.

9. Make a drawing (scale 3 ft. = 1 in.) to show that 5 strips of carpet each 3 ft. wide will be needed to carpet a room 15 ft. wide ?

10. Find the number of strips of carpet each 3 ft. wide required to carpet a room 18 ft. wide ; 24 ft. wide ; 26 ft. wide ; 28 ft. wide. If the room is in each case 10 yd. long, how many yards of carpet would be required to carpet it ?

11. How many yards of carpet 3 ft. wide are needed to carpet a room 12 ft. wide and 5 yd. long ?

12. Find the number of yards of carpet 3 ft. wide, needed to carpet each of the following rooms :

Width	Length	Width	Length
15 ft.	6 yd.	20 ft.	9 yd.
18 ft.	8 yd.	18 ft.	24 ft.

13. Make a drawing to show how many widths of matting $\frac{1}{2}$ yd. wide are needed to cover a room 6 yd. wide. How many ? This room is 8 yd. long, how many yards of matting are needed ?

14. Find the number of yards of matting, $\frac{1}{2}$ yd. wide, required to cover the floors of each of the following rooms :

Width	Length	Width	Length
5 yd.	6 yd.	12 yd.	18 yd.
6 yd.	8 yd.	16 yd.	18 yd.

Find the cost of the matting for the last room at 24 ¢ a yard.

15. Find the cost of the matting, $\frac{1}{2}$ yd. wide, needed to cover the floors of a hall 24 yd. wide and 30 yd. long, the matting costing 28 ¢ a yard.

16. If it cost $ 49.60 to carpet a room with carpet 48 ¢ a yard, what would it cost with carpet at 60 ¢ a yard ?

17. How many ounces in 1 lb. of sugar ? Of tea ? How many pounds in 32 oz. ? 64 oz.? 96 oz. ? 270 oz. ? 736 oz.?

18. How many pounds and ounces in 18 oz. ? 24 oz. ? 56 oz. ? 75 oz. ? 300 oz. ? 420 oz. ? 648 oz. ?

19. How many pounds and ounces in 1 yd. of carpet weighing 17 oz. to the yard ? 19 oz. ? 25 oz. ? 35 oz. ? 45 oz. ?

20. Find the number of pounds that 48 yd. of carpet will weigh at 29 oz. per yard.

21. I bought 84 yd. of Wilton carpet, weighing 45 oz. per yard. Find its weight.

22. The following live stock were received in one week in Chicago :

	CATTLE	CALVES	HOGS	SHEEP
Wednesday, Feb. 7	17,261	249	47,019	12,270
Thursday, Feb. 8	8,775	239	36,089	8,273
Friday, Feb. 9	1,569	212	24,288	4,529
Saturday, Feb. 10	159	6	19,224	1,505
Monday, Feb. 12	19,831	148	42,705	16,883
Tuesday, Feb. 13	3,207	807	33,991	13,275
Wednesday, Feb. 14 . . .	16,000	500	36,000	17,000

Find the total number of each kind.

23. Find from the following table the increase or decrease in the price in each of the following articles in July, 1899:

	AVERAGE WHOLESALE PRICE, JANUARY, 1890	AVERAGE WHOLESALE PRICE, JULY, 1899
Flour, per barrel	$ 5.50	$ 4.50
Lard, per pound	.056	.049
Beef loins, per pound	.19	.21
Beef ribs, per pound	.16	.17
Beef, salt, per barrel	9.75	9.00
Mess pork, per barrel	10.00	8.75
Bacon, per pound	.056	.052
Hams, per pound	.11¼	.10¾
Salt, per barrel	.65	.65
Sugar, granulated, per pound	.065	.052
Thread, spool	.032	.031
Candles, per pound	.12	.12
Coal, anthracite, per ton	4.20	3.85
Coal, bituminous, per ton	3.30	2.20
Matches, parlor, per gross	4.75	3.75
Nails, per keg	2.90	2.70
Cement, per barrel	.90	.80
Linseed oil, per gallon	.60	.39
Starch, silver gloss, per pound	.063	.058
Leather, per pound	.19	.22

24. In 1898 the cotton crop of Missouri was 1,247,128 bales, and the selling price $ 25 a bale. In the next year it was 1,212,-200 bales, and selling price $ 11.25 greater than in 1898. Find the increase in the value of the crop.

25. The fire department of the city of Chicago asked for the following sums for new buildings in 1902: $ 18,880; $ 12,700; $ 20,000; $ 22,100; $ 27,800; $ 23,400; $ 19,200. Find the total amount.

26. The following table shows the money allowed the fire department of the city of Chicago for 1901 and the estimate for 1902:

	1901	1902
General pay roll	$ 1,427,664	$ 1,445,984
Fire alarm pay roll	27,080	24,230
Repairs		29,690
Materials	40,000	40,000
Repairs, buildings, and boats	25,000	35,000
Supplies	148,000	203,000
Contingent fund	1,200	1,200
Rent	11,856	15,000
New buildings	18,382	366,400

Find the total amount in each case.

27. A Chicago daily paper states that there is a net increase of $ 466,000 in the estimate of the fire department for 1902 over 1901. Show that there is an error of $ 4678 in this statement.

28. 1 kilogram = 220 lb. 1 bu. of wheat weighs 60 lb. The duty on wheat imported into Germany is $ 1.19 per kilogram. Find the duty on 550 bu. of wheat.

CHAPTER VIII

COMPARISON OF NUMBERS

61. If $4 be multiplied in turn by the numbers 3 and 5, the products will be $12 and $20. Hence, taking $4 as the unit of measure, and noting how often the unit is repeated to produce $12 and $20, we find that these quantities are represented by 3 and 5 units respectively. Therefore $\frac{3}{5}$ is the ratio of $12 to $20.

What is the ratio of $8 to $20? Of $15 to $25? Of $25 to $15?

Exercise 65

1. What is the largest unit that will measure $24 and $30? How often in each case? What is the ratio of $24 to $30?

2. What is the largest unit that will measure $20 and $35? What is the ratio of $20 to $35? Of $35 to $20?

3. What is the ratio of $24 to $36? $36 to $24? $27 to $30? $30 to $27? $55 to $60? $60 to $36?

4. What is the ratio of 14 bu. to 35 bu.? 49 bu. to 63 bu.? 32 lb. to 56 lb.? 54 min. to 36 min.? 55 T. to 66 T.?

5. What is the ratio of the value of 48 bbl. of flour to 60 bbl. of flour? Of 24 bu. of oats to 42 bu. of oats? Of 70 bu. of wheat to 49 bu. of wheat?

6. What is the ratio of 36 lb. to 45 lb.? 45 lb. of tea cost $30; what part of $30 will 36 lb. cost? How much?

7. What is the ratio of 14 to 21? If 21 T. of hay cost $180, what will 14 T. cost?

8. If 36 yd. of cloth cost $ 28, what will 27 yd. of the same kind cost ?

9. If 27 T. of coal cost $ 180, what will 36 T. cost ?

10. What is the ratio of 9 to 12 ?   Of 12 to 9 ?

9 men can do a piece of work in 24 da.   Should you multiply 24 da. by $\frac{3}{4}$ or by $\frac{4}{3}$ to find how long it would take 12 men to do the same piece of work ?   How many days ?

11. 12 men can do a piece of work in 18 da.   Should you multiply 18 da. by $\frac{3}{4}$ or $\frac{4}{3}$ to find how long it would take 9 men to do the same piece of work ?   How many days ?

12. If 15 men can dig a ditch in 12 da., how long will it take 20 men to dig a ditch of the same size ?

62. (1) What is the ratio of $ 20 to $ 45 ?

> Let $ 5 be taken as the unit of measure.
> Then $ 20 is measured by 4 times the unit.
> And $ 45 is measured by 9 times the unit.
> ∴ $ 20 is $\frac{4}{9}$ of $ 45.

(2) If 22 yd. of cloth cost $16, what will 33 yd. cost at the same rate ?

> Take 11 yd. as the unit of length.
> Then 33 yd. = $\frac{3}{2}$ of 22 yd.
> ∴ 33 yd. cost $\frac{3}{2}$ of $ 16 or $ 24.

Exercise 66

1. What is the ratio of $ 18 to $ 24 ? $ 35 to $ 55 ? $ 28 to $ 63 ?

2. What is the ratio of 16 hr. to 56 hr. ?   72 hr. to 45 hr. ?

3. What is the ratio of 60 mi. to 25 mi. ?   99 A. to 55 A. ?

4. If 45 cd. of wood cost $ 162, what will 20 cd. cost ?

5. If 21 T. of hay cost $ 174, what will 70 T. cost ?

6. If 36 yd. of cloth cost $ 42, what will 24 yd. cost at the same rate ?

7. A miller sold 35 bbl. of flour for $ 126. How much will he receive for 15 bbl. at the same rate ?

8. A train runs 32 mi. in 48 min. At the same rate, what distance will it run in 54 min. ?

9. If 84 men can dig a trench in 36 da., how long will it take 108 men to dig a trench of the same size ?

10. If 88 horses eat 33 bu. of oats in 1 da., how many bushels will 48 horses eat in the same time ?

11. If 45 men can reap a field of 36 A. in a certain time, how many acres would 30 men reap in the same time ?

12. A bankrupt pays $ 35 out of every $ 63 owed. How much shall I receive if he owes me $ 81 ?

13. If 32 T. of coal cost $ 184, what will 88 T. cost at the same rate ?

14. If 56 men can do a piece of work in 21 da., how long will it take 24 men to do it ?

15. How many pounds of tea can be bought for $ 56, at the rate of $ 16 for 34 lb. ?

16. Tea is bought at 72 ¢ a pound, and sold for 84 ¢ a pound. The gain is what part of the cost price ?

17. The cost of fencing 132 rd. of railway is $ 117. What is the cost of fencing 88 rd. ?

18. If a 12-qt. pail is just filled by 6 units of milk, how many quarts are there in a pail which will hold 5 units ? What is the unit ?

19. Fifty-four minutes are represented by 9 units of time. How many minutes are there in 7 units ?

20. If 63 men can dig a trench in 16 da., how long will it take 18 men to dig it ?

SQUARE ROOT

63. Show by drawing a square whose side is 3 in. that a 3-in. square contains 9 sq. in. How many square inches does a 4-in. square contain? A 5-in. square?

64. The product of 3 and 3 is 9 ; of 5 and 5 is 25. The squares whose sides measure 3 and 5 units of length contain 9 and 25 units of square measure. We say that 9 is the square of 3 and that 25 is the square of 5; that 3 is the square root of 9 and 5 the square root of 25.

The square of 3 is written 3^2, and the square root of 9 is indicated thus: $\sqrt{9}$.

2 is called the **Exponent,** and $\sqrt{}$ the **Radical Sign.** 3^2 is also called the second **Power** of 3.

65. The **Square** of a number is the product found by multiplying the number by itself.

Thus the squares of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,
are   1, 4, 9, 16, 25, 36, 49, 64, 81, 100.

66. The square root of a number is one of its two equal factors.

Thus the square roots of 1, 4, 9, 16, 25, 36, 49, 64, 81, 100,
are   1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

* Note. — If preferred, this chapter may be postponed until Chapter XVII is reached.

How many digits in the square root of a number containing one or two digits?

67. Pupils should memorize the tables in the two preceding paragraphs and be able to answer instantly such questions as the following:

What is the first figure in the square root of 27? 58? 76? 43? 80?

Exercise 67

Write the following products as powers:

1. 5×5.   2. 7×7.   3. 24×24.

Write the following powers as products and find their values:

4. 8^2.   6. 46^2.   8. $\left(\frac{7}{8}\right)^2$.

5. 13^2.   7. $\left(\frac{3}{4}\right)^2$.   9. $\left(\frac{25}{34}\right)^2$.

Prove the following statements by multiplication:

10. $\sqrt{49} = 7 \ (7 \times 7 = ?)$.   14. $\sqrt{324} = 18$.   18. $\sqrt{\frac{289}{361}} = \frac{17}{19}$.

11. $\sqrt{169} = 13$.   15. $\sqrt{961} = 31$.   19. $\sqrt{\frac{1369}{1849}} = \frac{37}{43}$.

12. $\sqrt{729} = 27$.   16. $\sqrt{2025} = 45$.   20. $\sqrt{6\frac{1}{4}} = 2\frac{1}{2}$.

13. $\sqrt{6.25} = 2.5$.   17. $\sqrt{23.04} = 4.8$.   21. $\sqrt{53\frac{7}{9}} = 7\frac{1}{3}$.

Exercise 68

1. The square of a number of one digit contains how many digits? (See § 65.)

2. Find the squares of the numbers from 10 to 20 and commit the results to memory.

3. Find the squares of 25, 28, 54, 75, and 99.

4. From the results obtained in examples 2 and 3, state how many digits are found in the square of a number of 2 digits.

5. Find the squares of 175, 199, 246, 402, 814, 999.

6. From the results in Example 5 state how many digits are found in the square of a number of 3 digits.

7. Judging from the results obtained in Examples 2, 3, and 5, state the number of digits in the square root of a square number that contains 3 digits; 4 digits; 6 digits; 7 digits; 8 digits.

8. How many digits in the square correspond to **1** digit in the square root?

9. What is the square root of 400? Of 900?

10. The square root of 625 lies between what two numbers?

11. Find the square of 10, 20, 30, 40, 50, 60, 70, 80, and 90.

12. Between what numbers does the square root of **1225** lie? Of 4225? Of 2304? Of 8281? Of 8704?

13. What is the square of 200? Of 300?

14. The square root of 71,289 lies between what two numbers?

15. Find the squares of 100, 200, 300, etc., up to 900.

16. Between what two numbers does the square root of **271,441** lie? Of 795,664?

68. The following explanation will make clear the method of finding the square root of a number of 3 or 4 digits.

$$\begin{array}{r} 24 \\ 24 \\ \hline 16 \\ 80 \\ 80 \\ 400 \\ \hline 576 \end{array}$$

Thus 24, which is made up of two parts, 20 and 4, has for its square 576, which is seen to be made up of 400, the square of 20; 16, the square of 4; and twice the product of 20 and 4.

Now to recover 24 from 576, we know that its hundreds' digit 5, showing that the number is between 400 and 900, gives the tens' digit of the root, so that we know one of the parts of the root, viz., 20. The square of 20 is 400, and the rest of the given number, 176, must be 2 times 20, multiplied by the other part, together with the square of the other part. Multiplying 20 by 2, and using the product 40 as a divisor with 176 as dividend, we get the quotient 4. Multiplying 40 by 4 and subtracting the product 160 from 176, we get the remainder 16, which is the square of 4. Therefore 20 + 4 or 24 is the square root of 576.

The work of extracting the square root may be simplified by leaving out the unnecessary zeros, thus:

$$\begin{array}{r} 20\ \big|\ \overline{576}(20+4 \\ 400 \\ 40\ \big|\ \overline{176} \\ 160 \\ 4\ \big|\ \overline{16} \\ 16 \end{array}$$

$$\begin{array}{r} 2\ \big|\ \overline{5'76}(24 \\ 4 \\ 44\ \big|\ \overline{176} \\ 176 \end{array}$$

69. (1) The method of discovering the square root of a number of 5 or 6 digits is similar to that for finding the square root of numbers of 3 or 4 digits.

```
  246
  246
  ────
   36        Thus 246, which is made up of two parts, 240 and 6, has for
 1440     its square 60,516, which is seen to be made up of 57,600, the square
 1440     of 240 ; 36, the square of 6 ; and twice the product of 240 and 6.
57600
─────
60516
```

```
  2 | 6'05'16(240 + 6
    |   4
 44 | 205
    | 176
480 | 2916
    | 2880
  6 |   36
    |   36
```

(2) Hence proceeding with 605 as in § 68 with 576, we get in the square root 24 tens or 240. Multiplying 240 by 2 and using the product 480 as a divisor with 2916 as a dividend, the quotient is found to be 6.

Multiplying the 480 by 6, and subtracting the product 2880 from 2916 we have the remainder 36, which is the square of 6. Therefore 240 + 6 or 246 is the square root of 60516.

```
  2 | 6'05'16(246
    |   4
 44 | 205
    | 176
486 | 2916
    | 2916
```

Leaving out the unnecessary zeros, the work may be simplified as in the contracted form.

The number whose square root is to be extracted should be pointed off into groups of two figures, as in the preceding examples, beginning with the units' figure.

70. To find the square root of $\frac{289}{625}$.

These roots of 289 and 625 are found to be 17 and 25.
Hence $\sqrt{\frac{289}{625}} = \frac{17}{25}$.

Exercise 69

Find the square root and prove your answer correct:

1. 324.	**5.** 3025.	**9.** 71,824.	**13.** $\frac{64}{289}$.
2. 529.	**6.** 6889.	**10.** 101,761.	**14.** $\frac{729}{2704}$.
3. 841.	**7.** 4096.	**11.** 465,124.	**15.** $30\frac{1}{4}$.
4. 1156.	**8.** 9409.	**12.** 998,001.	**16.** $22\frac{9}{16}$.

17. Compare the process of extracting the square root of a number with that of long division.

Find the square root of:

18. 3136.	**20.** .4096.	**22.** 219.04.	**24.** 1176.49.
19. 72.25.	**21.** 7.8961.	**23.** 29.9209.	**25.** .469225.

71. (1) Find the side of a square containing 4225 sq. in.

The square is measured by 4225 units of 1 sq. in. ; therefore the side is measured by $\sqrt{4225}$, or 65 units of 1 in., *i.e.* the length of the square is 65 in.

(2) The sides of a rectangular field containing 735 sq. rd. are as 3 to 5. Find their length.

The field contains 3×5, or 15 units of area.

The area of one unit = 735 sq. rd. $\div$ 15, or 49 sq. rd.

The side of a square containing 49 sq. rd. = 7 rd.

∴ the sides of the field are 3×7 rd., or 21 rd., and 5×7 rd., or 35 rd.

Draw a rectangle whose sides are as 3 to 5, and divide it into 15 squares to illustrate this example.

Statement of Solution

First find the number of units of area in the field. (How many?) Divide this number into the area, and find the unit of area. (What is it?) Then find the unit of length, which is the length of the side of the unit of area. (What is it?)

Multiply the unit of length by 3 and 5 respectively, to find the sides of the field.

(3) *To find the area of a triangle, the lengths of whose sides are given, find one-half the sum of the number of units of length in the sides. Subtract from this the number of units of length in each side separately. Find the product of these four results. The square root of this product is the number of units of area in the given triangle.*

Find the area of a triangle whose sides are 5 in., 12 in., and 13 in. respectively.

The sum = $5 + 12 + 13 = 30$.

One-half this sum = 15.

$$15 - 5 = 10. \qquad 15 \times 10 \times 3 \times 2 = 900.$$
$$15 - 12 = 3. \qquad\qquad \sqrt{900} = 30.$$
$$15 - 13 = 2. \qquad \therefore \text{ the area of the triangle} = 30 \text{ sq. in.}$$

Exercise 70

1. Find the length of the side of a square park containing 5625 sq. yd.

2. How many rods are there in the side of a square field containing 1156 sq. rd.?

3. How many square inches in an oblong 9 in. wide and 16 in. long? How many inches in a square of the same size?

4. A flower bed is 9 ft. long and 4 ft. wide. Find the side of a square flower bed of the same size.

5. Find the side of a square that shall contain as many square feet as an oblong 264 ft. long and 66 ft. wide.

6. 7 sq. yd. 1 sq. ft. = ? sq. ft.

Find the side of a square containing 7 sq. yd. 1 sq. ft.

Find the side of a square containing 13 sq. yd. 4 sq. ft.

7. Find the length of the side of an enclosure in the form of a square containing 386 sq. yd. 7 sq. ft.

8. A garden bed contains 48 sq. ft., and it is 3 times as long as it is wide. Make a drawing of the bed. Find its length and width.

9. A park contains 9408 sq. yd., and it is 3 times as long as it is wide. Find its length and width.

10. A boy bought a number of oranges, paying as many cents for each orange as there were oranges. All cost 25 ¢. How many oranges did he buy, and at what price for one orange?

11. A merchant bought a number of yards of cloth, paying as many cents for each yard as there were yards. The entire cost was $56.25. How many yards did he buy, and at what price per yard?

12. What is one of the two equal factors of 36? Of 64? Of 121?

13. What is one of the two equal factors of 24,336?

14. A rectangular field, the sides of which are in the ratio of 4 to 7, contains 4032 sq. rd. Find the length of each side.

15. Find the perimeter of the field in the previous example and the cost of fencing the field at $ 4 a rod.

16. A body of soldiers in column form 567 ranks, 7 abreast. If they were drawn up in solid square, how many would there be on each side ?

17. Find the side of a square which is equal in area to the sum of the area of two squares, the sides of which are 6 in. and 8 in. long.

18. Draw two lines respectively 6 and 8 in. long, at right angles. Join their extremities by a straight line. Measure this line and show that it is equal to the side of the square found in question **17.**

19. Work examples similar to **17** and **18**, using the following as the lengths of the sides of the smaller squares : 3 in., 4 in. ; 5 in., 12 in. ; 8 in., 15 in.

20. From the preceding three questions make a rule showing how to find the length of the hypotenuse of a right triangle when the lengths of the other two sides are known.

21. What is the hypotenuse of a right triangle whose sides are 21 ft. and 28 ft. ? 15 ft., 36 ft. ? 56 ft., 105 ft. ?

22. Find the side of a square equal in area to the difference of the area of the two squares whose sides are 41 ft. and 9 ft.

23. What is the altitude of a right triangle whose hypotenuse and base are 34 ft., 16 ft. ? 205 ft., 45 ft. ? 136 ft., 64 ft. ?

24. The top of a ladder rests against the side of a building 84 ft. from the ground, and its foot is 35 ft. from the wall. Find the length of the ladder.

25. A ladder 51 ft. long stands close against a building. How far must the foot be drawn out that the top may be lowered 6 ft. ?

26. Find the diagonal of a rectangular field whose sides are 144 yd. and 60 yd.

27. Find the side of a square equal in area to a rectangle whose sides are 148 yd. and 333 yd. Find the difference between the perimeters of the rectangle and square.

28. Find the area of the largest rectangle which can be enclosed by a line 36 in. long.

29. A field in the form of a rectangle whose sides are as 3 to 4 contains 432 sq. rd. How much do I save by crossing along its diagonal instead of going along its two sides?

30. The sides of a triangle are 6 in., 8 in., and 10 in. Find its area. Draw the triangle. Does your result seem to correspond to the drawing?

31. The sides of a triangle are 8 in., 15 in., and 17 in. Find its area.

32. Find the area of a triangle whose sides are 12 in., 16 in., and 20 in.

3. Find the areas of the triangles whose sides are:

21 in., 28 in., 35 in.	4 in., 7.5 in., 8.5 in.
24 in., 45 in., 51 in.	3 in., 2.4 in., 1.8 in.
9 in., 40 in., 41 in.	4 in., 9.6 in., 10.4 in.

Miscellaneous Exercise 71

1. 3 ft. $\div \frac{1}{4}$ ft. = ?   4 lb. $\div \frac{1}{5}$ lb. = ?   6 T. $\div \frac{1}{3}$ T. = ?

2. Draw a line 4 ft. long, and show that it can be divided into 6 parts, each $\frac{2}{3}$ ft. long. 4 ft. $\div \frac{2}{3}$ ft. = ?

3. Make drawings to show that:

(1) 6 ft. $\div \frac{2}{3}$ ft. = 9.   (2) 6 ft. $\div \frac{3}{4}$ ft. = 8.

4. Into how many parts, each $\frac{1}{4}$ ft. long, can you divide a line 6 ft. long? How many of these 24 parts make $\frac{3}{4}$ ft.? How many parts, each equal to $\frac{3}{4}$ ft., can you count in these 24 parts?

5. In the second part of example 3, by what do you multiply 6 to get 24? By what do you divide to get 8? How, then, do you obtain the result on dividing 6 by $\frac{3}{4}$? Is it correct to divide by 3 first, and then multiply by 4? Which gives the smaller numbers to work with?

6. How do you divide 6 by $\frac{2}{3}$?   8 by $\frac{4}{5}$?   How do you divide any whole number by a fraction ? $8 \div \frac{4}{5} = 8 \times \frac{5}{4} = 10.$

7. Find the quotient in each of the following:

4 ft. $\div \frac{2}{3}$ ft.	36 A. $\div \frac{9}{8}$ A.	$63 \div 1\frac{1}{8}$
9 yd. $\div \frac{3}{5}$ yd.	32 A. $\div \frac{8}{9}$ A.	$51 \div 4\frac{1}{4}$
12 lb. $\div \frac{6}{5}$ lb.	42 yr. $\div 2\frac{1}{3}$ yr.	$35 \div 1\frac{3}{4}$
10 lb. $\div \frac{5}{6}$ lb.	72 yr. $\div 2\frac{1}{4}$ yr.	$98 \div 8\frac{1}{6}$
14 T. $\div \frac{7}{8}$ T.	84 yr $\div 2\frac{5}{8}$ yr.	$62 \div 4\frac{3}{7}$

8. How many yards of cloth will cost $6 at $\$\frac{3}{4}$ a yard?   At $\$\frac{2}{3}$ a yard?   At $\$1\frac{1}{5}$ a yard?

9. Make a drawing to show how many strips of carpet $\frac{3}{4}$ yd. wide are needed to carpet a room 6 yd. wide and 8 yd. long. (Scale 1 in. to 1 yd.)   How many strips?   How many yards?

10. How many strips of carpet $\frac{3}{4}$ yd. wide are needed to carpet a room 12 yd. wide?   If this room is 16 yd. long, how many yards of carpet are needed to carpet the room?

11. How many yards of carpet $\frac{3}{4}$ yd. wide are needed to carpet a room 9 yd. wide and 12 yd. long?   Find its cost at 75 ¢ a yard.

12. How many square yards in a piece of floor oil cloth, weighing 14 lb., at $3\frac{1}{2}$ lb. a square yard?

13. A wool carpet weighs $1\frac{1}{8}$ lb. to the yard.   How many yards in a roll weighing 18 lb.?

An ingrain carpet weighs $1\frac{1}{2}$ lb. to the yard.   How many yards in a roll weighing 24 lb.?

14. How many silver spoons weighing $1\frac{1}{4}$ lb. per dozen will weigh 10 lb.?

How many silver spoons weighing $1\frac{1}{2}$ lb. per dozen will weigh 12 lb.?

15. A double roll of wall paper weighs $1\frac{3}{4}$ lb.   How many double rolls weigh 21 lb.?   Find its value at $.45 a double roll.

16. The strength of the American and British navies is given below. Find the total number of ships in each case.

	COMPLETED		UNDER CONSTRUCTION	
	U. S.	Eng.	U. S.	Eng.
Battleships	5	52	8	12
Armored cruisers	2	18	—	8
Protected cruisers	14	95	1	24
Unprotected cruisers . . .	10	16	—	—
Coast-defence ships . . .	20	15	—	—
Torpedo vessels	—	35	—	—
Ships for special purposes .	1	3	—	—
Torpedo-boat destroyers . .	—	50	20	46
Torpedo boats	2	98	22	—

17. Find the increase or decrease in the relative price of the following on July 1, 1899:

	RELATIVE PRICE JAN. 1, 1890	RELATIVE PRICE JULY 1, 1899		RELATIVE PRICE JAN. 1, 1890	RELATIVE PRICE JULY 1, 1899
Beans . . .	86.9	64.5	Beef	97.5	105.7
Bread	97.1	97.8	Salt meat . . .	93.8	82.8
Butter	106.0	87.5	Bacon	89.5	82.5
Cheese	107.1	86.9	Ham	100.5	101.8
Coffee	101.8	36.2	Mutton	104.7	78.5
Eggs	107.4	70.9	Milk	105.5	88.0
Fish	99.9	90.7	Molasses . . .	114.6	99.6
Flour	92.7	75.8	Rice	94.0	105.7
Fruit (preserved)	93.8	73.1	Salt	94.3	90.4
Lard	92.6	80.5	Sugar	117.0	102.6
Cornmeal . . .	82.0	66.5			

18. Find the cost of 562 T. of hay at $11.50 a ton.

19. How many pounds does 1 bu. of wheat weigh? What is the ratio of its weight to 100 lb.? If the freight rate on wheat is 10¢ per 100 lb., what is the rate per bushel?

20. If the freight rate on wheat is 15¢ per 100 lb., what is the cost of shipping 3000 bu. of wheat?

21. Find quotients and remainders:

$82963 \div 48$	$60935 \div 66$	$889966 \div 96$
$70400 \div 24$	$10004 \div 82$	$384729 \div 99$

22. Find the square root of:

576	103041	692224
1849	10.3041	6922.24

23. Find the square root of: $\frac{16}{25}$; $\frac{225}{289}$; $\frac{49}{9}$; $5\frac{4}{9}$; $7\frac{1}{9}$; $16\frac{1}{4}$.

24. Find the side of a square lot that shall contain as many square feet as a lot 150 ft. long and 96 ft. wide.

CHAPTER X

GREATEST COMMON MEASURE AND LEAST COMMON MULTIPLE

72. Name all the units of length which will exactly measure 15 in.

They are 1 in., 3 in., 5 in., and 15 in.

73. Find all the different units of length that will exactly measure 12 ft. and 18 ft.

The measures of 12 ft. are 1, 2, 3, 4, 6, and 12 ft.

The measures of 18 ft. are 1, 2, 3, 6, 9, and 18 ft.

It is evident that all the *common* measures of 12 ft. and 18 ft. are 1, 2, 3, and 6 ft., and that the *greatest* common measure is 6 ft.

A **Common Measure** of two or more quantities is a unit that will exactly divide each of them.

The **Greatest Common Measure** (G. C. M.) of two or more quantities is the largest unit which will exactly divide each of them.

For convenience we speak of the common measure or the greatest common measure of two or more numbers.

74. Express the ratio of 54 ft. to 72 ft. in terms of the smallest numbers possible.

The ratio of 54 ft. to 72 ft. $= \dfrac{54}{72} = \dfrac{9}{12} = \dfrac{3}{4}$, dividing both terms of the ratio by 6 and then by 3.

Exercise 72

Express the ratios of the following quantities in terms of the smallest numbers possible:

1. Of 16 ft. to 48 ft.
2. Of $ 60 to $ 90.
3. Of 35 mi. to 42 mi.
4. Of 42 mi. to 105 mi.

5. Of 54 bu. to 81 bu.
6. Of 96 yr. to 144 yr.
7. Of 210 T. to 126 T.
8. Of 240 T. to 180 T.

Find the ratio of:

9. 20 to 24.
10. 45 to 60.
11. 56 to 63.
12. 50 to 75.

13. 90 to 108.
14. 64 to 24.
15. 125 to 75.
16. 168 to 210.

17. 36 to 60.
18. 64 to 80.
19. 189 to 168.
20. 42 to 182.

21. Find all the measures that can be used to measure the capacity of each of two baskets containing 20 qt. and 32 qt.

22. Find the lengths of the two longest boards that can be used to build a fence around a garden 30 ft. long and 24 ft. wide.

23. A farmer has 66 bu. of corn and 90 bu. of wheat, which he wishes to put into sacks of equal size, without mixing the two kinds of grain. How many bushels must each sack contain in order to be as large as possible?

PRIME NUMBERS

75. A **Prime Number** is one that can be divided only by unity and itself, as 5, 11, and 13.

Select the prime numbers: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.

The *prime factors* of a number are the prime numbers which when multiplied together give it; thus, 3, 3, and 5 are the prime factors of 45.

4 is the second power of 2, and 8 the third power. $4 = 2^2$; $8 = 2 \times 2 \times 2 = 2^3$; $9 = 3^2$; $27 = 3 \times 3 \times 3 = 3^3$; $16 = 2^4$.

76. Find the prime factors of 168.

$$\begin{array}{c|c} 8 & 168 \\ 3 & \overline{21} \\ \hline & 7 \end{array}$$

$\therefore 168 = 8 \times 3 \times 7 = 2^3 \times 3 \times 7.$

A number is exactly divisible:

By 2 if its right-hand figure is an even number or zero.

By 5 if its right-hand figure is 0 or 5.

By 10 if the right-hand figure is 0.

Select the numbers divisible by 2 or 5 or 10: 35, 24, 17, 40, 64, 85, 90, 180, 245.

Exercise 73

1. Name the even numbers from 1 to 50.

2. Name the odd numbers from 1 to 51.

3. Name the prime numbers from 1 to 100 and commit them to memory.

Find the prime factors of:

4. 30; 36; 56; 48; 84; 66; 196; 195; 231.

5. 86; 147; 104; 132; 78; 135; 342; 255.

6. 336; 408; 372; 564; 342; 484; 375; 861.

7. What prime factors are common to 30 and 36? 66 and 132? 147 and 336? 135 and 255?

8. Find the prime factors of all numbers from 1 to 100.

77. (1) A man owns a rectangular lot 210 ft. long and 144 ft. wide. Find the length of the longest board that can be used to fence it.

We are required to find the length of the longest board, *i.e.* the G. C. M. of 144 ft. and 210 ft.

$$144 = 9 \times 16 = 3^2 \times 2^4.$$
$$210 = 10 \times 21 = 2 \times 5 \times 3 \times 7.$$

Thus, the G. C. M. of 144 and 210 $= 2 \times 3 = 6$.

∴ the length of the longest board is 6 ft.

To prove the answer correct :

The number of boards required for the length $= 210 \div 6 = 35$.

The number of boards required for the width $= 144 \div 6 = 24$.

35 and 24 have no common measure except unity, ∴ 6 ft. is the correct answer.

(2) A certain school consists of 132 pupils in the high school, 154 in the grammar, and 198 in the primary grades. If each group is divided into sections of the same number containing as many pupils as possible, how many pupils will there be in each section?

We are required to find the number of pupils in each section, *i.e.* the G. C. M. of 132, 154, and 198 pupils.

$$\begin{array}{r|rrr} 2 & 132 & 154 & 198 \\ \hline 11 & 66 & 77 & 99 \\ \hline & 6 & 7 & 9 \end{array}$$

Since 2 and 11 are the only common factors, the G. C. M. of 132, 154, and 198 is 2×11, or 22.

∴ each section will contain 22 pupils.

Exercise 73 (a)

1. Draw two lines, one 15 in. and the other 21 in. long. What is the longest line that can be used to measure both lines?

2. What is the longest line that will exactly measure two lines 28 and 32 in. long?

3. What is the longest line that will exactly measure three lines respectively 20, 30, and 45 in. long?

4. What is the largest unit of capacity that can be used to measure the quantity of oil in each of two vessels, one containing 16 qt. and the other 36 qt.?

5. What is the largest unit of money that can be used to pay each of two debts, one of $45 and the other of $80?

6. Find the G. C. M. of 24 and 42; 30 and 45; 36 and 54.

7. Prove that your answer in each case, in example 6, is a common factor by dividing it into each of the numbers. Prove that it is the greatest common measure by examining your quotients and finding that they have no common measure except unity.

8. State how to find the G. C. M. of two numbers.

Find the G. C. M. of:

9. 40, 56.	**17.** 210, 455.
10. 42, 54.	**18.** 287, 369.
11. 81, 105.	**19.** 230, 506.
12. 108, 162.	**20.** 42, 72, 180.
13. 63, 91.	**21.** 60, 135, 165.
14. 90, 105.	**22.** 210, 462, 546.
15. 102, 114.	**23.** 395, 474, 632.
16. 75, 175.	**24.** 666, 738, 954.

LEAST COMMON MULTIPLE

78. The quantity 15 in. is measured by the unit 5 in., 3 times, and is therefore called a *multiple* of 5 in.

The quantity 18 lb. is exactly divisible by the units 1 lb., 2 lb., 3 lb., 6 lb., and 9 lb., and is a multiple of each one of them. Thus 18 lb. is equal to 18(1 lb.), 9(2 lb.), 6(3 lb.), or 3(6 lb.).

Select from the following quantities the multiples of the unit $3: $12, $16, $18, $25, and $27. Name all the units that will exactly measure the quantity 24 hr.

Any quantity is a *multiple* of a unit of measure when it is exactly divisible by the unit.

79. Thirty days is exactly divisible by the units 3 da. and 5 da., and is, therefore, a *common multiple* of 3 da. and 5 da.

One quantity is a *common multiple* of two or more units when the former is exactly divisible by each of the latter.

Thirty days is the least quantity that is exactly divisible by the units 6 da. and 10 da., and is, therefore, *the least common multiple* of 6 da. and 10 da.

The **Least Common Multiple** (L.C.M.) of two or more units is the least quantity that is exactly divisible by each of them.

For convenience, we speak of one number being a multiple of another, or a common multiple, or the least common multiple of two or more numbers.

80. The smallest number in the multiplication table of 6 and also of 9 is 18. Therefore 18 is the L. C. M. of 6 and 9.

Exercise 74

Find by the method of the preceding paragraph the L. C. M. of:

1. 4, 5.	**4.** 9, 12.	**7.** 2, 3, 6.	**10.** 5, 4, 10.
2. 6, 7.	**5.** 10, 12.	**8.** 2, 3, 4.	**11.** 6, 8, 12.
3. 6, 8.	**6.** 8, 12.	**9.** 4, 5, 6.	**12.** 3, 6, 9.

13. Find the L. C. M. of the denominators of these fractions:

$\frac{1}{2}, \frac{2}{3}$	$\frac{4}{5}, \frac{1}{6}$	$\frac{1}{2}, \frac{1}{3}, \frac{1}{5}$
$\frac{3}{4}, \frac{1}{6}$	$\frac{2}{3}, \frac{5}{6}$	$\frac{2}{3}, \frac{1}{4}, \frac{5}{6}$
$\frac{3}{4}, \frac{2}{3}$	$\frac{1}{6}, \frac{4}{9}$	$\frac{1}{3}, \frac{1}{6}, \frac{8}{9}$
$\frac{3}{10}, \frac{2}{5}$	$\frac{7}{10}, \frac{11}{12}$	$\frac{4}{5}, \frac{1}{6}, \frac{11}{12}$
$\frac{3}{7}, \frac{3}{4}$	$\frac{1}{8}, \frac{3}{10}$	$\frac{3}{8}, \frac{5}{6}, \frac{1}{9}$

81. (1) Find the shortest distance which can be exactly measured by two lines respectively 36 ft. and 48 ft. long.

We are here required to find the shortest distance, *i.e.* the L. C. M., of the units 36 ft. and 48 ft.

$$36 = 4 \times 9 \ = 2^2 \times 3^2.$$
$$48 = 3 \times 16 = 3 \times 2^4.$$

Thus the L. C. M. of 36 and 48 $= 2^4 \times 3^2 = 16 \times 9 = 144.$

$\therefore$ the shortest distance is 144 ft.

(2) Find the L. C. M. of 24, 30, 36.

```
6 | 24   30   36
2 |  4    5    6
  |  2    5    3
```

Here 6 and 2 are the factors common to two or more of the numbers, and 2, 5, and 3 are the factors not common.

$\therefore$ the L. C. M. $= 6 \times 2 \times 2 \times 5 \times 3 = 360.$

State how to find the L. C. M. of two or more numbers.
Show by division that 24, 30, and 36 are all factors of their L. C. M. 360.

(3) Find the L. C. M. of 14, 21, 54, 56, 84.

```
3 | 14   21   54   56   84
2 |           18   56   28
  |            9   28
```

$\therefore$ the L. C. M. $= 3 \times 2 \times 9 \times 28 = 1512.$

14 is erased since it is a factor of 56, and 21 since it is a factor of 84. In the second line, 28 is erased since it is a factor of 56.

Exercise 75

Find the L. C. M. of:

1. 4, 8, 16, 32.   4. 15, 18, 28, 36.   7. 65, 26, 56, 52.
2. 3, 6, 9, 12.   5. 12, 20, 21, 45.   8. 36, 48, 60, 54.
3. 24, 30, 36, 45.   6. 22, 33, 30, 44.   9. 33, 27, 55, 135.

10. 30, 21, 40, 28, 24, 56.   11. 56, 36, 63, 28, 72.

Find the L. C. M. of the denominators of these fractions:

12. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.   13. $\frac{6}{25}$, $\frac{7}{40}$, $\frac{19}{60}$.   14. $\frac{1}{4}$, $\frac{3}{7}$, $\frac{7}{12}$, $\frac{11}{18}$.

15. $\frac{2}{5}$, $\frac{5}{7}$, $\frac{11}{21}$, $\frac{8}{15}$.   17. $\frac{1}{3}$, $\frac{8}{15}$, $\frac{5}{12}$, $\frac{13}{42}$.

16. $\frac{4}{5}$, $\frac{2}{3}$, $\frac{1}{9}$, $\frac{14}{15}$.   18. $\frac{1}{3}$, $\frac{2}{5}$, $\frac{7}{12}$, $\frac{11}{18}$.

CHAPTER XI

FRACTIONS

82. If, on measuring a certain quantity with a unit 1 ft. long, I count 6 units in the quantity, then I know that the quantity is 6 times 1 ft., or 6 ft. Similarly, if I measure another quantity with a unit 2 ft. long and count 4 units in the quantity, I know that it is 4 times 2 ft., or 4 (2 ft.). If I measure a third quantity with a unit $\frac{1}{4}$ ft. long and count 3 units in the quantity, then I know that it is 3 times the unit $\frac{1}{4}$ ft., or $\frac{3}{4}$ ft. Thus when a measured quantity is denoted by the expression $\frac{3}{4}$ ft., the measuring unit is $\frac{1}{4}$ ft. and the number of units in the quantity is 3.

Let actual work be done by the class in measuring unknown quantities with fractional units.

Exercise 76

1. I measured a quantity with the unit $\frac{1}{3}$ ft. and counted 2 units. What was the quantity?

2. Name the quantities which, when measured by the following units, give the numbers indicated:

Unit	Number	Unit	Number
$\frac{1}{6}$ ft.	5	$\frac{1}{2}$ da.	5
$\frac{1}{12}$ ft.	7	$\frac{1}{8}$ gal.	7
$\frac{1}{8}$ mi.	5	$\frac{1}{12}$ doz.	18
$\frac{1}{16}$ lb.	13	$\frac{1}{9}$ bu.	7
$\frac{1}{4}$ lb.	9	$\frac{1}{10}$ T.	9
$\frac{1}{5}$ hr.	4	$\frac{1}{12}$ A.	11

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83. The expression $\frac{5}{6}$ ft. denotes that the quantity 1 ft. is conceived as made up of 6 equal parts or units, and that 5 of these parts or units have been taken to measure the quantity denoted by $\frac{5}{6}$ ft.

The primary unit, 1 ft., has been divided into 6 equal parts to give the measuring unit, which is $\frac{1}{6}$ ft., or 2 in. The number of these units in the given quantity is 5. The *ratio* of the given quantity to the measuring unit is 5.

84. The quantity represented by $\frac{5}{6}$ ft. contains 5 direct measuring units, and the primary unit, 1 ft., contains 6 of these units. *Hence, the fraction $\frac{5}{6}$ expresses the ratio of the quantity denoted by $\frac{5}{6}$ ft. to the primary unit, 1 ft.*
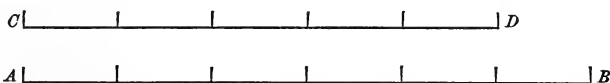
85. Draw a line, *AB*, 1 ft. long. Divide it into 6 equal parts, or units, each $\frac{1}{6}$ of a foot long. Draw a second line, *CD*, above the first, containing 5 of these units, and use these two lines to illustrate the preceding paragraph.

Exercise 77

In the following quantities name the measuring units, and state the number of units in each quantity:

|			
1. $\frac{3}{4}$ ft.	**5.** $\$\frac{3}{4}$.	**9.** $\frac{27}{36}$ sq. ft.	**13.** $\frac{3}{8}$ da.
2. $\frac{2}{3}$ yd.	**6.** $\$\frac{2}{5}$.	**10.** $\frac{5}{9}$ cu. yd.	**14.** $\frac{3}{4}$ da.
3. $\frac{7}{8}$ lb.	**7.** $\$\frac{6}{10}$.	**11.** $\frac{2}{7}$ wk.	**15.** $\frac{11}{15}$ hr.
4. $\frac{5}{9}$ sq. yd.	**8.** $\$\frac{13}{20}$.	**12.** $\frac{24}{73}$ yr.	**16.** $\frac{5}{6}$ min.

17. Make a drawing to show that the ratio of $\frac{3}{4}$ ft. to 1 ft. is $\frac{3}{4}$. Make a drawing to show that the ratio of $\frac{5}{9}$ yd. to 1 yd. is $\frac{5}{9}$.

18. Give the ratio of each quantity, in examples 1 to 16, to its primary unit.

86. A Fraction is a number in which the unit of measure is a definite part of some primary unit of the same kind.

The **denominator** shows into how many parts the primary unit is· divided to give the unit of measure; it also *names* this unit. The **numerator** shows the number of them that measures the quantity.

A **proper fraction,** as an expression of measured quantity, is one in which the numerator is less than the denominator. Select the proper fractions: $\frac{2}{3}$, $\frac{4}{7}$, $\frac{9}{5}$, $\frac{8}{8}$, $\frac{4}{5}$.
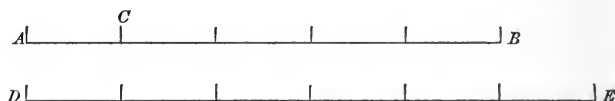
87. Let AB represent some quantity measured by 5 units, each equal to AC, and DE as measured by 6 units, each equal to AC. Then if we think of AB in relation to DE, we think of 5 units in relation to 6 units, and this relation or ratio is expressed by the fraction $\frac{5}{6}$.

Similarly, the fraction, or number $\frac{5}{6}$, expresses the ratio of $5 to $6, 5 hr. to 6 hr., 5 mi. to 6 mi., 5 (8 ft.) to 6 (8 ft.), 5 (12 lb.) to 6 (12 lb.), or, generally, 5 of any unit to 6 of the same unit.

Similarly, it expresses the ratio of 20 lb. (*i.e.* 5 × 4 lb.) to 24 lb. (*i.e.* 6 × 4 lb.), and so on.

Exercise 78

1. Name quantities whose ratio is $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{3}{8}$; $\frac{4}{9}$.

Write the fraction which expresses the ratio of the following quantities:

2. $4 to $9; $8 to $12; 1 dime to $1.

3. 3 qt. to 4 qt.; 2 qt. to 1 gal.

4. 12 yd. to 32 yd.; 4 in. to 2 ft.

5. If 32 yd. of cloth cost $48, what part of $48 will 12 yd. cost? How many dollars will 12 yd. cost?

6. What is the ratio of 24 men to 36 men? 18 men to 24 men?

7. If 18 men can do a piece of work in 32 da., in what part of 32 da. can 24 men do the same work? How many days?

88. Let AB represent some definitely measured quantity, as 4 ft. or 16 ft., and let it be divided as shown in the diagram.

It is evident from this diagram that $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, $\frac{6}{6}$, $\frac{8}{8}$, $\frac{10}{10}$, $\frac{12}{12}$, $\frac{16}{16}$, of a quantity measure it, and are all equal. It is also evident that $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$, $\frac{8}{16}$, of a quantity measure one-half of it, and are all equal.

Similarly, $\frac{1}{3}$, $\frac{2}{6}$, $\frac{4}{12}$, of a quantity measure one-third of it, and are equal. Similarly, $\frac{1}{5} = \frac{2}{10}$.

89. The fraction $\frac{6}{12}$ is got from the fraction $\frac{1}{2}$ by multiplying both numerator and denominator by 6. $\frac{8}{16}$ reduces to $\frac{1}{2}$ by dividing both numerator and denominator by 8. How

can you change the fraction $\frac{1}{2}$ to $\frac{5}{10}$? $\frac{1}{3}$ to $\frac{4}{12}$? $\frac{4}{8}$ to $\frac{1}{2}$? $\frac{2}{10}$ to $\frac{1}{5}$?

Multiplying or dividing both terms of a fraction by the same number does not change its value.

Exercise 79

1. By what must you multiply both numerator and denominator of $\frac{2}{3}$ to get $\frac{6}{9}$? By what must you divide both numerator and denominator of $\frac{6}{9}$ to reduce it to $\frac{2}{3}$?

2. Make a drawing to show that $\frac{2}{3} = \frac{6}{9}$.

Use the results you obtain in the following to verify the rule in the preceding paragraph:

3. Find $\frac{1}{2}$ of $8; $\frac{2}{4}$ of $8; $\frac{4}{8}$ of $8.

4. Find $\frac{1}{5}$ of $30; $\frac{2}{10}$ of $30; $\frac{3}{15}$ of $30.

5. Find $\frac{1}{3}$ of 36 ft.; find also respectively $\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$, $\frac{6}{18}$, and $\frac{12}{36}$ of 36 feet.

6. Find $\frac{2}{3}$ of 24 dimes; $\frac{4}{6}$ of 24 dimes; $\frac{8}{12}$ of 24 dimes.

7. Find $\frac{5}{7}$ of 28 lb.; $\frac{10}{14}$ of 28 lb.; $\frac{20}{28}$ of 28 lb.

8. Find $\frac{3}{4}$ of 32 da.; $\frac{6}{8}$ of 32 da.; $\frac{12}{16}$ of 32 da.; $\frac{24}{32}$ of 32 da.

9. $\frac{2}{3}$ da. = ? hr.; $\frac{8}{12}$ da. = ? hr.; $\frac{20}{28}$ da. = ? hr.; $\frac{5}{6}$ da. = ? hr.

10. $\frac{3}{5}$ hr. = ? min.; $\frac{18}{30}$ hr. = ? min.; $\frac{15}{20}$ hr. = ? min.; $\frac{3}{4}$ hr. = ? min.

11. What actual coins and how many of each kind are equal respectively to $\frac{1}{2}$, $\frac{2}{4}$, $\frac{5}{10}$, and $\frac{10}{20}$ of $1?

90. (1) Express 9 yd. as eighths of a yard.

$$A\underline{\quad|\quad|\quad|\quad|\quad|\quad|\quad|\quad}B .$$

Let the line AB be drawn to represent 1 yd. Think of 1 yd. as containing 8 units of length, as shown in the diagram.

Then 9 yd. will contain 9×8, or 72 of these units.

Therefore 9 yd. is equal to $\frac{72}{8}$ of 1 yd.

(2) Express $\$\frac{3}{4}$ as a fraction with 20 as a denominator.

Think of $\$1$ as containing 20 units of 5¢ each. Then $\$\frac{3}{4}$ contains $\frac{3}{4}$ of 20, or 15 of these units. Therefore, $\$\frac{3}{4} = \$\frac{15}{10}$.

Exercise 80

Express as fractions:

1. $\$8$ as 10ths.
2. $\$4$ as 20ths.
3. 9 yd. as 3ds.
4. 6 ft. as 12ths.
5. 14 gal. as 4ths.
6. 11 wk. as 7ths.

7. 5 yr. as 12ths.
8. 3 da. as 24ths.
9. 4 min. as 60ths.
10. 6 hr. as 60ths.
11. 8 pk. as 8ths.
12. 12 bu. as 4ths.

13. In each of these questions name the measuring unit in your fraction as an actual unit of measure in common use.

14. Express as fractions with 100 as denominator: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{25}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{9}{10}$.

Reduce, illustrating your work by diagrams:

15. $\frac{3}{4}$ ft. to 12ths.
16. $\frac{2}{3}$ yd. to 33ds.
17. $\frac{5}{8}$ lb. to 16ths.
18. $\frac{6}{7}$ gal. to 28ths.

19. $\frac{7}{8}$ da. to 24ths.
20. $\frac{2}{13}$ to 78ths.
21. $\frac{4}{15}$ to 45ths.
22. $\frac{11}{18}$ to 54ths.

23. What are the new units of measure in your results? Where possible, identify them with actual units in common use.

91. (1) Express 8 in. as a fraction of 1 ft.

$$8 \text{ in.} = \tfrac{8}{12} \text{ or } \tfrac{2}{3} \text{ of a foot} = \tfrac{2}{3} \text{ ft.}$$

(2) A man's capital is represented by 20 units of value. He invests $\frac{1}{4}$ of it in land and $\frac{1}{3}$ of the remainder in bank stock. How many units did he invest in bank stock, and what part is it of his entire capital?

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The amount invested in land = ¼ of 20 units, or 5 units.

The remainder = 20 − 5, or 15 units.

The amount invested in stock = ⅓ of 15, or 5 units.

∴ the amount invested in stock = $\frac{5}{20}$, or ¼ of his entire capital.

Exercise 81

1. Express as a fraction of a foot:

3 in., 4 in., 5 in., 6 in., 9 in., 10 in., 15 in., 16 in., 27 in.

2. At 8¢ a foot what is the cost of 9 in. of rubber tubing? Of 18 in.? Of 30 in.?

3. Express as a fraction of a pound Avoirdupois:

14 oz., 6 oz., 9 oz., 12 oz., 15 oz., 20 oz., 24 oz.

4. At 20¢ a pound what is the cost of 12 oz. of cheese? 20 oz.? 24 oz.?

5. Express as a fraction of a month:

12 da., 15 da., 18 da., 20 da., 25 da., 40 da., 45 da., 54 da.

6. A commercial traveller was away from home 27 da. in the month of April. What part of the month was he at home?

7. If 1 mi. is the measuring unit, what number measures each of the following:

640 rd.? 20 rd.? 35 rd.? 80 rd.? 120 rd.? 150 rd.?

8. If a bicyclist rides 1 mi. in 8 min., how long does it take him to ride 60 rd.?

9. Express as a fraction of a day:

6 hr., 9 hr., 12 hr., 15 hr., 18 hr., 24 hr., 32 hr., 36 hr.

10. If a baby sleeps 16 hr. a day, what part of the day is he awake?

11. Express as fractions of a dollar:

25, 50, 75, 60, 80, 100, 125, 150, and 160 cents.

12. At $1 a yard, how much cloth can I buy for $2.50?

13. If $4 is used as a measure of $4, what is the number expressing the measurement? If $4 is used as a measure of $3, what is the number?

14. If cloth is 40¢ a yard, how much can I buy for 30¢?

15. If oranges cost 30¢ a dozen, what part of a dozen can I buy for 25¢? How many oranges?

16. A man walks a certain distance in 4 hr. What part of it does he walk in 1 hr.? In 2 hr.? In ½ hr.? In ¼ hr.?

17. A can do a piece of work in 3 hr., B the same amount in 4 hr. If the work be measured by 12 units, how many units will A do in an hour? How many B? How many both working together? What part will their joint work for an hour be of the whole work?

18. A and B put $8000 into a business. B put three times as much as A. How many units measure is A's share? B's share? The entire amount? A put in what part of $8000? How much? B what part? How much?

19. A received a certain sum of money, B twice as much, and C as much as A and B together. How many units measure the entire amount? What part of the whole sum does each receive?

20. If, in the previous example, $24 was divided among A, B, and C, find the share of each.

21. Given that pure water contains 15 parts by weight of oxygen, and 2 parts of hydrogen, what part of the weight of a gallon of water is hydrogen?

22. How many pounds of oxygen and of hydrogen in 34 lb. of water?

23. Six brothers join in paying a debt of $700. The eldest pays $\frac{3}{7}$ of it, and each of the others $\frac{1}{5}$ of the remainder. How much does the eldest pay? How much does each of the five pay? This is what part of the whole debt?

24. If a pipe empties a tank at the rate of 12 gal. in 1 min., what is the rate per second?

25. $40 is divided among A, B, and C, giving A $\frac{3}{8}$ of it, B $\frac{2}{5}$, and C the remainder. What is the sum of A's and B's shares? What is C's share? C's share is what part of the whole sum?

26. The value of a mine is represented by 10 units of money. A man who owns $\frac{2}{5}$ of it sells $\frac{3}{4}$ of his share. How many units did he own? How many did he sell? What part of the whole mine did he sell?

27. The length of an oblong is 8 ft., and the width 4 ft. What part of the perimeter is the length?

28. An oblong 6 ft. wide 'and 8 ft. long is divided into strips each 1 ft. wide, made by drawing lines parallel to the length. What part of the area of the oblong is the area of one strip?

29. The area of an oblong is 16 sq. ft. What part of its area is that of a square whose side is 2 ft. ?

30. If 20 units measure the cost of a pound of tea sold at a gain of $\frac{3}{20}$ of the cost, how many units are gained by selling? What is the selling price? The cost price is what part of the selling price? What is the ratio of the selling price to the cost? The gain is what part of the selling price?

31. A has 12 marbles, and B has 3. They play together, and A loses $\frac{1}{4}$ of his marbles. How many has B now? What part are they of what A now has?

32. The value of a house is measured by 5 units. The lot on which it stands is worth $\frac{1}{5}$ as much as the house. What is the measure of the value of the house and lot? The value of the lot is worth what part of both together?

33. A and B set out at the same time from places 42 mi. apart, and meet at the end of 6 hr. A travels at the rate of 3 mi. an hour. How far does B travel? What is B's rate? A's rate is what part of B's?

34. Apples are sold at the rate of 12 for a dime, and bananas at the rate of 8 for a dime. Compare the value of an apple with that of a banana.

NOTE. — Let the dime be measured by 24 units.

35. A crew can row 6 mi. an hour in still water. What is the rate of rowing up stream in a current running at the rate of 2 mi. an hour? What is the rate down stream? What is the ratio of the rate down stream to that up stream?

REDUCTION OF FRACTIONS

92. A fraction is in its lowest terms when its numerator and denominator have no common factor.

(1) Reduce $\$\frac{315}{495}$ to its lowest terms.

The object of reduction is to give a more definite idea of the value of the quantity by expressing the ratio in the smallest numbers.

$$\$\tfrac{315}{495} = \$\tfrac{105}{165} = \$\tfrac{35}{55} = \$\tfrac{7}{11}.$$

The common factors are 3, 3, and 5.

The effect of dividing each term of the first fraction by 3 is to make each measuring unit 3 times as large, and to reduce the number of these units to one-third as many. Similarly, with the second division by 3, and the third by 5.

(2) A merchant paid $66 \not\!c$ a yard for cloth and sold it for $88 \not\!c$ a yard. What fraction of the cost was the selling price?

The selling price $= \frac{88}{66}$ or $\frac{4}{3}$ of the cost.

Exercise 82

Reduce to its lowest terms:

1. $\$\frac{16}{24}$.	**4.** $\$\frac{39}{48}$.	**7.** $\frac{14}{63}$.	**10.** $\frac{54}{78}$.	**13.** $\frac{24}{80}$.
2. $\$\frac{16}{34}$.	**5.** $\$\frac{14}{18}$.	**8.** $\frac{32}{56}$.	**11.** $\frac{35}{91}$.	**14.** $\frac{48}{156}$.
3. $\$\frac{14}{22}$.	**6.** $\frac{48}{64}$.	**9.** $\frac{18}{99}$.	**12.** $\frac{84}{96}$.	

15. If I pay $60 for a bicycle and afterward sell it for $45, what part of the cost do I sell it for?

16. A merchant sells 55 yd. of cloth from a piece containing 66 yd. What part of the whole piece does he sell?

17. A man buys a horse for $128 and sells it for $112. Find the selling price as a fraction of the cost.

18. On an investment of $88 a merchant gains $33. Find the ratio of the gain to the cost.

19. Sixty days is what fraction of a year?

20. Out of a farm containing 135 A., 81 A. were sold. Find what part of the farm was sold.

21. 1 ft. 8 in. = ? in. 2 ft. 6 in. = ? in. What is the ratio of 1 ft. 8 in. to 2 ft. 6 in. ?

22. What is the ratio of 2 ft. 9 in. to 3 ft. 8 in. ? Of 6 gal. 1 qt. to 7 gal. 2 qt. ? Of 2 lb. 8 oz. to 3 lb. 2 oz. ? Of 2 bu. 1 qt. to 3 bu. 8 qt. ?

23. If 2 lb. 3 oz. of cheese cost 30¢, what will 1 lb. 5 oz. cost?

24. If 3 doz. 6 eggs cost $.50, what will 5 doz. 3 eggs cost?

93. Make drawings to show the difference between $2\frac{3}{4}$ ft. and $\frac{11}{4}$ ft. In your first drawing how many units each equal to 1 ft. did you mark off? How many equal to $\frac{1}{4}$ ft. ? Can you count 5 units in this quantity? Why not? In your second drawing how many units, each $\frac{1}{4}$ ft., can you count?

94. Such an expression (as $2\frac{3}{4}$ ft., *e.g.*) denotes a quantity in which two units of measure of different values have been used : a primary unit and parts of this, a derived unit.

$2\frac{3}{4}$ is the number 11 in disguise. To make it *number* in the strict sense, we must express the quantity in the *smaller* unit of measure, *i.e.* as $\frac{11}{4}$ ft. ; this as a quantity can be *counted ;* $2\frac{3}{4}$ cannot be *counted.*

An *improper fraction* as an expression of measured quantity is one whose numerator is equal to or greater than its denominator ; as $\frac{7}{6}$, $\frac{9}{8}$, $\frac{8}{8}$, $\frac{10}{3}$.

95. Reduce to an improper fraction $5\frac{2}{3}$ yd.

Let   1 yd. = 3 units of $\frac{1}{3}$ yd. each.

Then   5 yd. = 15 units of $\frac{1}{3}$ yd. each.

                    $\frac{2}{3}$ yd. = 2 units of $\frac{1}{3}$ yd. each.

        ∴ $5\frac{2}{3}$ yd. = 17 units of $\frac{1}{3}$ yd. each, or $\frac{17}{3}$ yd.

Hence, if we divide the quantity $5\frac{2}{3}$ yd. into parts, each equal to $\frac{1}{3}$ yd., we can count 17 parts in the quantity.

Thus, we may multiply 3 by 5 and add 2, or, by the law of commutation, *multiply 5 by 3 and add 2* to get the numerator.

Exercise 83

Reduce to improper fractions:

1. $2\frac{1}{4}$ ft., $2\frac{3}{4}$ ft., $3\frac{1}{4}$ ft., $5\frac{3}{4}$ ft., $6\frac{1}{4}$ ft.

In this example by what number do you multiply each time?

2. $3\frac{2}{5}$ yd., $4\frac{3}{5}$ yd., $3\frac{4}{5}$ yd., $7\frac{1}{5}$ yd., $8\frac{4}{5}$ yd.

What is your multiplier here?

3. $\$4\frac{1}{4}$.	8. $8\frac{1}{3}$ yd.	13. $12\frac{3}{7}$.	18. $20\frac{3}{5}$.
4. $\$4\frac{3}{8}$.	9. $5\frac{7}{12}$ ft.	14. $9\frac{5}{9}$.	19. $21\frac{3}{4}$.
5. $\$7\frac{3}{5}$.	10. $20\frac{3}{4}$ gal.	15. $12\frac{2}{11}$.	20. $19\frac{4}{5}$.
6. $\$16\frac{1}{2}$.	11. $8\frac{5}{8}$ pk.	16. $6\frac{8}{15}$.	21. $7\frac{6}{13}$.
7. $\$9\frac{3}{10}$.	12. $5\frac{3}{16}$ lb.	17. $3\frac{4}{17}$.	22. $40\frac{7}{8}$.

23. What are the two units of measure in example 8? What is the unit of measure in the result? How many of these units must be counted to measure the quantity?

24. What is the quantity which is measured by the unit $\frac{2}{3}$ ft. and the number 9? If the same quantity be measured by the number 3, what is the measuring unit?

25. If 9 boys receive $\$\frac{4}{5}$ each, what sum was divided? If the same sum had been divided among 4 boys, what would each have received?

Exercise 84

Reduce to improper fractions:

1. $15\frac{3}{11}$.
2. $21\frac{9}{13}$.
3. $29\frac{15}{16}$.
4. $41\frac{3}{25}$.

5. $34\frac{5}{14}$.
6. $48\frac{15}{19}$.
7. $51\frac{25}{33}$.
8. $36\frac{43}{54}$.

9. $64\frac{21}{47}$.
10. $152\frac{8}{9}$.
11. $98\frac{4}{15}$.
12. $168\frac{5}{6}$.

13. $29\frac{14}{285}$.
14. $87\frac{282}{301}$.
15. $341\frac{17}{19}$.
16. $453\frac{241}{315}$.

96. (1) Which is greater, $\frac{5}{6}$ ft. or $\frac{6}{7}$ ft. ?

Consider the feet as having 42 measuring units, then $\frac{5}{6}$ of this is 35, and $\frac{6}{7}$ is 36 units.

∴ $\frac{6}{7}$ ft. is greater than $\frac{5}{6}$ ft. by 1 unit, or $\frac{1}{42}$ ft.

(2) Compare the quantities $\$\frac{3}{4}$, $\$\frac{7}{12}$, $\$\frac{5}{8}$.

Let $\$1$ be represented by 24 units of value. Then $\$\frac{3}{4}$, $\$\frac{7}{12}$, $\$\frac{5}{8}$, are respectively equal to 18, 14, and 15 units. Hence the first fraction is the greatest and the second the least.

State how to find the greatest and least of two fractions.

Exercise 85

Find the greatest and least of the following:

1. $\frac{2}{3}$ yd., $\frac{3}{4}$ yd.
2. $\$\frac{7}{8}$, $\$\frac{8}{9}$.
3. $\$\frac{3}{10}$, $\$\frac{1}{3}$, $\$\frac{7}{20}$.

4. $\frac{5}{6}$ ft., $\frac{3}{4}$ ft., $1\frac{1}{12}$ ft.
5. $\frac{3}{4}$, $\frac{8}{11}$, $\frac{17}{22}$.
6. $\frac{2}{3}$, $\frac{5}{8}$, $\frac{11}{16}$.

7. State how to find the greatest and least of a number of fractions.

97. Express $\frac{23}{5}$ yd. in terms of the primary and derived units of measure.

The primary unit 1 yd. contains 5 derived units of $\frac{1}{5}$ of a yard each.

Hence $\frac{23}{5}$ yd., *i.e.* 23 derived units = 4 primary and 3 derived units;

$$= 4\frac{3}{5} \text{ yd.}$$

Or, more simply,　　　　　　$5\overline{)23}$

$$4\frac{3}{5}$$

Exercise 86

Express in terms of the primary and direct measuring units:

1. $\frac{7}{2}$ ft.
2. $1\frac{1}{4}$ ft.
3. $1\frac{7}{4}$ yd.
4. $1\frac{3}{3}$ yd.
5. $\frac{20}{3}$ mi.
6. $1\frac{4}{5}$ mi.

7. \$ $\frac{43}{5}$.
8. \$ $\frac{89}{7}$.
9. $1\frac{13}{6}$ da.
10. $1\frac{24}{5}$ wk.
11. $\frac{187}{12}$ yr.
12. $\frac{293}{16}$ ft.

13. $\frac{284}{42}$ yd.
14. $\frac{276}{45}$ rd.
15. $\frac{385}{63}$ mi.
16. $\frac{3708}{87}$ oz.
17. $\frac{4849}{125}$ lb.
18. $\frac{8099}{343}$ lb.

19. What are the two units of measure in the answers to examples 1, 3, and 5 ?

Exercise 87

Reduce to integers and proper fractions:

1. $\frac{8048}{112}$.
2. $\frac{20835}{144}$.
3. $\frac{8943}{97}$.
4. $\frac{9947}{61}$.
5. $\frac{5793}{389}$.
6. $\frac{8679}{231}$.

98. Reduce $\frac{2}{3}$ ft., $\frac{3}{4}$ ft., $\frac{5}{6}$ ft. to fractions having for their common denominator the least common denominator (L.C.D.) of these fractions.

The L. C. D. of 3, 4, and 6 is evidently 12 ; the foot is considered as divided into 12 equal parts.
$\frac{2}{3}$ ft. = 8 parts = $\frac{8}{12}$ ft. ; $\frac{3}{4}$ ft. = 9 parts = $\frac{9}{12}$ ft. ; $\frac{5}{6}$ ft. = 10 parts = $\frac{10}{12}$ ft.
Make a drawing to show that this result is correct.

Exercise 88

1. Reduce \$ $\frac{1}{2}$ and \$ $\frac{2}{5}$ to tenths.
2. Reduce \$ $\frac{1}{6}$, \$ $\frac{4}{5}$, and \$ $\frac{3}{10}$ to thirtieths.
3. Reduce $\frac{1}{2}$ yd., $\frac{2}{3}$ yd., and $\frac{5}{9}$ yd. to eighteenths.

Reduce to their least common denominator:

4. $\frac{2}{3}$ ft., $\frac{3}{4}$ ft.   **10.** $\frac{1}{4}$, $\frac{3}{7}$.   **16.** $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$.

5. $\frac{3}{5}$ hr., $\frac{1}{6}$ hr.   **11.** $\frac{2}{5}$, $\frac{5}{9}$.   **17.** $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$.

6. $\frac{2}{3}$ da., $\frac{7}{12}$ da.   **12.** $\frac{3}{7}$, $\frac{4}{9}$.   **18.** $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{12}$.

7. $\frac{7}{9}$ yr., $\frac{5}{6}$ yr.   **13.** $\frac{3}{4}$, $\frac{9}{10}$.   **19.** $\frac{3}{4}$, $\frac{7}{8}$, $\frac{11}{16}$.

8. $\frac{5}{8}$, $\frac{3}{4}$.   **14.** $\frac{3}{10}$, $\frac{4}{15}$.   **20.** $\frac{5}{6}$, $\frac{3}{4}$, $\frac{7}{8}$.

9. $\frac{4}{5}$, $\frac{5}{6}$.   **15.** $\frac{5}{8}$, $\frac{1}{6}$.   **21.** $\frac{2}{7}$, $\frac{9}{14}$, $\frac{1}{4}$.

ADDITION OF FRACTIONS

99. The sum of 3 dimes and 4 dimes = 7 dimes.

The sum of $\$\frac{3}{10}$ and $\$\frac{4}{10} = \$\frac{7}{10}$.

The sum of 5 oz. and 8 oz. = 13 oz.

The sum of $\frac{5}{16}$ lb. and $\frac{8}{16}$ lb. = $\frac{13}{16}$ lb.

Here the measuring unit of value is $\$\frac{1}{10}$, or 1 dime, that of weight $\frac{1}{16}$ lb., or 1 oz.

Why cannot you add $\frac{1}{2}$ ft., $\frac{2}{3}$ ft., $\frac{3}{4}$ ft. by simply adding their numerators?

(1) Add $\frac{1}{2}$ ft., $\frac{2}{3}$ ft., $\frac{3}{4}$ ft.

The L. C. D. = 12.

$$\frac{1}{2} \text{ ft.} = \frac{6}{12} \text{ ft.} ; \ \frac{2}{3} \text{ ft.} = \frac{8}{12} \text{ ft.} ; \ \frac{3}{4} \text{ ft.} = \frac{9}{12} \text{ ft.}$$

$$\therefore \text{ the sum} = \frac{6 + 8 + 9}{12} \text{ ft.} = \frac{23}{12} \text{ ft.} = 1\frac{11}{12} \text{ ft.}$$

Prove the correctness of this result by drawing a line and measuring off $\frac{1}{2}$ ft., $\frac{2}{3}$ ft., and $\frac{3}{4}$ ft., and also $1\frac{11}{12}$ ft.

(2) Find the sum of $\$21\frac{3}{8}$, $\$15\frac{5}{6}$, $\$13\frac{4}{9}$, $\$8\frac{7}{12}$.

The L. C. D. = 72.

$$\$\frac{3}{4} + \$\frac{5}{6} + \$\frac{4}{9} + \$\frac{7}{12} = \$\frac{27 + 60 + 32 + 42}{72} = \$\frac{161}{72} = \$2\frac{17}{72}.$$

$$\$21 + \$15 + \$13 + \$8 = \$57.$$

$$\therefore \text{ the sum} = \$57 + \$2\frac{17}{72} = \$57\frac{17}{72}.$$

Exercise 89

Find the sum of:

1. $3\frac{3}{4}$, $\$\frac{1}{4}$.   3. $\frac{7}{16}$ oz., $\frac{2}{16}$ oz.   5. $\frac{4}{7}$ wk., $\frac{6}{7}$ wk.

2. $\frac{5}{12}$ ft., $\frac{4}{12}$ ft.   4. $1\frac{3}{24}$ da., $1\frac{1}{24}$ da.   6. $\frac{37}{60}$ min., $\frac{8}{60}$ min.

Exercise 90

In the following exercise, what is the least common denominator in each example?

Find the sum of:

1. $\$\frac{1}{2}$, $\$\frac{1}{4}$.   2. $\$\frac{1}{2}$, $\$\frac{3}{4}$, $\$\frac{5}{8}$.   3. $\frac{2}{3}$ ft., $\frac{3}{4}$ ft.   4. $\frac{1}{3}$ ft., $\frac{3}{4}$ ft., $\frac{5}{6}$ ft.

Prove your answers to 3 and 4 correct by measuring with a ruler.

5. $\frac{3}{4}$ lb., $\frac{5}{8}$ lb., $\frac{13}{16}$ lb.   6. $\frac{1}{2}$ gal., $\frac{3}{4}$ gal.   7. $\frac{3}{4}$ bu., $\frac{7}{8}$ bu., $\frac{11}{16}$ bu.

8. $\frac{2}{5}$ yr., $\frac{16}{73}$ yr.   9. $\frac{2}{3}$ da., $\frac{5}{8}$ da., $\frac{3}{4}$ da.

Prove your results correct by reducing each fractional quantity to a lower denomination (as $\$\frac{1}{2}$ to cents, $\frac{2}{3}$ ft. to inches, $\frac{3}{4}$ lb. to ounces, and so on) and also your result. Then add the integers.

10. What is the weight of $\frac{3}{4}$ lb. of tea and $\frac{7}{8}$ lb. of cheese?

11. A piece of ribbon can be cut into two parts, one $\frac{5}{6}$ yd. long and the other $\frac{7}{9}$ yd. Find its length.

Exercise 91

Find the sum of the following fractional parts of any unit:

1. $\frac{1}{2}$, $\frac{3}{4}$.   7. $3\frac{1}{4}$, $2\frac{2}{3}$.   13. $\frac{3}{10}$, $\frac{7}{15}$.

2. $\frac{2}{3}$, $\frac{5}{6}$.   8. $9\frac{3}{8}$, $3\frac{1}{2}$.   14. $\frac{4}{7}$, $\frac{1}{6}$.

3. $\frac{3}{8}$, $\frac{5}{6}$.   9. $4\frac{2}{3}$, $3\frac{5}{8}$.   15. $\frac{3}{14}$, $\frac{8}{21}$.

4. $\frac{2}{3}$, $\frac{5}{9}$.   10. $8\frac{1}{6}$, $4\frac{2}{9}$.   16. $2\frac{5}{12}$, $6\frac{7}{18}$.

5. $\frac{3}{4}$, $\frac{7}{10}$.   11. $9\frac{3}{4}$, $8\frac{5}{7}$.   17. $3\frac{8}{9}$, $2\frac{5}{12}$.

6. $\frac{2}{7}$, $\frac{3}{5}$.   12. $6\frac{7}{8}$, $4\frac{11}{12}$.   18. $5\frac{11}{15}$, $9\frac{17}{20}$.

19. State how to find the sum of two or more (1) proper fractions, (2) mixed numbers.

20. Explain clearly the principles involved in finding the sum of two fractions.

21. A boy had $6¼ in a bank and put in $3⅖. How much did he then have in the bank?

22. A child's dress was made from two remnants, one containing $2\frac{7}{8}$ yd., and the other $3\frac{3}{4}$ yd. How many yards did the dress contain?

23. Three brothers gathered walnuts. The first gathered $2\frac{1}{2}$ bu., the second $2\frac{3}{4}$ bu., and the third $3\frac{5}{6}$ bu. How many bushels all together?

24. If wheat sold at $61\frac{1}{2}$ ¢ per bushel in the morning and the price advanced $\frac{5}{8}$ ¢ during the day, find the price in the evening.

Exercise 92

Find the sum of:

1. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$.

2. $\frac{5}{8}, \frac{7}{12}, \frac{5}{6}$.

3. $\frac{5}{9}, \frac{3}{10}, \frac{4}{15}$.

4. $\frac{6}{7}, \frac{3}{4}, \frac{5}{14}$.

5. $\frac{7}{9}, \frac{2}{3}, \frac{11}{15}$.

6. $\frac{3}{5}, \frac{4}{15}, \frac{2}{3}, \frac{1}{6}$.

7. $\frac{3}{7}, \frac{9}{14}, \frac{5}{6}, \frac{11}{21}$.

8. $1\frac{3}{8}, 3\frac{4}{5}, 2\frac{9}{20}, 4\frac{2}{3}$.

9. $7\frac{5}{12}, 4\frac{8}{15}, 2\frac{13}{20}, 4\frac{7}{30}$.

10. $3\frac{3}{14}, 1\frac{5}{28}, 5\frac{10}{21}, 4\frac{37}{42}$.

11. $2\frac{7}{12}, 14\frac{16}{21}, 92\frac{3}{4}, 15\frac{9}{28}$.

12. $19\frac{2}{7}, 28\frac{11}{15}, 72\frac{17}{21}, 59\frac{12}{35}$.

Exercise 93

1. Three families ordered a carload of coal. The first got $8\frac{2}{3}$ T., the second $7\frac{5}{6}$ T., and the third $8\frac{3}{4}$ T. Find the amount of coal on the car.

2. A farmer raised $128\frac{9}{10}$ bu. of wheat from one field, $214\frac{7}{8}$ bu. from another, and from a third field $156\frac{5}{6}$ bu. Find his total crop of wheat.

3. A basket weighing $\frac{3}{16}$ lb. contains $3\frac{1}{2}$ lb. coffee, $4\frac{3}{4}$ lb. butter, and 5 lb. 6 oz. cheese. Find the total weight.

4. The sides of a field are $24\frac{2}{3}$ rd., $42\frac{5}{9}$ rd., $46\frac{7}{12}$ rd., and $38\frac{3}{8}$ rd. Find its perimeter.

5. What is the combined weight of three men, the first of whom weighs $125\frac{1}{4}$ lb., the second $147\frac{3}{8}$ lb., and the third $175\frac{3}{4}$ lb.?

6. A grocer has three barrels of oil; the first contains $18\frac{2}{3}$ gal., the second $24\frac{5}{6}$ gal., and the third $16\frac{3}{4}$ gal. Find how much there was in the three barrels.

7. Four farms join each other; the first contains $125\frac{3}{5}$ A., the second $78\frac{2}{3}$ A., the third $96\frac{1}{2}$ A., and the fourth $110\frac{3}{4}$ A. Find the total area.

8. I paid $\$125\frac{3}{4}$ for a horse and $\$57\frac{1}{2}$ more than that for a carriage. Find the cost of both.

9. A merchant sold four pieces of cloth containing respectively $12\frac{1}{2}$ yd., $15\frac{3}{4}$ yd., $16\frac{2}{3}$ yd., $24\frac{5}{6}$ yd. How many yards did he sell? What did he receive for the cloth at 48¢ a yard?

10. From a piece of cloth there were sold $3\frac{1}{4}$ yd., $7\frac{2}{3}$ yd., $8\frac{3}{4}$ yd., $12\frac{1}{2}$ yd. Find the total value of the cloth at 72¢ a yard.

SUBTRACTION OF FRACTIONS

100. (1) Find the difference between $\frac{5}{6}$ hr. and $\frac{8}{15}$ hr.

The L. C. D. $= 30$.

$$\frac{5}{6} - \frac{8}{15} = \frac{25 - 16}{30} = \frac{9}{30} \text{ or } \frac{3}{10}.$$

∴ the difference $= \frac{3}{10}$ hr.

Prove this result by expressing $\frac{5}{6}$, $\frac{8}{15}$, and $\frac{3}{10}$ hr. in minutes, and taking the difference of the first two.

(2) Find the value of $\$6\frac{29}{36} - \$4\frac{17}{48}$.

$$6 - 4 = 2.$$

The L. C. D. $= 144$.

$$\frac{29}{36} - \frac{17}{48} = \frac{116 - 51}{144} = \frac{65}{144}.$$

∴ the difference $= \$2\frac{65}{144}$.

(3) Subtract $6\frac{7}{12}$ da. from $9\frac{3}{8}$ da.

$$8 - 6 = 2 ; \quad 1\frac{3}{8} = \frac{11}{8}.$$

$$\frac{11}{8} - \frac{7}{12} = \frac{33 - 14}{24} = \frac{19}{24}.$$

$\therefore$ the difference $= 2\frac{19}{24}$ da.

On reducing $\frac{7}{12}$ and $\frac{3}{8}$ to 24ths, it is found that $\frac{7}{12}$ is greater than $\frac{3}{8}$. Hence we break $9\frac{3}{8}$ up into $8 + 1\frac{3}{8}$, and then subtract 6 from 8 and $\frac{7}{12}$ from $1\frac{3}{8}$ or $\frac{11}{8}$.

Exercise 94

Find the value of:

1. $\$\frac{9}{10} - \$\frac{5}{10}$.

2. $\frac{7}{12}$ ft. $- \frac{3}{12}$ ft.

3. $\frac{7}{8}$ pk. $- \frac{3}{8}$ pk.

4. $\frac{3}{4}$ gal. $- \frac{1}{4}$ gal.

5. $\frac{31}{60}$ hr. $- \frac{16}{60}$ hr.

6. $\frac{15}{24}$ da. $- \frac{3}{24}$ da.

7. What is the direct unit of measure in each question? Express your answer in two ways.

8. Prove results by reducing each fraction to the next lower denomination and then subtracting.

Exercise 95

Find the difference as a fraction of any unit of measure:

1. $\frac{3}{4} - \frac{2}{3}$.

2. $\frac{7}{8} - \frac{5}{6}$.

3. $\frac{6}{7} - \frac{2}{3}$.

4. $\frac{8}{15} - \frac{3}{10}$.

5. $\frac{11}{12} - \frac{8}{9}$.

6. $\frac{13}{18} - \frac{5}{12}$.

7. Give the three steps required in subtracting one proper fraction from another.

8. In Exercise 88, examples 4 through 15, subtract the smaller fraction from the larger.

9. $12 - \frac{4}{9}$.

10. $5 - \frac{7}{13}$.

11. $8 - \frac{5}{31}$.

12. $7 - 4\frac{3}{43}$.

13. $9 - 6\frac{13}{51}$.

14. $12 - 8\frac{15}{64}$.

15. $8\frac{9}{16} - 5\frac{5}{12}$.

16. $9\frac{27}{32} - 4\frac{13}{48}$.

17. $18\frac{11}{15} - 12\frac{3}{20}$.

18. $4\frac{17}{22} - 2\frac{31}{33}$.

19. $5\frac{11}{12} - 3\frac{27}{28}$.

20. $4\frac{5}{18} - 1\frac{37}{42}$.

21. $46\frac{4}{9} - 39\frac{6}{7}$.

22. $95\frac{2}{3} - 84\frac{3}{4}$.

Exercise 96

1. A dealer bought chickens at $9\frac{1}{2}\cancel{c}$ per pound and sold them for $12\cancel{c}$ a pound. Find his gain per pound. Find his gain on selling a 6-lb. chicken.

2. A dealer bought turkeys at $10\frac{3}{4}\cancel{c}$ per pound and sold them at $14\cancel{c}$ per pound. Find his gain on a 12-lb. turkey.

3. Dec. 9, 1899, wheat sold in Minneapolis at $66\frac{3}{4}\cancel{c}$ per bushel, and in Toledo at $68\frac{1}{2}\cancel{c}$ per bushel. Find the difference in price.

4. On the same date wheat sold in New York at $74\frac{7}{8}\cancel{c}$ per bushel, and in Liverpool at $83\frac{3}{8}\cancel{c}$ per bushel. Find the difference in price.

5. Dec. 9, 1899, wheat sold in Chicago at $69\frac{3}{4}\cancel{c}$ per bushel, and a year previous to that date at $65\frac{1}{8}\cancel{c}$ per bushel. Find the increase in price.

6. The prices of wheat per bushel Friday, March 19, 1900, and one year previous to that date, are given in the following table. Find the difference in price in each case:

	Friday	Year Ago		Friday	Year Ago
Chicago . . .	$0.66¼	$0.70¼	Duluth . . .	$0.66	$0.69
Minneapolis . .	.63⅞	.68¼	Toledo . . .	.72⅝	.73¼
New York . .	.72¾	.75½	Liverpool . .	.83	.79⅜
St. Louis . . .	.69½	.74¼			

7. I bought wheat at $63\frac{3}{4}\cancel{c}$ per bushel and sold it for $65\frac{1}{2}\cancel{c}$. Find my gain on 8000 bu.

8. The average selling price of wool in Montana in 1899 was $17\frac{1}{2}\cancel{c}$ a pound, and in 1898 was $16\frac{3}{4}\cancel{c}$. The farmers of that state sold 7,000,000 lb. in 1899; how much did they gain on account of the increase in price?

Exercise 97

1. $\frac{2}{5}$ of the value of a horse = $\$60$.

$\frac{1}{5}$ of the value of a horse = ?

$\frac{5}{5}$ of the value of a horse = ?

$\therefore$ the horse is worth $\$$?

Fill out the blanks.

By what do you divide $\$60$? By what do you multiply the result?

2. If $\frac{3}{4}$ of the value of a farm is $\$9000$, what is the value of $\frac{1}{4}$ of the farm? What is the farm worth?

3. A person sold a cow, gaining $\frac{2}{7}$ of the cost price. If he gained $\$12$, what did the cow cost him?

4. A man lost in business $\frac{5}{6}$ of his property. His loss was $\$4500$; what was his property worth?

5. A boy lost $\frac{3}{8}$ of his marbles, and then had 60 left. What fraction of his marbles did he have left? How many had he at first?

6. If $\frac{5}{4}$ of the cost of a gallon of wine is $\$3$, what was the cost?

7. A merchant sold potatoes at $75\rlap{/}c$ a bushel, gaining $\frac{1}{4}$ of the cost. The selling price was what fraction of the cost? What was the cost price of the potatoes per bushel?

8. A merchant sold cloth at $90\rlap{/}c$ a yard and gained $\frac{1}{5}$ of the cost. Find the cost.

9. A merchant sold cloth for $80\rlap{/}c$ a yard, thereby losing $\frac{1}{5}$ of the cost. Find the cost.

MULTIPLICATION OF FRACTIONS

101. (1) Find the cost of 12 yd. of cloth at $\$\frac{3}{4}$ per yard.

We are required to find the quantity measured by the number **12** and the measuring unit $\$\frac{3}{4}$.

$$12 \text{ yd. cost } \tfrac{12}{1} \times \$\tfrac{3}{4} = \$9.$$

(2) Find the cost of $\frac{5}{6}$ yd. at $12 a yard.

The cost is measured by the number $\frac{5}{6}$ and the unit $12.

$$\frac{5}{6} \text{ yd. costs } \frac{5}{6} \text{ of } \$12 = \$10.$$

(3) Find the area of a floor 12 ft. long and 9 ft. 9 in. wide.

The area is measured by the unit $9\frac{3}{4}$ sq. ft., which is the area of 1 strip, and the number 12.

The area of 1 strip $= 9\frac{3}{4}$ or $\frac{39}{4}$ sq. ft.

∴ the total area $= \frac{12}{1} \times \frac{39}{4}$ sq. ft. $= 117$ sq. ft.

(4) Reduce $\frac{8}{9}$ ft. to inches.

$$\frac{8}{9} \text{ ft.} = \frac{8}{\underset{3}{9}} \times \frac{\overset{4}{12}}{1} \text{ in.} = \frac{32}{3} \text{ in.} = 10\frac{2}{3} \text{ in.}$$

Exercise 98

Find the cost of:

1. 10 yd. at $\$\frac{3}{5}$ per yard.   **3.** 9 yd. at $\$2\frac{1}{2}$ a yard.

2. 12 yd. at $\$1\frac{3}{4}$ per yard.   **4.** $\frac{4}{7}$ yd. at $\$6$ a yard.

5. If the cost price is measured by the number $\frac{2}{3}$ and the unit $5, find the cost.

Find the cost of:

6. A 4-lb. chicken at $9\frac{1}{2}¢$ a pound.

A 5-lb. pail of butter at $24\frac{1}{2}¢$ a pound.

A 13-lb. turkey at $11\frac{1}{2}¢$ a pound.

9 lb. cheese at $12\frac{3}{4}¢$ a pound.

A bale of cotton (1 bale = 500 lb.) at $9\frac{3}{4}¢$ a pound.

A bale of cotton at $9\frac{3}{16}¢$ a pound.

233 bales of cotton at $8\frac{3}{4}¢$ a pound.

A 10-lb. ham at $8\frac{3}{4}¢$ a pound.

8 lb. fish at $6\frac{3}{4}¢$ a pound.

12 doz. eggs at $12\frac{1}{2}¢$ a dozen.

Reduce to inches:

7. $\frac{5}{6}$ ft.; $\frac{3}{8}$ ft.   **8.** $2\frac{3}{4}$ ft.; $1\frac{5}{9}$ ft.   **9.** $5\frac{3}{7}$ ft.; $2\frac{4}{5}$ ft.

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Reduce to yards:

10. 8 rd.; 10 rd.; 12 rd.; 3 rd.; 9 rd.; 17 rd.

Reduce to feet:

11. 4 rd.; 8 rd.; 10 rd.; 3 rd.; 7 rd.; 13 rd.

12. Find the cost of 4 rd. wire fencing at $2\frac{3}{4}$ ¢ a foot.

13. Find the cost of the wire fencing at $3\frac{3}{4}$ ¢ a foot, needed to enclose a square garden 2 rd. on a side.

Reduce to quarts:

14. $3\frac{5}{8}$ gal.; $2\frac{5}{12}$ gal.; $5\frac{3}{16}$ gal.; $12\frac{11}{32}$ gal.

15. Find the cost of $2\frac{7}{8}$ gal. of milk at 6 ¢ a quart.

Find the areas of the floors of the following rooms:

16.	Length	Width	Length	Width
	24 ft.	14 ft. 10 in.	19 ft.	16 ft. 4 in.
	14 ft.	12 ft. 6 in.	22 ft.	16 ft. 9 in.

Find the area of the walls of a room whose:

17. Perimeter is 80 ft., height 9 ft. 6 in.

18. Perimeter is 63 ft., height 10 ft. 8 in.

19. Perimeter is 66 ft., height 8 ft. 9 in.

102. (1) Find the cost of $12\frac{3}{4}$ yd. of cloth at $\$1\frac{3}{8}$ a yard.

$12\frac{3}{4}$ yd. cost $12\frac{3}{4} \times \$1\frac{3}{8} = \frac{51}{4} \times \$\frac{11}{8} = \$\frac{561}{32} = \$17\frac{17}{32} = \$17.53$.

(2) Find the area of the four walls of a room whose perimeter is 62 ft. 8 in., and height 8 ft. 9 in.

The perimeter $= 62$ ft. 8 in. $= 62\frac{2}{3}$ ft. $= \frac{188}{3}$ ft.

The height $= 8$ ft. 9 in. $= 8\frac{3}{4}$ ft. $= \frac{35}{4}$ ft.

$\therefore$ the area $= \frac{\overset{47}{\cancel{188}}}{3} \times \frac{35}{\cancel{4}}$ sq. ft. $= 548\frac{1}{3}$ sq. ft.

(3) Reduce $\frac{8}{9}$ rd. to yards.

$\frac{8}{9}$ rd. $= \frac{8}{9} \times 5\frac{1}{2}$ yd. $= \frac{8}{9} \times \frac{11}{\underset{}{\cancel{2}}}^{\,4} = \frac{44}{9}$ or $4\frac{8}{9}$ yd.

Exercise 99

Find the value of:

1. $3\frac{3}{4} \times \$1\frac{3}{5}$ $\frac{7}{8} \times \frac{2}{3}$ lb. $\frac{25}{33} \times 1\frac{9}{35}$

     $2\frac{1}{4} \times \$5\frac{1}{3}$ $\frac{5}{6} \times \frac{4}{9}$ oz. $2\frac{2}{9} \times 4\frac{4}{5}$

     $4\frac{2}{3} \times \$1\frac{2}{7}$ $\frac{3}{5} \times \frac{6}{7}$ yd. $3\frac{6}{25} \times 1\frac{19}{36}$

     $1\frac{7}{8} \times \$2\frac{2}{5}$ $\frac{7}{8} \times \frac{7}{8}$ yd. $4\frac{9}{10} \times 1\frac{11}{14}$

     $3\frac{8}{9} \times \$2\frac{1}{5}$ $\frac{9}{10} \times \frac{9}{10}$ mi. $5\frac{3}{5} \times 1\frac{4}{21}$

2. Find the cost of a remnant of oilcloth containing $5\frac{1}{3}$ yd. at $6\frac{3}{4}\cancel{\phi}$ a yard.

Find the cost of:

3. $8\frac{1}{2}$ lb. cheese at $9\frac{3}{4}\cancel{\phi}$ a pound.

     $12\frac{1}{2}$ lb. turkey at $12\frac{1}{2}\cancel{\phi}$ a pound.

     $11\frac{1}{2}$ lb. turkey at $10\frac{3}{4}\cancel{\phi}$ a pound.

     $20\frac{3}{4}$ lb. chickens at $10\frac{1}{2}\cancel{\phi}$ a pound.

     $8\frac{1}{2}$ lb. veal at $7\frac{1}{2}\cancel{\phi}$ a pound.

     $4\frac{1}{2}$ lb. butter at $22\frac{1}{2}\cancel{\phi}$ a pound.

4. I bought cotton at $8\frac{1}{4}\cancel{\phi}$ a pound and sold it for $9\frac{3}{8}\cancel{\phi}$ a pound. Find my gain on one bale (1 bale = 500 lb.).

5. I bought cotton at $8\frac{1}{4}\cancel{\phi}$ a pound and sold it for $9\frac{7}{16}\cancel{\phi}$ a pound. Find my gain on 600 bales.

Reduce to yards:

6. $\frac{6}{11}$ rd.; $1\frac{9}{11}$ rd.; $\frac{4}{5}$ rd.; $\frac{3}{4}$ rd.; $\frac{7}{8}$ rd.; $2\frac{1}{3}$ rd.; $5\frac{6}{11}$ rd.; $3\frac{6}{13}$ rd.

7. Find the cost of $9\frac{9}{11}$ rd. of wire fencing at $11\frac{1}{2}\cancel{\phi}$ a yard.

8. Find the cost of $\frac{8}{11}$ rd. of wire fencing at $2\frac{3}{4}\cancel{\phi}$ a foot.

Find the cost of the following:

9. $8\frac{1}{8}$ yd. at $\$3\frac{1}{5}$ a yard; $4\frac{3}{7}$ yd. at $\$2\frac{3}{4}$ a yard;

     $20\frac{2}{3}$ yd. at $\$7\frac{1}{2}$ a yard; $5\frac{1}{4}$ yd. at $\$2\frac{2}{3}$ a yard.

10. $21\frac{3}{5}$ lb. of sugar at $5\frac{1}{4}$ ¢ a pound;

$13\frac{1}{3}$ lb. of sugar at $5\frac{1}{2}$ ¢ a pound.

11. $17\frac{2}{9}$ yd. of cotton at $11\frac{1}{4}$ ¢ a yard.

12. $15\frac{1}{2}$ doz. of eggs at $14\frac{3}{4}$ ¢ a dozen.

13. $8\frac{1}{4}$ T. of hay at $\$11\frac{5}{9}$ a ton.

14. Find the area of a floor whose dimensions are: (1) $10\frac{2}{3}$ ft., $9\frac{3}{4}$ ft.; (2) $14\frac{1}{4}$ ft., $13\frac{1}{3}$ ft.; (3) $15\frac{3}{4}$ ft., $12\frac{2}{3}$ ft.

15. Find the area of the ceiling of a room whose dimensions are: (1) 12 ft. 6 in., 10 ft. 8 in.; (2) 16 ft. 4 in., 11 ft. 3 in.; (3) 18 ft. 8 in., 15 ft. 3 in.

16. At 40 ¢ a square yard find the cost of plastering the ceiling of a hall $16\frac{2}{3}$ yd. long and $12\frac{3}{4}$ yd. wide.

17. Find the area of the four walls of a room whose perimeter and height are respectively: (1) 52 ft. 6 in., 9 ft. 4 in.; (2) 63 ft. 4 in., 10 ft. 6 in.

18. Draw a line 16 in. long. Mark off $\frac{5}{8}$ of it. The remainder is what part of the line? Mark off $\frac{2}{3}$ of the remainder. This is what part of the whole line? $\frac{2}{3}$ of $\frac{3}{8} = ?$

19. If I draw a line and mark off $\frac{1}{4}$ of it, and then mark off $\frac{2}{3}$ of the remainder, what part of the whole line will I mark off the second time? Illustrate this by drawing a line and marking it off. $\frac{2}{3}$ of $\frac{3}{4} = ?$

20. On $\frac{3}{4}$ of a field I planted potatoes; on $\frac{2}{3}$ of the remainder I sowed wheat. What part of the field did I sow with wheat?

21. I withdrew from the bank $\frac{5}{8}$ of my deposit and then $\frac{4}{9}$ of the remainder. What part of the original deposit did I take out the second time?

22. A man who owns $\frac{9}{16}$ of a ship sells $\frac{1}{3}$ of his share. What fraction of his former share does he still own? What fraction of the ship? If he had sold $\frac{2}{3}$ of his share, what part of the ship would he have still owned?

23. A grain dealer invested $\frac{3}{8}$ of his money in wheat, and $\frac{4}{5}$ of the remainder in oats. What part of his money did he invest in oats? If he invested $3000 in oats, how much did he have at first?

24. The owner of a farm valued at $12,000 sells $\frac{3}{5}$ of it to one man, and $\frac{1}{2}$ of the remainder to another. What part of the farm does he sell to the second man, and what should he get for it?

25. Four brothers enter into partnership; the eldest puts in $\frac{1}{3}$ of the capital and the others the remainder in equal shares. What part of the entire capital does each of the younger brothers put in? If they each put in $2000, what is the entire capital?

26. If I own $\frac{3}{5}$ of $\frac{5}{6}$ of a business, what part do I own? If I sell $\frac{1}{4}$ of my share, what part of the business do I still own?

27. A man left his farm to be divided among his three sons; the oldest got 80 A., the second $\frac{1}{3}$ of the farm, and the youngest $\frac{2}{5}$ as much as the other two. Prove that the farm contained 210 A.

28. If the loss is measured by the number $\frac{2}{3}$ and the unit $17\frac{1}{4}$, what is the loss?

If the gain is measured by the number $\frac{2}{7}$ and the unit $11\frac{2}{3}$, what is the gain?

CANCELLATION

103. (1) Find the product of $1\frac{2}{35} \times 6\frac{2}{9} \times 7\frac{11}{12}$.

Reducing to improper fractions and cancelling, the product

$$= \frac{\overset{}{12}}{\underset{7}{35}} \times \frac{\overset{8}{56}}{9} \times \frac{\overset{19}{95}}{12} = \frac{152}{9} = 16\frac{8}{9}.$$

(2) Find the volume of a solid whose dimensions are $2\frac{3}{5}$ in., $2\frac{2}{3}$ in., and $4\frac{1}{2}$ in.

The volume $= 2\frac{3}{5} \times 2\frac{2}{3} \times 4\frac{1}{2}$ cu. in.

$= \frac{13}{5} \times \frac{8}{3} \times \frac{9}{2}$ cu. in.

$= \frac{156}{5}$ or $31\frac{1}{5}$ cu. in.

How do you find the volume of a rectangular solid?

Exercise 100

In the following exercise, indicate the operations as in the preceding paragraph, and cancel where possible:

Find the volume of the solids whose dimensions are:

1. 2 in., 4 in., $6\frac{1}{2}$ in.

2. $1\frac{1}{5}$ in., $2\frac{1}{2}$ in., $2\frac{2}{3}$ in.

3. $2\frac{1}{12}$ in., $2\frac{2}{35}$ in., $4\frac{2}{3}$ in.

4. $\frac{3}{16}$ ft., $2\frac{2}{5}$ ft., $4\frac{4}{9}$ ft.

5. $\frac{2}{3}$ ft., $\frac{6}{7}$ ft., $\frac{5}{12}$ ft.

6. $\frac{6}{35}$ yd., $2\frac{6}{11}$ yd., $8\frac{1}{4}$ yd.

Find the product of:

7. $\frac{14}{3} \times \frac{6}{7} \times \frac{5}{12}$.

8. $\frac{3}{13} \times \frac{5}{6} \times \frac{26}{33}$.

9. $\frac{10}{21} \times \frac{1}{8} \times \frac{28}{15}$.

10. $\frac{17}{56} \times 2\frac{3}{23} \times 13\frac{1}{7}$.

11. $\frac{4}{27} \times 2\frac{1}{7} \times 3\frac{15}{16}$.

12. $16\frac{1}{4} \times 4\frac{4}{15} \times 2\frac{3}{26}$.

13. Find the weight of a solid $4\frac{1}{2}$ in. long, $1\frac{1}{4}$ in. wide, $\frac{2}{3}$ in. thick, if 1 cu. in. weighs $\frac{4}{5}$ oz.

14. Find the weight of a piece of lumber 10 ft. long, $\frac{2}{3}$ ft. wide, $\frac{1}{6}$ ft. thick, if 1 cu. ft. weighs $21\frac{3}{5}$ lb.

15. Find the weight of $\frac{3}{4}$ of a piece of lumber 16 ft. long, $\frac{5}{6}$ ft. wide, $\frac{1}{6}$ ft. thick, if 1 cu. ft. weighs $15\frac{9}{10}$ lb.

16. What is the weight of a piece of floor oilcloth $1\frac{1}{2}$ yd. long, $1\frac{1}{3}$ yd. wide, if 1 sq. yd. weighs $3\frac{1}{2}$ lb. ?

17. What is the weight of one dozen floor oilcloths, each $2\frac{1}{2}$ yards square, weighing $3\frac{1}{2}$ lb. to the square yard ?

18. If an acre of land produces 42 bu. of potatoes, how much will $\frac{2}{3}$ of $\frac{5}{7}$ of an acre produce ? What is their value at 40 ¢ a bushel ?

19. If an acre of land produces $43\frac{1}{5}$ bu. of potatoes, what is the value of the potatoes grown on $\frac{3}{4}$ of $\frac{2}{3}$ A. at 35 ¢ a bushel?

20. If a certain number of men can do a piece of work in 24 da., how long will it take them to do a similar piece of work $\frac{5}{6}$ as hard and $\frac{2}{3}$ as great ? How long if $\frac{3}{4}$ as hard and $1\frac{1}{2}$ times as great ?
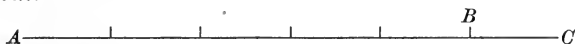
21. If a certain number of men can do a piece of work in 18 da., how many days will they take to do a similar piece $1\frac{1}{3}$ times as hard and $1\frac{1}{2}$ times greater, if they work $\frac{9}{10}$ as long each day?

DIVISION OF FRACTIONS

A————|————|————|————|————B

104. The quantity AB, in the above drawing, is represented by $\frac{5}{5}$ of itself. We think of the quantity as made up of 5 units, and the unit as equal to $\frac{1}{5}$ of the quantity. Hence we may think of any quantity as equal to $5 \times \frac{1}{5}$ (*i.e. five times* one-fifth) of itself.

What, then, is the meaning of the terms 5 and $\frac{1}{5}$ considered separately? 5 shows the ratio of the quantity to the *unit of measure*, and $\frac{1}{5}$ shows the ratio of the unit of measure to the quantity. Numbers thus mutually related are said to be *reciprocal*.

A————|————|————|————|——B—|———C

Again, let the line AB, which is divided into 5 equal parts, represent the primary unit, \$1. Then AC represents the quantity denoted by \$$\frac{6}{5}$. Hence it is evident from the diagram that $\frac{6}{5}$ is the ratio of AC to AB, *i.e.* of the quantity denoted by \$$\frac{6}{5}$ to the primary unit, \$1; also, that its reciprocal $\frac{5}{6}$ is the ratio of AB to AC, *i.e.* of the primary unit, \$1, to the quantity denoted by \$$\frac{6}{5}$. Similarly, the ratio of \$1 (which contains 2 units) to the quantity \$$\frac{3}{2}$ (which contains 3 units) is equal to $\frac{2}{3}$, *i.e.* to the *reciprocal* of $\frac{3}{2}$.

Exercise 101

1. Draw a line, AB, divided into 6 equal parts. The line AB is represented by how many units? by what fraction of itself? What is the ratio of AB to the unit? of the unit to AB? 6 and $\frac{1}{6}$ are called *reciprocals* of each other.

2. What is the reciprocal of 5? 3? 4? 6? 8? 9?

3. What is the reciprocal of $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{6}$? $\frac{1}{8}$? $\frac{1}{4}$? $\frac{1}{12}$?

4. What is the reciprocal of 3? $\frac{1}{4}$? $\frac{1}{9}$? 7? $\frac{1}{10}$? $\frac{1}{15}$?

5. What is the ratio of $\frac{1}{6}$ ft. to 1 ft.? Of 1 ft. to $\frac{1}{6}$ ft.? Show by a drawing.

6. What is the ratio of $\frac{1}{3}$ ft. to 1 ft.? Of 1 ft. to $\frac{1}{3}$ ft.? Of \$$\frac{1}{4}$ to \$1? Of \$1 to \$$\frac{1}{4}$? Of 1 yd. to $\frac{1}{6}$ yd.? Of $\frac{1}{6}$ yd. to 1 yd.?

7. What is the ratio of 1 qt. to $\frac{1}{8}$ qt.? Of $\frac{1}{9}$ gal. to 1 gal.? Of $\frac{1}{8}$ mi. to 1 mi.? Of 1 T. to $\frac{1}{5}$ T.? Of 1 da. to $\frac{1}{12}$ da.? Of $\frac{1}{10}$ min. to 1 min.? Of $\frac{1}{11}$ to 1? Of 1 to $\frac{1}{15}$?

8. Multiply $\frac{2}{3}$ by the reciprocal of 6.

9. Multiply $\frac{3}{4}$ by the reciprocal of each of the following numbers: 3, 6, 9, 15, 2, 18, 5, 11.

10. Multiply $1\frac{2}{3}$ by the reciprocal of $\frac{1}{6}$.

11. Multiply $2\frac{3}{4}$ by the reciprocal of each of the following numbers: $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{5}$, $\frac{1}{18}$, 7, $\frac{1}{24}$.

12. Multiply $\frac{4}{5}$ by the reciprocal of each of these numbers: 4, $\frac{1}{5}$, $\frac{1}{10}$, 6, $\frac{1}{3}$, 8, 12, $\frac{1}{4}$.

13. Draw a line, AB, 1 ft. long. Mark off AC equal to $\frac{3}{4}$ ft. What is the ratio of AC to AB? What is the ratio of AB to AC? What is the ratio of $\frac{3}{4}$ ft. to 1 ft.? Of 1 ft. to $\frac{3}{4}$ ft.? $\frac{3}{4}$ and $\frac{4}{3}$ are called reciprocals of each other.

14. What is the ratio of $\frac{2}{3}$ ft. to 1 ft.? Of 1 ft. to $\frac{2}{3}$ ft.? Show by a drawing.

15. What is the ratio of $\frac{5}{6}$ yd. to 1 yd.? Of $\frac{7}{8}$ lb. to 1 lb.? Of $\frac{5}{9}$ da. to 1 da.? Of $\frac{8}{7}$ wk. to 1 wk.?

16. What is the ratio of 1 mi. to $\frac{4}{5}$ mi.? Of 1 T. to $\frac{9}{10}$ T.? Of 1 A. to $\frac{4}{3}$ A.? Of 1 A. to $\frac{3}{2}$ A.?

17. What is the ratio of $\frac{3}{5}$ lb. to 1 lb.? Of 1 oz. to $\frac{5}{6}$ oz.? Of 1 gal. to $\frac{9}{7}$ gal.? Of $\frac{3}{8}$ qt. to 1 qt.? Of 1 doz. to $1\frac{1}{6}$ doz.? Of \$1 to \$$\frac{8}{5}$?

18. What is the reciprocal of $\frac{2}{3}$? $\frac{3}{4}$? $\frac{5}{6}$? $\frac{3}{8}$? $\frac{7}{3}$? $\frac{5}{4}$? $\frac{3}{5}$? $\frac{12}{7}$? $1\frac{5}{7}$? $2\frac{1}{2}$? $1\frac{3}{4}$? $4\frac{1}{2}$?

19. Multiply $\frac{2}{3}$ by the reciprocal of $\frac{4}{3}$.   Multiply $\frac{5}{6}$ by the reciprocal of $\frac{15}{4}$.

20. Multiply $\frac{3}{4}$ by the reciprocal of each of the following fractions: $\frac{3}{2}$, $\frac{3}{8}$, $\frac{5}{6}$, $\frac{9}{4}$, $\frac{7}{5}$, $2\frac{1}{2}$, $\frac{4}{3}$, $2\frac{1}{4}$, $1\frac{1}{8}$, $\frac{1}{6}$.

Exercise 102

1. Make drawings to show that (1) 1 ft. ÷ $\frac{1}{4}$ ft. = 4.   (2) 1 ft. ÷ $\frac{1}{6}$ ft. = 6.

2.   1 ft. ÷ $\frac{1}{3}$ ft. = ?   1 A. ÷ $\frac{1}{6}$ A. = ?   1 da. ÷ $\frac{1}{12}$ da. = ?

1 yd. ÷ $\frac{1}{2}$ yd. = ?   1 T. ÷ $\frac{1}{10}$ T. = ?   1 yr. ÷ $\frac{1}{15}$ yr. = ?

1 lb. ÷ $\frac{1}{4}$ lb. = ?   1 bu. ÷ $\frac{1}{8}$ bu. = ?

3. How many meals will 1 bu. of oats last a horse, if he has $\frac{1}{8}$ bu. a meal?

4. Into how many lots, each $\frac{1}{8}$ A. can you divide 1 A.?

5. Make drawings to show that

(1) 3 ft. ÷ $\frac{1}{4}$ ft. = 12   (2) $2\frac{1}{2}$ ft. ÷ $\frac{1}{4}$ ft. = 10

(3) $2\frac{2}{3}$ ft. ÷ $\frac{1}{3}$ ft. = 8   (4) $1\frac{2}{3}$ ft. ÷ $\frac{1}{6}$ ft. = 10

6. Multiply 3 by the reciprocal of $\frac{1}{4}$; $2\frac{1}{2}$ by the reciprocal of $\frac{1}{4}$; $2\frac{2}{3}$ by the reciprocal of $\frac{1}{3}$; $1\frac{2}{3}$ by the reciprocal of $\frac{1}{6}$.   What is the result in each case?

By what do you multiply $2\frac{3}{4}$ to find the value of $2\frac{3}{4} \div \frac{1}{8}$?

7.   2 ft. ÷ $\frac{1}{6}$ ft. = ? $3\frac{3}{4} \div \frac{1}{8}$ = ?

$3\frac{1}{3}$ ft. ÷ $\frac{1}{6}$ ft. = ? $7\frac{1}{2} \div \frac{1}{6}$ = ?

$4\frac{1}{2}$ yd. ÷ $\frac{1}{2}$ yd. = ? $3\frac{1}{4} \div \frac{1}{2}$ = ?

$5\frac{1}{2}$ hr. ÷ $\frac{1}{4}$ hr. = ? $2\frac{1}{6} \div \frac{1}{3}$ = ?

$2\frac{5}{6}$ da. ÷ $\frac{1}{12}$ da. = ? $3\frac{7}{8} \div \frac{1}{4}$ = ?

8. Show by a drawing that $2\frac{1}{6} \div \frac{1}{3} = 6\frac{1}{2}$.

9. A merchant put up $7\frac{1}{2}$ lb. tea in packages, each containing $\frac{1}{4}$ lb.   Find the number of packages.

10. A piece of ribbon $3\frac{1}{2}$ yd. long is cut into parts each $\frac{1}{2}$ yd. long. How many parts?

11. Draw a line 4 ft. long. Divide it into parts each $\frac{1}{3}$ ft. long. How many? How many of these 12 parts make $\frac{2}{3}$ ft.? How many parts, each $\frac{2}{3}$ ft. long, can you count in these 12 parts?

$$4 \text{ ft.} \div \tfrac{2}{3} \text{ ft.} = ? \qquad\qquad 6 \text{ ft.} \div \tfrac{2}{3} \text{ ft.} = ?$$

12. In the first part of example 11, by what did you multiply 4 to get 12? By what did you divide 12 to get 6?

13. How can you divide a number like 4 by a fraction? (*To divide a number by a fraction multiply the number by the reciprocal of the fraction.*)

14. Find the quotient in each of the following examples:

9 ft. $\div \frac{3}{4}$ ft.	\$6 $\div$ \$$\frac{3}{5}$	5 $\div$ $\frac{2}{3}$
8 yd. $\div \frac{2}{3}$ yd.	\$12 $\div$ \$ $\frac{4}{3}$	7 $\div$ $\frac{3}{4}$
12 mi. $\div \frac{6}{7}$ mi.	\$10 $\div$ \$ $\frac{5}{8}$	9 $\div$ $\frac{6}{7}$
16 lb. $\div \frac{4}{9}$ lb.	\$10 $\div$ \$1$\frac{2}{3}$	21 $\div$ $3\frac{1}{2}$
12 T. $\div \frac{3}{5}$ T.	\$15 $\div$ \$2$\frac{1}{2}$	16 $\div$ $2\frac{2}{5}$

15. How many strips of carpet $\frac{3}{4}$ yd. wide are needed to carpet a room 6 yd. wide? If the room is 8 yd. long how many yards of carpet are needed for it?

16. What part of a yard is 32 in.? How many strips of hemp carpet 32 in. wide are needed to carpet a hall 16 yd. wide? If the hall is 20 yd. long, how many yards of carpet are needed?

17. As in examples 5 and 8, make a drawing to show that $2\frac{1}{4}$ ft. $\div \frac{3}{4}$ ft. = 3.

18. How can you find the value of $2\frac{1}{4} \div \frac{3}{4}$, without actually dividing one quantity by another?

19. *To divide one fraction by another, multiply the first fraction by the reciprocal of the second.*

20. Find the value of:

$\frac{2}{3} \div \frac{1}{6}$	$3\frac{3}{5} \div \frac{3}{10}$	$26\frac{1}{4} \div 5\frac{8}{11}$
$\frac{3}{4} \div \frac{3}{8}$	$4\frac{3}{8} \div 5\frac{1}{4}$	$41\frac{1}{4} \div 4\frac{2}{5}$
$\frac{6}{5} \div \frac{2}{9}$	$7\frac{1}{3} \div 5\frac{1}{2}$	$1\frac{53}{63} \div \frac{29}{42}$
$\frac{7}{15} \div 1\frac{4}{5}$	$5\frac{5}{8} \div 4\frac{1}{6}$	$9\frac{5}{7} \div 2\frac{1}{8}$
$\frac{4}{11} \div \frac{5}{2}$	$23\frac{3}{8} \div 2\frac{1}{4}$	$45\frac{5}{6} \div 14\frac{2}{3}$

21. At $12\frac{1}{2}\cent$ a pound, how many pounds of chicken will cost $50\cent$?

22. Find the number of square yards in a piece of floor oil-cloth, weighing $7\frac{1}{2}$ lb. at $2\frac{1}{2}$ lb. per square yard. At $1\frac{1}{2}$ lb. per square yard.

23. A double roll of wall paper weighs $1\frac{3}{4}$ lb. Find how many double rolls weigh $8\frac{3}{4}$ lb.

24. How many double rolls of wall paper weigh $10\frac{1}{2}$ lb.? Find their cost at $25\cent$ a double roll.

25. Find the cost of a parcel of wall paper, containing $15\frac{3}{4}$ lb., at $20\cent$ a double roll.

26. Find the quotients:

$33\frac{1}{3} \div 10$	$330\frac{3}{4} \div 21$	$352\frac{1}{3} \div 9\frac{1}{3}$
$37\frac{4}{5} \div 9$	$233\frac{1}{3} \div 8\frac{1}{3}$	$230\frac{5}{8} \div 11\frac{1}{4}$

105. (1) How do you reduce feet to yards?

Reduce $2\frac{1}{2}$ ft. to a fraction of 1 yd.

$$2\frac{1}{2} \text{ ft.} = 2\frac{1}{2} \div 3 \text{ or } \frac{5}{2} \times \frac{1}{3} \text{ yd.} = \frac{5}{6} \text{ yd.}$$

(2) Find the number of strips required to carpet a room 12 ft. wide with carpet $2\frac{1}{2}$ ft. wide.

The number of strips $= 12 \text{ ft.} \div 2\frac{1}{2} \text{ ft.} = \frac{12}{1} \times \frac{2}{5} = 4\frac{4}{5}$, *i.e.* 5.

$\therefore$ 5 strips are required.

106. (1) 2 yd. of cloth cost 72 ¢, what will 1 yd. cost?

(2) 3 yd. of cloth cost 75¢, what will 1 yd. cost?

(3) $\frac{3}{4}$ yd. of cloth cost 48¢, what will 1 yd. cost?

In example (1) what is the divisor of 72 ¢? In example (2) what is 3? In example (3) what is $\frac{3}{4}$?

$\frac{3}{4}$ yd. costs 48 ¢.

1 yd. costs 48 ¢ ÷ $\frac{3}{4}$ = 48 ¢ × $\frac{4}{3}$ = 64 ¢.

In this example the whole measured quantity is 48 ¢, and the number is $\frac{3}{4}$. We find the measuring unit by dividing 48 ¢ by $\frac{3}{4}$.

Exercise 103

Find the number of pounds of:

1. Veal at $7\frac{1}{2}$ ¢ a pound that cost 45 ¢.

2. Chicken at $8\frac{3}{4}$ ¢ a pound that cost $52\frac{1}{2}$ ¢.

3. Cheese at $12\frac{3}{4}$ ¢ a pound that cost \$1.27$\frac{1}{2}$.

4. Turkey at $12\frac{1}{2}$ ¢ a pound that cost \$ 1.56$\frac{1}{4}$.

Find the number of:

5. Barrels of flour at \$ $5\frac{3}{4}$ a barrel that cost \$103$\frac{1}{2}$.

6. Rods of barbed wire fencing in a roll weighing $25\frac{1}{2}$ lb., if 1 rd. weighs $1\frac{1}{16}$ lb.

7. Yards of carpet, weighing 24 oz. to 1 yd., in a roll weighing $11\frac{1}{4}$ lb.

8. $\frac{3}{4}$ of the time I took to do a piece of work is $\frac{9}{10}$ of an hour. How long did I take to do it? ($\frac{9}{10}$ hr. ÷ $\frac{3}{4}$.)

9. $\frac{2}{3}$ yd. of cloth cost \$ $\frac{2}{5}$, what will 1 yd. cost?

10. $\frac{3}{4}$ yd. of cloth cost \$ $\frac{6}{5}$, what will 1 yd. cost?

11. $\frac{4}{5}$ yd. of carpet weighs $\frac{5}{4}$ lb., find the weight of 1 yd. How many ounces?

12. · If $\frac{5}{6}$ yd. of carpet weighs $1\frac{1}{4}$ lb., how many ounces will 1 yd. weigh?

13. If $2\frac{1}{2}$ doz. silver forks weigh $3\frac{1}{8}$ lb., how many pounds will 1 doz. weigh? How many ounces?

14. Reduce to the fraction of a foot: $1\frac{1}{5}$ in., $2\frac{1}{4}$ in., $4\frac{1}{2}$ in., $2\frac{2}{3}$ in.

15. Reduce to the fraction of a yard: $1\frac{1}{2}$ ft., $2\frac{2}{3}$ ft., $1\frac{1}{5}$ ft., $2\frac{1}{7}$ ft.

16. What is the cost of a piece of carpet $2\frac{1}{4}$ ft. long at 80 ⊄ a yard?

17. Reduce to fractions of one dollar: \$ .12$\frac{1}{2}$, \$ .37$\frac{1}{2}$, \$ .62$\frac{1}{2}$, \$ .87$\frac{1}{2}$, \$ .16$\frac{2}{3}$, \$ .06$\frac{1}{4}$, \$ .06$\frac{2}{3}$.

18. At one dollar a day, how long will it take a boy to earn \$ .37$\frac{1}{2}$? \$ 1.62$\frac{1}{2}$?

19. Find the lengths of the following rooms:

Area 300   sq. ft., width 15 ft.

Area $466\frac{2}{3}$ sq. ft., width 20 ft.

Area $242\frac{1}{2}$ sq. ft., width 15 ft.

Area $681\frac{1}{4}$ sq. ft., width 25 ft.

Area $877\frac{1}{2}$ sq. ft., width 27 ft.

20. Reduce to the fraction of a rod: $2\frac{3}{4}$ yd., $1\frac{3}{8}$ yd., $4\frac{1}{2}$ yd., $6\frac{7}{8}$ yd., $4\frac{3}{17}$ yd.

21. Find and prove by actual measurement the following: $2\frac{1}{3}$ ft. $\div \frac{1}{3}$ ft.; $1\frac{1}{4}$ ft. $\div \frac{1}{6}$ ft.; $2\frac{1}{4}$ ft. $\div \frac{1}{8}$ ft.; $1\frac{7}{8}$ ft. $\div \frac{1}{4}$ ft.; $4\frac{1}{6}$ ft. $\div \frac{5}{12}$ ft.

22. Find the number of strips of carpet required to carpet each of the following rooms:

Width, 22 ft. 6 in.; width of carpet, 27 in.   ($22\frac{1}{2}$ ft. $\div 2\frac{1}{4}$ ft.)

Width, 15 ft. 9 in.; width of carpet, 27 in.

Width, 10 ft. 8 in.; width of carpet, 32 in.

Width, 14 ft. 8 in.; width of carpet, 32 in.

23. How many yards of carpet are needed to carpet the last room in the preceding example, if its length is 5 yd.? 6 yd.? 21 ft.? 24 ft.?

24. 27 in. is what part of a yard ? How many yards of carpet are needed to carpet a room 8 yd. long and $6\frac{3}{4}$ yd. wide with carpet 27 in. wide, the carpet running lengthwise ?

Make a drawing to illustrate this. (Scale 1 in. to 1 yd.)

25. How many yards of hemp carpet, 32 in. wide, are needed to carpet a room 12 yd. long and 10 yd. 2 ft. wide, the carpet running lengthwise ? Find the cost of this carpet at $24 \not\! c$ a yard.

26. Find the length of each of the following rooms :

Area of floor, $126\frac{3}{4}$ sq. yd. ; width, 8 yd. 2 ft.

Area of floor, $133\frac{19}{32}$ sq. yd. ; width, $9\frac{3}{8}$ yd.

Area of floor, $334\frac{11}{16}$ sq. ft. ; width, 15 ft. 9 in.

Area of floor, $253\frac{1}{3}$ sq. ft. ; width, 10 ft. 8 in.

27. Find the perimeter of each of the following rooms :

Area of the Walls	Height
$533\frac{1}{3}$ sq. ft.	8 ft. 4 in.
$704\frac{2}{3}$ sq. ft.	9 ft. 4 in.
$877\frac{1}{5}$ sq. ft.	10 ft. 9 in.
$65\frac{7}{9}$ sq. yd.	2 yd. 2 ft.

107. What is the ratio of 4 da. to 6 da. ?

The ratio of 4 da. to 6 da. $= 4 \div 6 = \frac{4}{6} = \frac{2}{3}$.

What is the ratio of $2\frac{1}{2}$ da. to $3\frac{1}{3}$ da. ?

The ratio of $2\frac{1}{2}$ da. to $3\frac{1}{3}$ da. $= 2\frac{1}{2} \div 3\frac{1}{3} = \frac{5}{2} \div \frac{10}{3} = \frac{5}{2} \times \frac{3}{10} = \frac{3}{4}$. What is the ratio of $3\frac{1}{3}$ da. to $2\frac{1}{2}$ da. ?

Exercise 104

1. What is the ratio of :

$1\frac{1}{3}$ to 4	$3\frac{3}{4}$ to $2\frac{1}{2}$	$\frac{8}{9}$ to $\frac{5}{6}$	$7\frac{11}{27}$ to $17\frac{7}{9}$
$2\frac{2}{3}$ to $6\frac{2}{3}$	$3\frac{1}{3}$ to $3\frac{3}{4}$	$\frac{5}{22}$ to $\frac{4}{11}$	$18\frac{3}{4}$ to $13\frac{1}{3}$
$6\frac{2}{3}$ to $2\frac{2}{3}$	$\frac{2}{3}$ to $\frac{3}{4}$	$6\frac{1}{4}$ to $7\frac{1}{2}$	$16\frac{1}{5}$ to $23\frac{1}{3}$
$2\frac{1}{2}$ to $3\frac{3}{4}$	$\frac{8}{9}$ to $\frac{5}{6}$	$17\frac{1}{4}$ to $3\frac{5}{6}$	$52\frac{1}{2}$ to $18\frac{2}{3}$

2. What is the ratio of $2\frac{2}{3}$ to $4\frac{4}{5}$? If $4\frac{4}{5}$ T. of coal cost \$27, what part of \$27 will $2\frac{2}{3}$ T. cost? How much?

3. What is the ratio of $\frac{5}{6}$ to $3\frac{1}{3}$? Of $3\frac{1}{3}$ to $\frac{5}{6}$?

If a boy rides 7 mi. in $\frac{5}{6}$ hr., how far can he ride in $3\frac{1}{3}$ hr. at the same rate?

4. A piece of matting, weighing $2\frac{1}{2}$ lb. to the square yard, weighs 120 lb. Find the weight of a piece of the same size, weighing $3\frac{1}{2}$ lb. to the square yard.

5. I paid \$20.80 for silver spoons, at \$$6\frac{2}{5}$ a dozen. Find the cost of the same number at \$$7\frac{1}{5}$ a dozen.

6. If $6\frac{2}{5}$ T. of hay cost \$42, what part of \$42 will $1\frac{13}{35}$ T. cost? How much will $1\frac{13}{35}$ T. cost?

7. If \$$\frac{2}{25}$ is represented by unity, what number will represent \$$4\frac{2}{35}$?

8. Find the value of (1) $5\frac{1}{7} \div 3\frac{3}{8}$; (2) $7\frac{7}{8} \div 4\frac{1}{12}$; (3) $2\frac{5}{14} \div 5\frac{19}{27}$; (4) $1\frac{37}{38} \div 2\frac{11}{57}$.

108. Simplify $\dfrac{4\frac{2}{3}}{5\frac{1}{4}}$.

$$\frac{4\frac{2}{3}}{5\frac{1}{4}} = 4\frac{2}{3} \div 5\frac{1}{4} = \tfrac{14}{3} \times \tfrac{4}{21} = \tfrac{8}{9}.$$

Or thus, multiplying the numerator and denominator by 12, the L. C. M. of 3 and 4, we have

$$\frac{4\frac{2}{3}}{5\frac{1}{4}} = \frac{56}{63} = \frac{8}{9}.$$

Exercise 104 (a)

Simplify, using either method:

1. $\dfrac{\frac{2}{3}}{\frac{5}{6}}$.   **3.** $\dfrac{2\frac{3}{4}}{5\frac{1}{2}}$.   **5.** $\dfrac{14\frac{2}{3}}{30\frac{1}{4}}$.   **7.** $\dfrac{6\frac{4}{7}}{17\frac{1}{4}}$.

2. $\dfrac{\frac{3}{4}}{1\frac{3}{5}}$.   **4.** $\dfrac{5\frac{5}{8}}{3\frac{8}{9}}$.   **6.** $\dfrac{4\frac{2}{13}}{1\frac{16}{65}}$.   **8.** $\dfrac{1\frac{3}{4}}{25\frac{2}{3}}$.

109. (1) A owns a farm containing $81\frac{3}{8}$ A., B owns $96\frac{7}{12}$ A., and C $64\frac{11}{15}$ A. How many acres do they own all together?

Here we are required to find the whole quantity measured by the parts, $81\frac{3}{8}$ A., $96\frac{7}{12}$ A., $64\frac{11}{15}$ A.

$$81 + 96 + 64 = 241.$$

$$\tfrac{3}{8} + \tfrac{7}{12} + \tfrac{11}{15} = \frac{45 + 70 + 88}{120} = \tfrac{203}{120} = 1\tfrac{83}{120}.$$

$$\therefore \text{ the sum} = 241 \text{ A.} + 1\tfrac{83}{120} \text{ A.} = 242\tfrac{83}{120} \text{ A.}$$

(2) A sum of money is divided among 4 persons. The first receives $\frac{1}{3}$, the second $\frac{1}{4}$, the third $\frac{1}{5}$, and the fourth the remainder. It is found that the first received $700 more than the fourth. Find the sum received by each.

Consider the sum of money as made up of 60 units.

The first receives $\frac{1}{3}$ of 60 or 20 units; the second 15, and the third 12 units.

The first three receive $20 + 15 + 12$ or 47 units.

The fourth receives $60 - 47$ or 13 units.

The first receives $20 - 13$ or 7 units more than the fourth.

$$7 \text{ units} = \$700.$$

$$1 \text{ unit } = \$100.$$

$\therefore$ the first receives 20 units or \$2000, the second \$1500, the third \$1200, and the fourth \$1300.

Exercise 105

1. What is the combined weight of three sheep, the first of which weighs $133\frac{1}{4}$ lb., the second $127\frac{3}{8}$ lb., and the third $135\frac{3}{4}$ lb.?

2. A grocer had three barrels of sugar; the first contained $28\frac{2}{3}$ lb., the second $44\frac{5}{6}$, and the third $16\frac{3}{4}$. Find how much there was in the three barrels.

3. If three crocks contain respectively $8\frac{1}{4}$, $12\frac{3}{8}$, and $14\frac{15}{16}$ lb. of butter, how much do the first two contain more than the third?

4. Four farms join each other; the first contains $125\frac{2}{3}$ A., the second $85\frac{2}{3}$ A., the third $156\frac{1}{2}$ A., and the fourth $120\frac{3}{4}$ A. Find the total area.

5. A lady had three dresses made. The first contained $12\frac{1}{2}$ yd., the second $14\frac{3}{4}$ yd., and the third $16\frac{1}{8}$ yd. Find the average number of yards in a dress.

6. A merchant sold to four customers, respectively, $15\frac{1}{2}$ yd., $14\frac{3}{4}$ yd., $16\frac{2}{3}$ yd., and $18\frac{7}{8}$ yd. from the same piece of cloth. Find the selling price at 72 ¢ a yard.

7. A person paid $\$165\frac{3}{4}$ for a horse, and $\$23\frac{1}{2}$ more than that for a carriage, and shortly after sold them at a loss of $\$46\frac{5}{8}$. What was the selling price?

8. How much greater than unity is $1\frac{2}{3}$? What fraction subtracted from the sum of $\frac{3}{4}$ and $\frac{5}{7}$ will leave unity for remainder?

9. If $\frac{2}{3}$ of a school term exceed $\frac{1}{2}$ of it by $13\frac{1}{3}$ da., how many days are there in the whole term?

10. I am the owner of $\frac{1}{5}$ of a ship worth $\$30,000$, and sell $\frac{1}{6}$ of the ship. What part of it will then belong to me, and what will it be worth?

11. Add together the greatest and least of the fractions $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$, and subtract their sum from the sum of the other two fractions.

12. From a piece of cloth containing 45 yd. there were sold $12\frac{3}{4}$ yd., $13\frac{3}{4}$ yd., and $15\frac{3}{4}$ yd. The remainder was put on the remnant table and sold for $\$1.65$. Find the number of yards it contained and what it sold for per yard.

13. An estate worth $\$10,000$ is left to A, B, and C; $\frac{3}{8}$ to A, $\frac{2}{5}$ to B, and the remainder to C. Find C's portion and its value.

14. I paid 15 ¢ for cheese at 20 ¢ a pound. How many ounces did I buy?

15. If during the day I pay out $\frac{1}{2}$, then $\frac{1}{3}$, next $\frac{1}{12}$, and lastly $\frac{1}{15}$ of the money I had in the morning, what fraction of it have I left? If the sum left amounts to $\$1.54$, what sum had I at first?

16. At a public meeting $\frac{3}{5}$ of those present were voters. If there were 135 voters present, how many attended the meeting?

17. At an entertainment $\frac{1}{3}$ of the seats were reserved and sold at 50 ¢ each. The remainder brought $40 at 25 ¢ each. Find the number who attended and the total amount received.

18. In a certain subscription list, $\frac{1}{8}$ of the number of subscriptions are for $5 each, $\frac{1}{6}$ are for $4 each, $\frac{1}{5}$ are for $2 each, $\frac{1}{3}$ are for $1 each, and the remaining subscriptions, amounting to $10.50, are for 50 ¢ each. Find the whole number of subscribers, and the total amount of their subscriptions.

19. A man lost $\frac{1}{4}$ of his property in speculation; he afterwards purchased a partnership in business for $16,000, and had still $6000 left. What was he worth at first?

20. A house and lot cost $3600; the value of the lot is $\frac{1}{5}$ that of the house. Find the value of each.

21. What must be the length of a plot of ground, if the breadth is $15\frac{3}{4}$ ft., that its area may contain 35 sq. yd.?

22. A has $6 more than $\frac{1}{3}$ of the whole of a sum of money; B has $8 more than $\frac{1}{4}$ of the whole; and C has $12 more than $\frac{1}{5}$ of the whole. Find the sum divided.

110. (1) A owns $\frac{2}{5}$ of a ship and B the remainder, and $\frac{3}{4}$ of the difference between their shares is $1500. What is the vessel worth?

$$\text{B's share} = \tfrac{3}{5} \text{ of the ship.}$$
$$\text{The difference} = \tfrac{1}{5} \text{ of the ship.}$$
$$\tfrac{3}{4} \text{ of the difference} = \tfrac{3}{4} \text{ of } \tfrac{1}{5} \text{ or } \tfrac{3}{20} \text{ of the ship.}$$
$$\tfrac{3}{20} \text{ of the ship} = \$1500.$$
$$\therefore \text{ the value of the ship} = \$1500 \times \tfrac{20}{3} = \$10,000.$$

(2) A man lost $\frac{2}{5}$ of the value of his horse by selling it for $60. For what should he have sold it to gain $\frac{2}{5}$ of its value?

$$\text{Let the value of the horse} = 5 \text{ units of money.}$$
$$\text{Then the loss on selling} = 2 \text{ units of money.}$$
$$\text{The first selling price} = 3 \text{ units of money.}$$
$$\text{The second selling price} = 7 \text{ units of money.}$$
$$\therefore \text{ the second selling price} = \tfrac{7}{3} \text{ of } \$60 = \$140.$$

(3) A person who has $\frac{2}{5}$ of a mine sells $\frac{3}{4}$ of his share for $ 6000. What is the value of the whole mine?

$\frac{3}{4}$ of $\frac{2}{5}$ or $\frac{3}{10}$ of the value of the mine = $ 6000.

∴ the value of the mine = $ 6000 × $\frac{10}{3}$ = $ 20,000.

Exercise 106

1. A grocer buys tea at 64 ¢ a pound, and sells it so as to gain $\frac{3}{16}$ of the cost price. Find what he receives on selling a chest of tea containing 88 lb.

2. If I own $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of a ship worth $ 20,000, and sell $\frac{1}{4}$ of the ship, what will the part I have left be worth?

3. Find the number of square feet in a garden $9\frac{1}{3}$ yd. long and $5\frac{1}{2}$ yd. wide.

4. Find the cost of sodding a piece of ground $8\frac{1}{3}$ yd. long and $6\frac{1}{2}$ yd. wide with sod costing $1\frac{1}{3}$ ¢ per square foot.

5. A city lot $83\frac{1}{3}$ ft. long and $66\frac{1}{2}$ ft. wide sold for $ $\frac{3}{5}$ a square foot. Find what the lot sold for.

6. A man has $ 4000 in the bank. He drew out $\frac{3}{20}$ of it, and then $\frac{1}{5}$ of the remainder, and afterwards deposited $\frac{1}{8}$ of what he had drawn out. How much had he then in the bank?

7. A man divided a farm among three sons; to the first he gave 80 A., to the second $\frac{4}{9}$ of the whole, and to the third $\frac{3}{4}$ as much as to both the others. How many acres did the farm contain?

8. Divide $ 65.80 between two persons, so that one shall receive one-third as much again as the other.

9. Five brothers join in paying a sum of money; the eldest pays $\frac{2}{7}$ of it, and the others pay the remainder in equal shares. It is found that the eldest brother pays $ 300 more than a younger brother's share. Find the sum of money.

10. Show by measuring the diameter and the circumference of a circle (as a plate, etc.) that the circumference is about $3\frac{1}{7}$ times the diameter.

11. What is the circumference of each of the following circles, the diameter being:

7 in. ?	$17\frac{1}{2}$ in. ?	$12\frac{1}{4}$ in. ?	3.5 in. ?
21 in. ?	$15\frac{2}{5}$ in. ?	$\frac{7}{8}$ ft. ?	1.26 in. ?
$10\frac{1}{2}$ in. ?	$31\frac{1}{2}$ in. ?	$\frac{2}{3}$ ft. ?	6.51 in. ?

12. How can you find the circumference of a circle when the diameter is given? How would you find the diameter if the circumference were given?.

13. What is the diameter of each of the following circles, the circumference being:

44 in. ?	$5\frac{1}{2}$ in. ?	$1\frac{1}{10}$ ft. ?	8.8 ft. ?
66 in. ?	$3\frac{2}{3}$ in. ?	$15\frac{5}{7}$ ft. ?	13.2 ft. ?
99 in. ?	$8\frac{4}{5}$ in. ?	$3\frac{1}{5}$ ft. ?	17.6 ft. ?

14. Show clearly that both terms of a fraction can be multiplied by the same number, without changing the value of the fraction.

15. A man commenced business with a capital of $ 8000; the first year he gained $\frac{2}{5}$ of his capital, and added his gain to his capital; the second year he gained $\frac{1}{4}$, adding his gain as before; the third year he lost $\frac{1}{7}$ of his last capital. How much did he make in the three years?

16. If $\frac{2}{3}$ of $\frac{3}{7}$ of an acre produces 16 bu. of potatoes, how many bushels will an acre produce?

17. A paid $ 60 per acre for his farm, which was $\frac{5}{9}$ as much as B paid per acre for his farm of 150 A. Find the entire cost of B's farm.

18. A piece of cloth, when measured with a yard measure which is $\frac{1}{2}$ in. too short, appears to be 4 yd. long. How much shorter is it than 4 yd.? How long is it?

19. A piece of cloth, when measured with a yard measure which is $\frac{2}{3}$ of an inch too short, appears to be $10\frac{1}{2}$ yd. long. What is its true length?

20. I had a sum of money, of which I paid away $\frac{1}{5}$, then $\frac{1}{2}$ of the remainder, and found that I had still left $46. What sum had I at first?

21. A man who owns $\frac{10}{21}$ of a mill sells $\frac{2}{5}$ of his share. What fraction of the mill does he still own? Had he sold $\frac{2}{5}$ of the mill, what fraction of the mill would he still have owned?

111. (1) A man earns 4\frac{1}{5}$ a day, and his daily expenses are 1\frac{7}{8}$. How many days will it take him to save enough money to buy a bicycle costing 58\frac{1}{8}$?

> The sum saved each day $= \$4\frac{1}{5} - \$1\frac{7}{8} = \$2\frac{13}{40}$.
>
> $\therefore$ the number of days $= 58\frac{1}{8} \div 2\frac{13}{40} = 25$.

(2) A farm of 340 A. was divided between two sons, so that $\frac{3}{4}$ of the youngest son's share was equal to $\frac{2}{3}$ of the eldest son's share. Find the size of each farm.

> $\frac{3}{4}$ of the youngest son's share $= \frac{2}{3}$ of the eldest son's share.
>
> The youngest son's share $= \frac{2}{3} \div \frac{3}{4}$, or $\frac{8}{9}$ of the eldest son's share.
>
> $\therefore \frac{9}{9} + \frac{8}{9}$, or $\frac{17}{9}$ of the oldest son's share $= 340$ A.
>
> $\therefore$ the oldest son's share $= 340$ A. $\div \frac{17}{9} = 180$ A.
>
> $\therefore$ the youngest son's share $= \frac{8}{9}$ of 180 A. $= 160$ A.
>
> Complete the following solution of question (2):
>
> Let $\frac{3}{4}$ of the youngest son's share $= 6$ units.

(3) Sold tea at $92\cent$ per pound, having gained $\frac{3}{20}$ of the cost. Find the selling price per pound if he had lost $\frac{3}{20}$.

> Let the cost of 1 lb. be measured by 20 units.
>
> Then $\frac{3}{20}$ of the cost price $= 3$ units.
>
> The first selling price $= 23$ units.
>
> The second selling price $= 17$ units.
>
> $\therefore$ the second selling price $= \frac{17}{23}$ of the first.
>
> $= \frac{17}{23}$ of $92\cent = 68\cent$.

Exercise 107

1. What part of a dollar is 75¢? 33⅓¢? 62½¢? 87½¢? 6¼¢? 16⅔¢?

2. At 12½¢ each, how many pillow cases will cost $2? (2 ÷ ⅛ = ?)

3. I paid $18 for cloth to make a dress, at $1.50 a yard. Find the number of yards. (18 ÷ 1½ = ?)

4. At 33⅓¢ a pair, how many pairs of stockings will cost $3?

5. At $2.75 a pair, how many pairs of curtains will cost $11?

6. At 37½¢ a pair, how many dozen sheets will cost $18?

7. At $3.75 a pair, how many pairs of curtains will cost $26.25?

8. At 3 for 50¢, how many handkerchiefs will cost $3.50?

9. At $10.50 each, how many rugs will cost $42?

10. At $5.75 per barrel, how many barrels of flour will cost $51.75?

11. At 62½¢ a bushel, how many bushels of wheat will cost $250?

12. At 37½¢ a bushel, how many bushels of oats will cost $36?

13. How many cans, each containing 1½ qt., can be filled from a barrel containing 31½ gal. of syrup?

14. The bottom of a box measures 4 ft. by 2½ ft. How deep must it be to contain 20 cu. ft.?

15. The bottom of an excavation measures 7 ft. 6 in. by 3 ft. 2 in. How deep must it be to contain 95 cu. ft. of water?

16. What is the cost price of goods sold for $1.20, at a gain of ⅓ of the cost? Of ⅕ of the cost? Of ¼ of the cost? Of ⅑ of the cost? Of 1/11 of the cost? Find the gain in each case.

17. A man sold 24 horses for $150 each; on half of them he gained ¼ of what they cost; and on the remainder he lost ⅕ of what they cost. Find his whole gain or loss.

18. The sum paid for three copies of a certain book, including a duty of $\frac{1}{4}$ of the cost, was $2.25. Find the original cost and the duty on one book.

19. The sum paid for 494 gal. of oil, including a duty on each gallon which amounted to $\frac{1}{5}$ of the cost price of a gallon, was $1719.12. Find the duty on each gallon.

20. By selling oranges at the rate of $2.60 for 4 doz., it was found that $\frac{5}{8}$ of the cost was gained. Find the price at which each orange ought to have been sold in order to gain 50% of the cost.

21. A person who has $\frac{2}{5}$ of a mine sells $\frac{3}{4}$ of his share for $6000. What is the value of the whole mine?

22. A grocer in selling goods sells $15\frac{3}{4}$ oz. for 1 lb. How much does he cheat a customer who buys to the amount of $40?

23. A cannon ball travels at the rate of 1500 ft. in $1\frac{1}{2}$ sec. How far will it have gone in $\frac{11}{15}$ of a minute?

24. Given, that pure water is composed of oxygen and hydrogen in the proportion by weight of 15 to 2, find the weight of each in a cubic foot of water. (A cubic foot of water weighs 1000 oz.)

25. A tank is 9 ft. long, 3 ft. 4 in. wide, and 2 ft. 6 in. deep. Find the number of cubic feet of water it will hold. How many gallons? (1 cu. ft. = 7.48 gal.)

26. A tank is 6 ft. 8 in. long, 2 ft. 6 in. wide, and 1 ft. 6 in. deep. Find the number of gallons of water it will hold.

27. A man willed $\frac{1}{3}$ of his property to his wife, $\frac{2}{3}$ of the remainder to his daughter, and the rest to his son. The difference between the wife's and daughter's shares was $2500. Find the value of his property.

28. A grain dealer paid $61\frac{3}{8}$ ¢ a bushel for corn, and sold it for $60\frac{5}{8}$ ¢ a bushel. He lost $375 on the transaction. Find the number of bushels.

Miscellaneous Exercise 108

1. The citizens of Madison, Wis., have raised the following sums of money by private subscription for public improvements in the seven years preceding 1900: $6888.86, $655, $995, $1580, $10,160, $2171, $3231.50. Find the total amount.

2. The following sums of money were collected under the War Revenue Act from June 13, 1898, to March 31, 1900, cents omitted. Find the total amount collected:

OBJECTS OF TAXATION	AMOUNT COLLECTED
Cigars	$5,202,691
Cigarettes	2,442,020
Snuff	1,641,281
Tobacco, chewing and smoking	27,070,113
Dealers in leaf tobacco	127,170
Dealers in manufactured tobacco	30,637
Manufacturers of tobacco	39,193
Manufacturers of cigars	446,724
Miscellaneous collections relating to tobacco	773,175
Fermented liquors	56,936,631
Additional collections on fermented liquors stored in warehouse	197,936
Mixed flour	14,154
Bankers, capital not exceeding $25,000	712,426
Bankers, capital exceeding $25,000, for each additional $1000 in excess of $25,000	6,066,155
Billiard rooms	583,443
Brokers, stocks, bonds, etc.	559,356
Brokers, commercial	277,016
Brokers, custom house	11,860
Brokers, pawn	71,776
Bowling alleys	90,626
Circuses	28,929
Exhibitions not otherwise provided for	148,759
Theatres, museums, and concert halls	97,729
Legacies	2,896,306
Schedule A	66,781,776
Schedule B	8,693,881
Excise tax on gross receipts	1,463,547

3. The following statement shows the imports from and the exports to the Philippines, including gold and silver, for the four months ending Oct. 31, 1899.   Find the total imports and exports:

	IMPORTS FROM	EXPORTS TO
United States	$635,495	$1,518,748
Canada	174	None
United Kingdom	1,515,893	1,683,806
Spain	1,008,813	541,416
France	112,944	62,953
Germany	550,425	19,961
Belgium	48,299	None
Gibraltar	None	252,044
Netherlands	119,597	None
Russia	48,867	None
Switzerland	77,470	None
Italy	45,904	1,720
Denmark	7,953	None
Austria	15,281	None
Japan	82,098	486,324
China	4,165,949	1,463,638
British East Indies	876,177	193,594
Dutch East Indies	22,593	7,384
Australia	424,380	180,345
German Oceanica	None	693
British Africa	None	2,510

4. As the result of a freight rate war over wool, the rate from Denver to Boston was reduced from $1.54 to $1.15 per 100 lb. Find the freight saved on a shipment of 135,000 lb.

5. Wire fencing is bought at $4.90 a bale of 20 rd., and sold at 2¢ a foot.   Find the gain on 12 bales.

6. Find the value of 6 beeves, average weight 1500 lb., at $5.85 per 100 lb.

7. The Chicago real estate sales for the week ending Aug. 25, 1899, were $1,336,455, and for the corresponding week of the preceding year were $889,709.   Find the increase.

8. The freight on grain from Chicago to New York is 15¢ per 100 lb. What is the rate per bushel? On 8000 bu.?

9. What is the ratio of 150 lb. to 100 lb. Find the selling price of a hog weighing 150 lb. at $4.75 per 100 lb.

10. Find the selling price of a hog weighing 180 lb. at $4.85 per 100 lb.

11. Find the selling price of 50 hogs, average weight 120 lb., at $4.15 per 100 lb.

12. Find the cost of a 190-lb. lamb at $6 per 100 lb.

13. Find the cost of 250 128-lb. lambs at $6.20 per 100 lb.

14. The record of the temperature at Chicago for March 10–11, 1900, is given in the following table. Find the average temperature during the day:

4 P.M.	.	.	.	.	.	42	4 A.M.	.	.	.	.	. 32
5 P.M.	.	.	.	.	.	44	5 A.M.	.	.	.	.	. 32
6 P.M.	.	.	.	.	.	42	6 A.M.	.	.	.	.	. 32
7 P.M.	.	.	.	.	.	40	7 A.M.	.	.	.	.	. 32
8 P.M.	.	.	.	.	.	34	8 A.M.	.	.	.	.	. 34
9 P.M.	.	.	.	.	.	33	9 A.M.	.	.	.	.	. 34
10 P.M.	.	.	.	.	.	32	10 A.M.	.	.	.	.	. 34
11 P.M.	.	.	.	.	.	32	11 A.M.	.	.	.	.	. 35
12 midnight		.	.	.	.	32	12 M.	.	.	.	.	. 38
1 A.M.	.	.	.	.	.	32	1 P.M.	.	.	.	.	. 39
2 A.M.	.	.	.	.	.	33	2 P.M.	.	.	.	.	. 39
3 A.M.	.	.	.	.	.	32	3 P.M.	.	.	.	.	. 39

15. The rate of insurance on a building, which was insured for $25,000, was advanced from $1.85 to $2.34 per $100 insured. Find the increase in the cost of insuring the building.

CHAPTER XII

DECIMALS *

Exercise 109

1. With a metric stick measure a distance 2 meters (2 m.) long; 1 meter 6 decimeters (1 m. 6 dm.); 6 decimeters 5 centimeters (6 dm. 5 cm.); 5 centimeters 8 millimeters (5 cm. 8 mm.).

2. Measure the following distances:

1 m. 6 dm.	4 dm. 8 cm.	1 m. 2 dm. 4 cm.
2 m. 4 dm.	6 cm. 5 mm.	2 dm. 4 cm. 5 mm.
9 dm.	1 m. 9 cm.	6 dm. 1 cm. 7 mm.
5 dm. 6 cm.	4 dm. 8 mm.	1 m. 4 dm. 6 cm.

3. How many decimeters in 1 m. ?   How many centimeters in 1 m. ?   How many millimeters in 1 m. ?

4. Measure the following distances:

1.2 m.	.54 m.	.265 m.	.04 m.
1.8 m.	.73 m.	.492 m.	.045 m.
.6 m.	.475 m.	.309 m.	.005 m.

5. Read the following expressions: 4.56 m. (four and fifty-six hundredths meters), 2.35 m., 1.64 mi., 4.25 yr., .28 da., .16 hr., .08 min., 2.425 lb., 6.405 bu., .345 yd., .248 yd., .049 ft., .009 in., .006 in., 2.243, .065, .06, .005.

6. Give the *place value* of each figure in the preceding example, thus: 2.45 m. = two meters, four-tenths of a meter, five-hundredths of a meter.

* If the teacher has not a metric stick, the work in decimals may be based on dollars, cents, and mills.

7. Measure these distances: 1.2 m., .34 m., .48 m., .425 m., .369 m., .04 m., .08 m., .005 m., .009 m.

112. The standard unit of money is $1. One dime is one-tenth of $1, and 1 ¢ is one-tenth of 1 dime. Hence we write $1, 1 dime, and 1 ¢ thus, using the dollar sign: **$1.11**.

Write in terms of $1, the sum of:

(1) $1, 2 dimes, and 4 ¢.

(2) 1 ten-dollar bill, $1, 1 dime, and 1 ¢.

(3) 1 one-hundred-dollar bill, 1 ten-dollar bill, $1, 1 dime, and 1 ¢.

113. In the metric system of measures, the standard unit is 1 meter.

One decimeter is one-tenth of 1 meter, 1 centimeter is one-tenth of 1 decimeter, and 1 millimeter is one-tenth of 1 centimeter. Since the same relation holds between these units as between 1 dime and $1, 1 ¢ and 1 dime, we can express the results of measurements in terms of 1 meter, as we express money in terms of $1.

Hence if we measure with the metric stick a distance equal to 1 meter, 1 decimeter, and 1 centimeter, we can express the distance thus: **1.11 meters**.

Measure the following distances and express them in meters:

(1) 2 meters, 3 decimeters, and 4 centimeters.

(2) 1 meter, 2 decimeters, 3 centimeters, and 5 millimeters.

(3) 1 meter, 1 decimeter, 1 centimeter, and 1 millimeter.

114. NOTATION AND NUMERATION. — Consider the number 111: the first 1, beginning at the right, denotes one unit; the second, *one* ten or *ten* units; the third, *one* hundred or *one hundred* units.

The third 1 is equivalent to one hundred times the first 1, and to ten times the second 1; the second is equivalent to ten times the first 1, and to one-tenth of the third 1; the first 1 is equivalent to one-tenth of the second 1, and to one-hundredth of the third 1.

Now rewrite the number 111, place a point after the first 1 to indicate that this 1 is to be regarded as representing the standard unit, and then place after the point three 1's, so that we have

<div align="center">111.111.</div>

We may ask what each of these 1's should mean, if the same relation is to hold among successive digits that we have supposed hitherto to hold. The 1 after the point would naturally mean one tenth. The next 1 to the right would naturally mean one hundredth. It is one-tenth of the preceding one — that is, one-tenth of one tenth.

Similarly, the next 1 would signify one thousandth, and would equal one-hundredth of the one tenth or one-tenth of the one hundredth. Thus, the number 111.111 may be written as follows: One hundred, one ten, one unit, one tenth, one hundredth, and one thousandth.

Again, the one to the extreme right is 1 thousandth: the next 1 is, from its position, equivalent to 10 thousandths; and the next 1 is 100 thousandths. So that to the right of the point we have 111 thousandths. The whole number may now be read one hundred eleven, and one hundred eleven thousandths.

115. A **Decimal Fraction** or a decimal is one which has for its denominator 10, 100, 1000, or some power of 10.

The **Power** of a number is the product found by multiplying the number by itself one or more times; thus, 100 or 10^2 is the second power of 10; 1000 or 10^3, the third power of 10.

The denominator of a decimal fraction is never expressed; thus $\frac{5}{10}$ and $\frac{57}{100}$ are written as decimals, .5 and .57.

The point placed to the right of the one-unit and between it and the tenth-unit is called the *decimal* point.

116. *To change a decimal to a common fraction* in its lowest terms.

$$.12 = \tfrac{12}{100} = \tfrac{3}{25}; \qquad .425 = \tfrac{425}{1000} = \tfrac{85}{200} = \tfrac{17}{40}.$$

$$2.75 = 2\tfrac{75}{100} = 2\tfrac{3}{4}; \qquad 6.036 = 6\tfrac{36}{1000} = 6\tfrac{9}{250}.$$

Conversely, $\qquad 5\tfrac{29}{100} = 5.29; \qquad 92\tfrac{25}{1000} = 92.025.$

Exercise 110

1. Read the numbers in Exercises 111 and 112, expressing them in terms of different units, as 1 m., 1 mi., 1 lb.

2. Read .5; .05; .005; .0005; .004; .075; 2.008; 3.029; .0006.

3. Read the following expressions: 2.548 T.; 2.917 A.; 4.73 A.; 8.04 mi.; 3.04 yr.; 4.007 da.; .346 mo.; .85 hr.; .008 bu.; .0009 T.; 4.444; 5.032; 6.2059; 8.0063; 7.2906; .043.

4. Read the *place value* of each figure in the previous example.

Reduce the following decimals to common fractions in their lowest terms:

5. .5; .8; .75; .45; .72; .05; .125; .625.

6. 2.4; 7.25; 4.375: 3.875; 3.0875; 6.325.

7. 2.125 T.; $.0625; 8.225 A.; $9.375.

8. Find the cost of:

        12 bars of soap at 6.25¢ a bar.

        16 lb. sugar at 5.125¢ a pound.

        2 doz. pairs rubbers at $.375 a pair.

        64 sheep at $4.875 each.

9. A farmer gave .875 of his farm of 400 A. to his three sons. How many acres did he keep for himself?

10. A merchant sold cloth at an advance of .375 of the cost, and gained 24 ¢ a yard. What did his customers pay a yard for the cloth?

Express in figures:

11. Four tenths; two, and fifty-six hundredths; three, and five hundredths; sixty-six hundredths; six hundredths; three hundred sixty-eight thousandths; seventy-nine thousandths; eight thousandths.

12. Seventy-five, and twenty-eight thousandths; three hundred twenty-nine, and ninety-four thousandths; two, and five hundredths; seven, and seven thousandths; two hundred, and two hundredths; two thousand, and two thousandths.

13. Five thousand nine hundred forty-two ten-thousandths; eight hundred seventy-one ten-thousandths; forty-five ten-thousandths; six ten-thousandths.

14. Express the following fractions as decimals: $\frac{7}{10}$; $\frac{6}{10}$; $\frac{43}{100}$; $\frac{6}{100}$; $\frac{243}{1000}$; $\frac{29}{1000}$; $\frac{7}{1000}$; $9\frac{8}{10}$; $6\frac{3}{100}$; $69\frac{54}{1000}$; $76\frac{3189}{10000}$; $6\frac{57}{10000}$; $17\frac{9}{1000}$.

ADDITION OF DECIMALS

117. What is the sum of 4.9, 6.084, 24.32, and .8976?

4.9	In arranging the numbers, be careful to put the decimal
6.084	points directly under each other, thus bringing units under
24.32	units, tenths under tenths, etc. Then begin at the lowest
.8976	order and add as if the figures were integers, putting the
36.2016	decimal between the unit and the tenths' place.

Exercise 111

Find the sum and prove your answers correct by adding down the columns:

1.	53.674	2.	9.72	3.	.8592
	4.009		20.492		913.7451
	821.646		.0487		21.0106
	2.182		918.0006		47.9

4. Explain step by step the process of addition in the first three examples.

Write in columns and add:

5. $6.5 + 32.47 + 2.048 + 59$.

6. $.452 + 4.08 + .646 + .06 + 49.027$.

7. $4.0406 + 213.939 + 2.91 + 3.04$.

8. $89432.1 + 7.65439 + .0084 + 8400$.

9. $27.064 + .0012 + 394.2001 + .819$.

10. How many yards are there in five pieces of cloth, the first of which contains 37.5 yd., the second 26.75, the third 14.375, the fourth 36.5, and the fifth 63.125 ?

11. Four sections of land contain the following areas: 24.729 sq. mi., 92.04 sq. mi., 8.007 sq. mi., and 36.429 sq. mi. Find the total area.

12. Find the sum of sixty-one ten-thousandths; eight, and seven thousand six hundred ninety-five ten-thousandths; nine thousand seven hundred eighty-six ten-thousandths.

13. During the year 1899, in the city of Chicago, 11.32 mi. of streets were paved with asphalt, 10.17 mi. with cedar, 13.07 mi. with brick, 1.59 mi. with granite, and 8.09 mi. with macadam. Find the total.

14. The north side of the city of Chicago has parks containing the following number of acres: 320 A., 2.3 A., .53 A., .2 A., .19 A., .46 A., .0225 A., .0482 A. Find their total area.

15. The west side of the city of Chicago has parks containing the following number of acres: 200.62 A., 185.87 A., 179.79 A., 14.8 A., 15.79 A., 5.42 A., 4.51 A., 4.89 A., 5.5 A., 6.06 A., 3.65 A., .94 A., .25 A., 2.38 A., .68 A., 1.08 A., 2 A., 1.28 A. Find their total area.

16. A man paid a state tax of $ .45, a city tax of $ 1.12, and a school tax of $ 1.78 on every $ 100 property. Find his total tax on each $ 100 property.

17. The tax rates for the west town of the city of Chicago for 1899 and 1898 were for each $ 100 property as follows:

	1899	1898		1899	1898
State	$ 0.42	$ 0.56	Sanitary. . . .	$ .893	$ 1.50
County	.548	.78	Park	.847	1.10
Town	.035	.29	Boulevard . . .	.05	.05
City	1.27	2.65	Town bond . . .	.059	.10
Library	.049	.06	School	1.829	2.77

Find the total tax on $ 100 property.

SUBTRACTION OF DECIMALS

118. From 29.364 take 3.87049.

```
   29.36400                    29.364
    3.87049                    3.87049
```

As in addition of decimals, place the decimal points under each other, thus placing units under units, tenths under tenths, etc.

As the value of the decimal is not changed by annexing zeros to the right of the decimal, annex in this case two zeros. Subtract as in whole numbers, and place the decimal point in the remainder between the unit and the tenths' place.

Exercise 112

From	**1.** 8.43	**2.** 13.47016	**3.** .503	**4.** .52
Take	2.95	2.0984	.28914	.13064

5. Explain step by step the process of subtraction in the first three examples.

Find the difference and prove your answers correct:

6. .62 − .47.   **10.** .07 − .059.   **13.** .7304 − .67.

7. .73 − .35.   **11.** 8.9 − 3.4265.   **14.** 4.8295 − 3.9998.

8. .894 − .406.   **12.** 39.42 − 15.9879.   **15.** 2.03 − .00428.

9. .74 − .365.

N

16. The parks on the north side of Chicago contain 323.7507 A., and those on the west side 615.45 A. How many more acres of parks on the west than on the north side of the city?

17. The height of the barometer at Davenport, Ia., May 3, 1900, was 30.96. How much greater is this than the height at each of the following places?

Albany . . 29.62   Nashville . . 30.04   Buffalo . . 29.82
Cleveland . 29.96   Montreal . . 29.54   Oklahoma . 30.16

18. Explain whether .067 or .068 is nearer .06748, and express in words the difference in each case.

19. The length of a seconds pendulum is 39.1392 in., and that of a meter is 39.371 in. Find the difference in their lengths.

20. Find the difference between the length of 1 meter and 1 yard.

21. From a piece of cloth containing 35.5 yd., a merchant sold 12.75 yd. How much was left?

22. Find the difference between $\$ 11\frac{33}{100}$ and 35 ¢.

23. Monday the barometer read 29.86, Tuesday it rose .12, and Wednesday it fell .28. What was the height of the barometer Wednesday evening?

24. The mercury in a barometer rose .121 in., .073 in., and .019 in. in three successive days; it fell .054 in. and .065 in. during the two following days, rose .053 in. on the sixth day, and fell .028 in. on the seventh day. If its height at the beginning of the first day was 30.078 in., what was its height at the close of the seventh day?

MULTIPLICATION OF DECIMALS

119. (1) $\frac{7}{10}$ multiplied by $10 = 7$, and therefore .7 multiplied by $10 = 7$.

$\frac{627}{100}$ multiplied by $10 = \frac{627}{10}$, or 62.7, and therefore 6.27 multiplied by $10 = 62.7$.

(2) $\frac{75}{100}$ multiplied by $100 = 75$, and therefore .75 multiplied by $100 = 75$.

$\frac{6275}{1000}$ multiplied by $100 = \frac{6275}{10} = 627.5$, and therefore 6.275 multiplied by $100 = 627.5$.

(3) $\frac{62758}{10000}$ multiplied by $1000 = \frac{62758}{10} = 6275.8$, and therefore 6.2758 multiplied by $1000 = 6275.8$.

In the preceding examples how many places to the right was the decimal point moved on multiplying by 10 ? 100 ? 1000 ?

Exercise 113

1. Multiply each of the following numbers by 10:

.6; .8; .84; .95; .842; .763; 1.75; 2.439; 20.4.

2. State how to multiply a decimal by 10, without actually doing the work of multiplication.

3. Multiply by 10: .06; .04; .005; .0123; .0044; .0001.

4. Multiply by 10: 42.3; 5.69; .478; 54.793; 2.9342.

5. Multiply by 100: .84. 9.65; .763; .003; .04; .246.

6. State how to multiply a decimal by 100, without actually doing the work of multiplication; by 1000.

7. Reduce to pounds: 2.34 cwt.; 6.42 cwt.; .345 cwt.; .125 cwt.; .24 cwt.; .06 cwt.; 3.46 cwt.; 2.468 cwt.

8. Find the number of pounds in 1.24 cwt. of sugar. Find its value at 6 ¢ a pound.

9. Multiply by 1000: .982; .0642; .0009; .008; .0123; .0004.

10. Multiply by 100: .86; 8.6; .9; .060; 9.8; .4; 6.245; .005.

11. Multiply by 1000: .594; 5.94; 59.4; .007; .07; .7; 3.14; 2.5.

12. State how to divide a decimal by 10. By 100. By 1000.

13. Divide by 10: 27; 82.19; 4.8; 52.93; .4; .06; .009.

14. Divide by 100: 482; 76; 415.62; 8.1; .78; .4; .09; 789.46.

15. Reduce to hundredweight: 400 lb.; 237 lb.; 575 lb.; 832 lb.; 3475 lb.; 4759 lb.; 67 lb.; 25 lb.; 87 lb.; 95 lb.

16. 69 lb. = ? cwt. Find the cost of a 69-lb. lamb at $5 per hundredweight.

17. Find the number of hundredweight in 16 85-lb. lambs; in 24 97-lb. sheep.

18. Divide by 1000: 643; 2459.7; .69; 2.31; .03; 6009; 17643.

19. Divide 8436 by 1000; by 2000.

20. Divide by 2000: 6436; 2974; 3298; 7935; 428; 526; 439; 24; 56; 77.

21. Reduce to tons: 5436 lb.; 3138 lb.; 624 lb.; 7296 lb.; 518 lb.; 14324 lb.

22. Find the number of tons in a carload of coal weighing 39,374 lb.

23. During the ten years preceding July, 1899, the wholesale price of bacon per pound decreased from $.056 to $.052. Find the decrease in the wholesale price of 1 lb.; of 1000 lb.

24. The average wholesale price of the following articles, January, 1890, and July, 1899, are given below. Find the difference in price on an order of 1000 lb. in each case.

	JANUARY, 1890	JULY, 1899
Lard per pound	$.056	$.049
Ham per pound	.1125	.1075
Sugar, granulated, per pound	.065	.052
Thread, spool	.032	.031
Starch, silver gloss, per pound	.063	.058

25. If sheep are quoted at $5.25 per hundredweight (*i.e.* per 100 lb.), find the price per pound as a decimal of a dollar.

26. Live stock is quoted at the following prices per hundredweight. Find in each case the price per pound as a decimal of a dollar: $5.45; $2.25; $4.10; $4.50; $8.75; $5.37½; $5.42½; $7.10; $5.32½.

27. A sheep weighs 87 lb. Express this as a decimal of 100 lb., *i.e.* of 1 cwt. A cow weighs 769 lb. This is how many hundredweight?

28. How many hundredweight will 12 89-lb. Texas sheep weigh? Find their value at $5 per hundredweight.

29. How many thousand shingles in 5000? 6200? 4375? 9360? 478?

30. How many tons of coal in 6000 lb.? 4800 lb.? 3468 lb.? 7296 lb.?

31. Write as decimals: 4%; 6%; $2\frac{1}{2}\%$; $4\frac{1}{2}\%$; 8%; $5\frac{1}{4}\%$; $6\frac{1}{4}\%$; $3\frac{1}{2}\%$; 9%; 37%; 25%; $23\frac{1}{2}\%$; 16.4%; 27.6%; 8.2%; $18\frac{2}{5}\%$.

120. (1) Multiply 6.24 by 46.

$$\begin{array}{r} 6.24 \\ 46 \\ \hline 37.44 \\ 249.6 \\ \hline 287.04 \end{array}$$

Here we multiply 4 hundredths by 6, and the product is 24 hundredths, or 2 tenths 4 hundredths. Again, we multiply 2 tenths by 6, and, adding in the 2 tenths, the result is 14 tenths, or 1 unit 4 tenths, and so on. Next, multiplying by 4, we must write the results one place to the left, as in the multiplication of integers. In multiplying, it is as well to omit the decimal points from the partial products 37.44 and 249.6.

(2) Multiply 6.24 by 4.6.

$$\begin{array}{r} 6.24 \\ 4.6 \\ \hline 3744 \\ 2496 \\ \hline 28.704 \end{array}$$

This differs from the former question only in that the multiplier 4.6 is one-tenth of 46, and therefore the product is one-tenth as large, or 28.704.

121. From this and other similar problems the rule can be deduced:

To multiply two decimals, proceed as if they were integers, and mark off in the product as many decimal places as there are in both the multiplier and the multiplicand.

122. (1) Multiply 2.56 by .94.

$$\begin{array}{r} 2.56 \\ .94 \\ \hline 1024 \\ 2304 \\ \hline 2.4064 \end{array}$$

(2) Multiply .249 by .035.

$$\begin{array}{r} .249 \\ .035 \\ \hline 1245 \\ 747 \\ \hline .008715 \end{array}$$

EXPLANATION. —

$$.249 \times .035 = \tfrac{249}{1000} \times \tfrac{35}{1000} = \tfrac{8715}{1000000} = .008715.$$

123. What is the interest * on $468 for 1 yr. at 6%?

$$\begin{array}{l} \$468 \text{ principal} \\ \underline{\$.06 \text{ rate per unit}} \\ \$28.08 \text{ interest for 1 yr.} \end{array}$$

∴ The interest on $468 for 1 yr. at 6% = $28.08. To find the interest for 6 mo. take .03 of $468, or divide $28.08 by 2.

Exercise 114

Multiply:

1.	3.26	5.29	3.148	.4378	.8664	.0581
	4	.6	.9	.6	.5	.8
2.	58.9	.685	.072	.093	.182	2.98
	2.4	.36	.41	.56	.75	3.5
3.	3.1416	59.38	.0075	.094	.006	9271
	.72	3.12	9.8	.04	.006	.004

* See § 207 for definitions of principal and interest.

Find the interest for 1 yr. on:

4. $26 at 5%; $248 at 6%; $16 at 4%.

5. $46.50 at 8%; $894.75 at 3%; $2389.20 at 7%.

6. $324 at $2\frac{1}{2}$%; $704.60 at $3\frac{1}{2}$%; $852.94 at $4\frac{1}{2}$%.

Find the interest on:

7. $237 for 6 mo. at 6%; $628 for 3 mo. at 8%.

8. $22.58 for 8 mo. at 6%; $69.50 for 9 mo. at 8%.

9. $6438 for 4 mo. at $7\frac{1}{2}$%; $523.60 for 8 mo. at $7\frac{1}{2}$%; $250 for 10 mo. at 6%; $495.80 for 1 mo. at 6%.

10. Find .375 of 45 mi.; .0625 of 640 A.; .0875 of $415.60.

Multiply:

11. 4.8 × 5.12; .21 × 4.67.   **16.** .8 × .8; .09 × .09.

12. 3.1416 × .02; 1.46 × .39.   **17.** .1 × .1; .01 × .01.

13. .004 × 99; .004 × .005.   **18.** .2 × .2; .7 × .7

14. $249 × 1.04.   **19.** .375 × 2.15; .0375 × 2.15.

15. .84 × .251; 2.04 × .0037.   **20.** .051 × .042; .014 × .0038.

21. I sold an article which cost me $265 at a gain of 23%. Find the gain. Find the selling price.

22. I sold an article that cost $226.50 at a loss of 34%. Find the loss and the selling price.

23. Measure the diameter of some circular object, and also the circumference. Multiply the diameter by 3.1416, in which case the product should equal the circumference.

24. *To find the circumference of a circle multiply the diameter by* 3.1416.

25. Find the circumferences of the circles whose diameters are:

8 in.	2.8 in.	9.21 in.	.98 in.
9 in.	3.4 in.	4.37 in.	.45 in.
10 in.	7.2 in.	2.06 in.	.75 in.

26. Find the number of rods of barbed wire required to enclose a circular pond whose diameter is 5 rd., there being 4 rows of wire in the fence.

27. Find the cost of the woven wire for fencing in a circular pond 10 rd. in diameter, the fencing costing $ 6.72 per bale of 20 rd.

Exercise 115

1. Reduce to inches: .62 ft.; .75 ft.; .375 ft.; .245 ft.

2. Reduce to feet: .85 yd.; .46 yd.; .7 yd.; .09 yd.; .768 yd.

3. Reduce to yards (multiply by 5.5): .84 rd.; .72 rd.; .456 rd. .324 rd.; .07 rd.

4. Find the cost of .8 rd. wire cloth at $ .20 a yard.

5. Reduce to feet (multiply by 16.5): .4 rd.; .96 rd.; .44 rd.; .25 rd.; .375 rd.; .225 rd.

6. Find the cost of .6 rd. wire fencing at $ .20 a foot.

7. Multiply $ 240 by 1.05 twice in succession.

8. Multiply $ 325 by 1.06 twice in succession.

9. Multiply $ 415.80 by 1.04 and the result by 1.02.

10. Multiply $ 75 by 1.045 twice in succession.

11. Multiply $ 975 by 1.065 twice in succession.

12. Multiply 3.1416 by 8; by 7.5; by .04.

13. Multiply 7.48 by 9.1; by .04; by .006.

14. Multiply the square of 5 by 3.1416.

15. A railroad engineer gets 3.85¢ per mile. How much will he earn in a month if his runs amount to 3000 mi.?

16. One meter = 39.371 in. Find the number of inches in a distance equal to 4 m.

17. The length of a wall, according to the French metric system, is 9.48 meters. Find its length in inches, the length of 1 meter being 39.371 in.

18. Multiply the sum of 2.616, .00132, and 1.0448 by .626.

19. Find the length of the fence enclosing an oblong garden 2.5 rd. long and 1.5 rd. wide. What will it cost to fence it at 50 ¢ a rod ?

20. A piece of land is 63.5 rd. long and 27.75 rd. wide. What will it cost to fence it at $.875 per rod ?

21. A person sold .15 of an estate to one person, and then $\frac{5}{17}$ of the remainder to another person. What part of the estate did he still retain ?

22. If a business produces an annual return of $12,000, and of three partners one has .465 and another .28 share of the profits, how much money falls to the share of the third partner ?

23. A merchant sells 28.5 yd. of cloth which cost him 25¢ a yard, for 37.5¢ a yard. What was his gain ?

24. Find the weight of 25 lambs, averaging 65 lb. each. How many hundredweight ? Find their value at $5.75 per hundredweight.

25. Find the value of :

> 30 109-lb. lambs at $6.25 per hundredweight.
>
> 27 122-lb. sheep at $5.60 per hundredweight.
>
> 75 125-lb. pigs at $4.80 per hundredweight.
>
> 400 170-lb. hogs at 5.16\frac{1}{2}$ per hundredweight.
>
> 16 185-lb. hogs at 5.42\frac{1}{2}$ per hundredweight.
>
> 11 1330-lb. beeves at $4.90 per hundredweight.
>
> 19 1450-lb. beeves at $5.60 per hundredweight.

26. Find the cost of 6350 shingles at $2.80 per 1000.

27. How many tons of coal in 6240 lb. (write as a decimal)? Find its value at $6.25 a ton.

28. Find the value of 8466 lb. of coal at $5.75 a ton; of 744 lb. at $6.50 a ton.

29. Find the cost of 976 lb. hay at $11.50 a ton.

DIVISION OF DECIMALS

124. (1) Reduce 19.2 in. to feet.

$$19.2 \text{ in.} = 19.2 \div 12 \text{ or } 1.6 \text{ ft.}$$

(2) May 30, 1900, the Philadelphia baseball team had won 19 games and lost 10. Find the percentage of games won.

The total number of games $= 19 + 10 = 29$.
The percentage of games won $= 19 \div 29 = .655$.

$$
\begin{array}{r}
.655 \\
29\overline{)19.000} \\
174 \\
\hline
160 \\
145 \\
\hline
150 \\
145 \\
\hline
5
\end{array}
$$

Exercise 116

1. Reduce to feet: 13.2 in.; 56.4 in.; 4.32 in.; 5.4 in.; 39.84 in.; 274.08 in.; 3.2412 in.

2. Reduce to yards: 2.4 ft.; 1.68 ft.; 4.32 ft.; 10.8 ft.; .48 ft.; 241.35 ft.

3. Reduce to yards: 19.44 in.; 75.24 in.; 17.28 in.; 247.68 in.; 15.48 in.

4. Reduce to gallons: 24.8 qt.; 1.732 qt.; 3.456 qt.; .712 qt.; .312 qt.; .516 qt.

5. Reduce to weeks: 34.3 da.; 5.39 da.; .245 da.; 29.12 da.; 456.4 da.

6. Reduce to days: 19.2 hr.; 3.84 hr.; 736.8 hr.; .456 hr.; 196.8 hr.

7. Reduce to bushels: 22.4 qt.; 4.16 qt.; 8.32 qt.; 238.08 qt.; 2.848 qt.

8. How many cubic inches in 1 cu. ft. ? In 1 gal. ?

9. Show by dividing 231 cu. in. into 1728 cu. in. that 1 cu. ft. $= 7.48$ gal., nearly.

10. How many gallons in 3 cu. ft. ? 8 cu. ft. ? 15 cu. ft. ? 36 cu. ft. ? 160 cu. ft. ? 1 cu. ft. = 7.48 gal.

11. A cistern contains 720 cu. ft. How many gallons of water will it hold ?

12. A baseball team won 29 games and lost 26. Find the percentage of games won.

13. The record of the following baseball teams on June 25, 1900, is given below. Find in each case the percentage of games won:

	Won	Lost		Won	Lost
Brooklyn	33	17	Chicago	24	27
Philadelphia	32	19	Cincinnati	22	27
Pittsburg	25	27	St. Louis	20	27
Boston	23	25	New York	19	29

125.

$$8\overline{)24} \qquad 8\overline{)2.4} \qquad 8\overline{).24}$$
$$3 \qquad\qquad .3 \qquad\qquad .03$$

That is, 24 divided by 8 = 3.

24 tenths divided by   8 = 3 tenths = .3.

24 hundredths divided by 8 = 3 hundredths = .03.

$$6\overline{).0018} \qquad\qquad 46\overline{)251.62}$$

$$
\begin{array}{r}
5.47 \\
46)251.62 \\
230 \\
\hline
216 \\
184 \\
\hline
322 \\
322 \\
\hline
\end{array}
$$

$$6\overline{).0018}$$
$$.0003$$

That is, 18 ten-thousandths ÷ 6 = 3 ten-thousandths = .0003 ; and 25,162 hundredths ÷ 46 = 547 hundredths = 5.47.

$$5\overline{)15} \qquad 50\overline{)150} \qquad 500\overline{)1500} \qquad 5000\overline{)15000}$$
$$3 \qquad\quad 3 \qquad\quad 3 \qquad\qquad 3$$

Therefore, if we multiply both divisor and dividend by 10, 100, 1000, and so on, the quotient remains unchanged.

126. (1) Find the value of 4.1262 ÷ .69.

The quotient of 4.1262 ÷ .69 is the same as that of 412.62 ÷ 69.   Here we multiplied each number by 100.

$$
\begin{array}{r}
5.98 \\
\hline
69)\overline{412.62} \\
345 \\
\hline
676 \\
621 \\
\hline
552 \\
552 \\
\hline
\end{array}
$$

∴ 4.1262 ÷ .69 = 5.98.

In this division the 412 is 412 units, and the quotient 5 is therefore 5 units; 676 is 676 tenths, and the quotient 9 is therefore 9 tenths; 552 is 552 hundredths, and the quotient 8 is therefore 8 hundredths.

(2) Find correct to the third decimal place the quotient of 8.94 ÷ 3.1416.

$$
\begin{array}{r}
3.1416)8.94 \\
\\
2.845 \\
\hline
31416)\overline{89400} \\
62832 \\
\hline
265680 \\
251328 \\
\hline
143520 \\
125664 \\
\hline
178560 \\
157080 \\
\hline
21480 \\
\end{array}
$$

∴ 8.94 ÷ 3.1416 = 2.845, correct to three decimal places.

Exercise 117

Divide, proving your answer correct to every third question:

1. 25.68 ÷ 3.21.   3. 8.54 ÷ .07.

2. 10.836 ÷ 5.16.   4. $ 49.92 ÷ .065.

5. 246.48 ÷ .003; 61.725 ÷ .075.

6. $ 64.26 ÷ 102; $ 5100 ÷ 1.02.

7. $ 5306.04 ÷ 102 twice in succession.

8. $ 84.3648 ÷ 1.04 twice in succession.

9. $ 54.75 ÷ .98, correct to three decimal places.

$ 75.60 ÷ .99, correct to three decimal places.

10. $ 16989.7728 ÷ 1.04 twice in succession and the result by 1.02.

11. 2450.90 ÷ .998, correct to three decimal places.

12. $ 11.679 ÷ $ 4.8665.

13. .00081 ÷ 27, and 1.77089 by 4.735.

14. 1 ÷ .1; 1 ÷ .01; 1 ÷ .0001.

15. 31.5 ÷ .126; 5.2 ÷.32.

16. 12.6 ÷ .0012, and .065341 ÷ .000475.

17. 3.012 ÷ .0006.

18. 130.4 ÷ .0004 and .004; 46.634205 ÷ 4807.65.

19. 1.69 ÷ 1.3, by .13, by 13, and by .013.

20. 816 ÷ .0004.

21. .00005 ÷ 2.5, by 25, and by .0000025.

22. 32.5 ÷ 8.7; .02 ÷ 1.7, correct to four decimal places.

23. .009384 ÷ .0063, correct to four decimal places.

24. 37.24 ÷ 2.9; .0719 ÷ 27.53, correct to four decimal places.

25. Reduce to rods: 38.5 yd.; 10.45 yd.; 4.4 yd.; 7.315 yd.

26. Reduce to rods: 3.3 ft.; .495 ft.; 379.5 ft.; .33 yd.

Exercise 118

1. A conductor whose runs amount to 4000 mi. a month, gets $ 90 a month. This is how many cents a mile?

2. During the month of April, 1900, the United States exported to Europe cotton to the value of $ 24,684,078, the average price being 9.3 ¢ a pound. Find the number of pounds.

3. Find the cost of 7225 lb. coal at $ 7.25 per ton of 2000 lb.

4. A creditor receives $ 1.50 for every $ 4 of what was due to him, and thereby loses $ 301.05. What was the sum due?

5. Divide .0075 by 6.4, and explain the reason for fixing the position of the decimal point in the quotient.

6. A merchant expended $ 280.60 in purchasing cloth at 95 ¢ a yard, at $ 1.37 a yard, and at 73 ¢ a yard, buying the same quantity of each. Find the entire number of yards purchased.

7. Find the earth's equatorial diameter in miles, supposing the sun's diameter, which is 111.454 times as great as the equatorial diameter of the earth, to be 883,345 mi.

REDUCTION OF DECIMALS

127. (1) Reduce .275 to a common fraction.

$$.275 = \tfrac{275}{1000} = \tfrac{55}{200} = \tfrac{11}{40}.$$

(2) Reduce to a common fraction .08$\frac{1}{3}$.

$$.08\tfrac{1}{3} = \frac{8\frac{1}{3}}{100} = \frac{25}{300} = \frac{1}{12}.$$

Exercise 119

Reduce to common fractions in their lowest terms:

1. .5.	**6.** .625.	**11.** .33$\frac{1}{3}$.	**16.** 8.9375.
2. .25.	**7.** .125.	**12.** .03$\frac{1}{3}$.	**17.** 29.975.
3. .75.	**8.** .0625.	**13.** .06$\frac{1}{4}$.	**18.** 18.06$\frac{2}{3}$.
4. .60.	**9.** .875.	**14.** .66$\frac{2}{3}$.	**19.** 6.00$\frac{4}{5}$.
5. .375.	**10.** .16$\frac{2}{3}$.	**15.** .14$\frac{2}{7}$.	**20.** 249.075.

21. My gain on selling a pound of tea was .125 of the cost, which was 72 ¢. What was my gain on 1 lb.?

22. A crate of berries which cost $ 1.35 was sold at a gain of .22$\frac{2}{9}$ of the cost; find the gain.

23. I bought a farm for $ 4800, and sold it at a loss of .375 of the cost price. Find the selling price.

24. A merchant sold coffee at a gain of .33⅓ of the cost. His gain on a quantity of coffee was $ 12; what did it cost him ?

25. A grain merchant sold wheat for $ 3400, gaining .06¼ of the cost. Find the cost price.

128. (1) Divide 7 by 8, expressing the result as a decimal.

$$8)\overline{7.000}$$
$$.875$$

Now $7 \div 8 = \frac{7}{8}$. ∴ $\frac{7}{8} = .875$.

PROOF

$.875 = \frac{875}{1000} = \frac{175}{200} = \frac{7}{8}$.

(2) Reduce $\frac{15}{16}$ to a decimal.

$$
\begin{array}{r}
.9375 \\
16)\overline{150} \\
144 \\
\hline
60 \\
48 \\
\hline
120 \\
112 \\
\hline
80 \\
80 \\
\hline
\end{array}
$$

∴ $\frac{15}{16} = .9375$.

(3) Reduce $9\frac{18}{23}$ to a decimal correct to four decimal places.

$$
\begin{array}{r}
.7826 \\
23)\overline{180} \\
161 \\
\hline
190 \\
184 \\
\hline
60 \\
46 \\
\hline
140 \\
138 \\
\hline
2 \\
\end{array}
$$

∴ $9\frac{18}{23} = 9.7826$, correct to four decimal places.

Exercise 120

Reduce to decimals:

1. $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$.   2. $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.   3. $\frac{1}{16}$, $\frac{5}{16}$, $\frac{9}{16}$.   4. $\frac{9}{25}$, $\frac{19}{20}$.

5. $\frac{66}{128}$, $\frac{54}{125}$, $\frac{1}{160}$.   6. $6\frac{11}{32}$, $3\frac{13}{16}$.   7. $\frac{5}{4}$, $\frac{9}{8}$, $\frac{32}{25}$, $\frac{31}{20}$.

8. Find correct to four decimal places the value of $\frac{4}{13}$, $\frac{15}{17}$, $\frac{14}{29}$, $\frac{16}{21}$.

9. Butter bought for $25 ¢$ a pound was sold for $28 ¢$ a pound. Express the gain as a decimal of the cost.

10. I bought a store for $ 6912, and sold it for $ 5184. Express the selling price as a decimal of the cost.

11. A real estate agent sold land which cost him $ 6240 at a loss of $ 2340. What was his loss on each $ 1000 invested?

12. A chain contains 66 ft., and a mile 5280 ft. What decimal part of a mile is a chain?

CHAPTER XIII

COMPOUND QUANTITIES

129. Quantities like 4 yd., $3\frac{1}{2}$ lb., and $6\frac{1}{4}$ gal. are called *simple quantities*, because they are expressed in terms of a single unit of measurement.

Quantities like 3 lb. 8 oz. 6 gal. 1 qt. are called *compound quantities*, because they are expressed in terms of two or more units of measurement.

130. The units of money are the units which are used to measure the *values* of things. The one dollar gold piece is at present (November, 1901) the prime unit or standard of value in the United States and Canada.

131.   UNITS OF VALUE

UNITED STATES MONEY

10 mills (m.) = 1 cent (ct. or ¢)
10 cents   = 1 dime (d.)
10 dimes   = 1 dollar ($)
10 dollars   = 1 eagle (E.)

The coins of the United States are :

Bronze : the cent.
Nickel : the five-cent piece.
Silver : the dime, quarter-dollar, half-dollar, and dollar.
Gold : the quarter-eagle, half-eagle, eagle, and double eagle.

132. Sterling Money is the money of Great Britain and Ireland.

The prime unit is 1 pound, whose value is $4.8665.

The pound, when coined, is called the sovereign.

4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound (£)
5 shillings	= 1 crown
21 shillings	= 1 guinea

133. The unit of *French Money* is 1 franc, which is worth 19.3¢.

The unit of *German Money* is 1 mark, which is worth 23.85¢.

Exercise 121

1. How many mills are there in 2¢? 3¢? ½¢? 1½¢? 2½¢?

2. How many cents are there in 40 mills? 60 mills? 15 mills? 5 mills? 25 mills?

3. State orally the table of English Money.

4. Reduce to farthings: 3d.; 6d. 2 far.; 9d. 3 far.

5. Reduce to pence: 4s.; 8s. 5d.; 12s. 6d.

6. Reduce to shillings: £7; £2 12s.; £9 7s.

7. How many pence are there in ½s.? ¾s.? ⅔s.? ⅚s.? ⅜s.?

8. How many shillings and pence are there in £⅔? £⅓? £⅚? £⁷⁄₇?

9. How many shillings are there in £.3? £.7? £.25? £.33⅓?

10. What fraction of a shilling is 3d.? 4d.? 8d.? 9d.? 10d.?

11. What is the value of £1 in American money? Of £10? Of £100?

12. How many shillings and pence are there in 60d.? 84d.? 39d.? 58d.? 112d.?

13. How many pounds and shillings are there in 80s.? 65s.? 120s.? 48s.?

14. How many pounds and shillings in 1 guinea? 4 guineas? 6 guineas? 9 guineas?

15. What decimal of a pound is 10s.? 12s.? 17s.? 24s.?

16. What part of a crown is 1s.? How many crowns in 10s.? 20s.?

17. Express 2 guineas in sovereigns and shillings.

18. What is the value of 10 francs in United States money?

19. What is the value of 100 marks in United States money?

20. What is the cost in cents of 3 books at 1 franc each?

21. What is the difference in value between 100 marks and 100 francs?

UNITS OF WEIGHT

134. **Avoirdupois Weight** is used for weighing everything except jewels, precious metals, and medicines when dispensed.

The prime unit of weight is 1 pound Avoirdupois.

AVOIRDUPOIS WEIGHT

$$16 \text{ ounces (oz.)} = 1 \text{ pound (lb.)}$$
$$100 \text{ pounds} = 1 \text{ hundredweight (cwt.)}$$
$$20 \text{ hundredweight} = 1 \text{ ton (T.)}$$

In the United States Custom House, and in weighing iron and coal at the mines, the long hundredweight and the long ton are used.

$$112 \text{ pounds} = 1 \text{ long hundredweight}$$
$$2240 \text{ pounds} = 1 \text{ long ton}$$

One pound Avoirdupois = 7000 grains
One ounce Avoirdupois = $437\frac{1}{2}$ grains

Exercise 122

1. State orally the table of Avoirdupois Weight.

2. Reduce to ounces: 1 lb. 8 oz.; 2 lb. 4 oz.

3. Express 1 lb. 8 oz. as a fraction of 2 lb. 4 oz.

4. Express 1 lb. 8 oz. as a decimal of 2 lb. 4 oz.

5. What part of 1 lb. is 4 oz.? 12 oz.? 2 oz.? 8 oz.? Express your results also as decimals.

6. What part of 1 T. is 400 lb.? 800 lb.? 1500 lb.?

7. A coal dealer buys coal by the carload at the mines. How many more pounds of coal does he get for 1 T. than he gives?

8. Show by dividing 7000 gr. by 16, that 1 oz. Avoirdupois is equal to $437\frac{1}{2}$ gr.

9. One ounce is what part of 1 lb.? $\frac{4}{7}$ of an ounce is what part of 1 lb.?

10. A farmer sells 3 cows whose united weight is 1 T. 5 cwt. What is the average weight of the cows?

135. Troy Weight is chiefly used for weighing gold, silver, and jewels.

<div style="text-align:center">TROY WEIGHT</div>

24 grains (gr.)   = 1 pennyweight (pwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces   = 1 pound (lb.)

One pound Troy = 5760 grains
One ounce Troy = 480 grains

Exercise 123

1. State orally the table of Troy Weight.

2. Reduce to grains: 1 pwt. 16 gr.; 2 pwt. 12 gr.

3. What is the ratio of 2 pwt. 12 gr. to 1 pwt. 16 gr.?

4. Express 1 pwt. 16 gr. as a decimal of 2 pwt. 12 gr.

5. Reduce 1 oz. to grains.

6. Divide 5760 gr. by 12, and verify your result in question 5.

7. By how many grains is 1 lb. Avoirdupois heavier than 1 lb. Troy?

8. By how many grains is 1 oz. Troy heavier than 1 oz. Avoirdupois?

9. How many ounces and pennyweights are there in $\frac{1}{4}$ lb.? $\frac{3}{8}$ lb.? $\frac{5}{6}$ lb.? $\frac{8}{9}$ lb.?

10. How many pennyweights are there in .25 oz.? .4 oz.? .35 oz.?

11. What part of a pound is 8 oz.? 1 oz.? $\frac{2}{3}$ oz.? $\frac{4}{5}$ oz.?

12. If coal is worth $6 a ton, how many pounds can be bought for $3? $2?

13. A cubic foot of water contains 1000 oz. How many pounds does a cubic foot of water weigh?

136. Druggists buy their medicines by Avoirdupois Weight, but use Apothecaries' Weight in mixing and in selling medicines.

APOTHECARIES' WEIGHT

20 grains	= 1 scruple (℈)
3 scruples	= 1 dram (ʒ)
8 drams	= 1 ounce (℥)
12 ounces	= 1 pound (℔)

One pound Apothecaries' Weight = 5760 grains
One ounce Apothecaries' Weight = 480 grains

Exercise 124

1. State orally the table of Apothecaries' Weight.

2. How many ounces in 16 ʒ? 40 ʒ? 72 ʒ?

3. How many drams in 9 ℈ 18 ℈? 54 ℈?

4. What part of a pound is 4 ℥? 9 ℥? 10 ℥?

5. How many scruples in 3 ʒ and 2 ℈? In 40 gr.? In 120 gr.?

UNITS OF LENGTH

137. The prime or standard unit of length is **1 yard.**

Long Measure

$$
\begin{aligned}
12 \text{ inches (in.)} &= 1 \text{ foot (ft.)} \\
3 \text{ feet} &= 1 \text{ yard (yd.)} \\
5\tfrac{1}{2} \text{ yards or } 16\tfrac{1}{2} \text{ feet} &= 1 \text{ rod (rd.)} \\
320 \text{ rods} &= 1 \text{ mile (mi.)}
\end{aligned}
$$

1 mi. = 320 rd. = 1760 yd. = 5280 ft.

A hand, used in measuring the height of horses, = 4 in. ; a knot, used in navigation, = 6086 ft. or 1.15 mi.

A fathom, used in measuring depth at sea, = 6 ft.

138. Surveyor's Linear Measure is used by surveyors in measuring land. The prime unit is **1 chain,** called *Gunter's Chain.*

Surveyor's Linear Measure

$$
\begin{aligned}
100 \text{ links (l.)} &= 1 \text{ chain (ch.)} \\
80 \text{ chains} &= 1 \text{ mile (mi.)}
\end{aligned}
$$

1 ch. = 4 rd. = 22 yd. = 66 ft. = 792 in.
1 link = 7.92.

139. Mark off in the schoolroom 1 ft., 1 yd., and 1 rd. Locate two points exactly **1 mi.** apart.

Exercise 125

1. How many inches are there in 1 yd.? $\frac{1}{4}$ yd.? $\frac{3}{4}$ yd.? $\frac{1}{6}$ yd.? $\frac{5}{6}$ yd.?

2. Reduce to yards: 4 rd.; 8 rd.; 32 rd.; 320 rd.; 1 mi.

3. Reduce to feet: 18 yd.; 76 yd.; 176 yd.; 1760 yd.; 1 mi.

4. Show that 1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63,360 in.

5. How many yards are there in .3 mi.? .8 mi.? .25 mi.? .06$\frac{1}{4}$ mi.?

6. What part of a foot is 3 in.? 6 in.? 8 in.? 10 in.?

7. What part of a yard is 12 in.? 18 in.? 24 in.? 27 in.?

8. What part of a rod is 1 yd.?   3 yd.?   5 yd.? $5\frac{1}{2}$ yd.?

9. What part of a mile is 40 rd.?   200 yd.?   280 yd.?

10. How many yards are there in 1 rd.?   How many feet? How many inches?

11. How many chains are there in 320 rd.?   How many rods are there in 1 ch.?

12. Show that 1 ch. = 4 rd. = 22 yd. = 66 ft. = 792 in.

13. How many inches are there in 100 links?   In 1 link?

UNITS OF SURFACE OR SQUARE MEASURE

140. Surface has two dimensions, — length and breadth.

141. The prime unit of area is 1 square yard, which, like 1 square inch, 1 square foot, 1 square rod, and 1 square mile, is derived from the corresponding unit of linear measure.

The measure of 1 ft. is 12, the unit being 1 in.
The measure of 1 sq. ft. is 144, the unit being 1 sq. in.
∴ 1 sq. ft. = 144 sq. in.
The measure of 1 yd. is 3, the unit being 1 ft.
The measure of 1 sq. yd. is 9, the unit being 1 sq. ft.
∴ 1 sq. yd. = 9 sq. ft.
The measure of 1 rd. is $5\frac{1}{2}$, the unit being 1 yd.
The measure of 1 sq. rd. is ($5\frac{1}{2} \times 5\frac{1}{2}$), or $30\frac{1}{4}$, the unit being 1 sq. yd.
∴ 1 sq. rd. = $30\frac{1}{4}$ sq. yd.

Illustrate the above by drawing 1 sq. ft., 1 sq. yd., and 1 sq. rd., and dividing each into the next lower units of area.

In the case of 1 sq. rd. draw according to the scale of 4 in. to 1 rd.

Surface or Square Measure

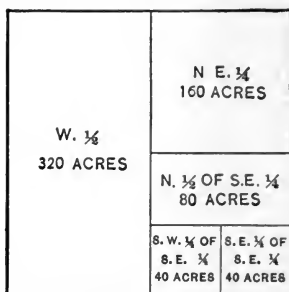|	
144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)
10 square chains = 1 acre ;	1 acre = 4840 square yards

142. A township is 6 mi. square, and is divided, as in the accompanying figure, into 36 sections, each 1 mi. square.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

TOWNSHIP.

SECTION.

Locate sections 8, 22, and 36 in the drawing. Draw a township on a scale of 1 in. to 1 mi., and divide it into 36 sections, numbering each section. Divide one of these sections into 4 square farms of 160 A. each, and name each farm according to its position in the section. Divide a second section into 8 rectangular farms of 80 A., and a third into 16 square farms of 40 A., and locate each farm as before.

Exercise 126

1. How many square inches are there in 2 sq. ft. ? 4 sq. ft. ? 9 sq. ft. ?

2. How many square feet in 5 sq. yd. ? ½ sq. yd. ? ¼ sq. yd. ?

3. 1 sq. rd. is equal to how many square yards ? 4 sq. rd. ? 16 sq. rd. ? 160 sq. rd. ? 1 A. ?

4. What part of a square rod is 1 sq. yd. ?

5. Reduce to square rods: 484 sq. yd.; 151¼ sq. yd.

6. What part of an acre is 80 sq. rd. ? 120 sq. rd. ?

7. How many square chains are there in 160 sq. rd. ? 1 sq. ch. equals how many square rods ?

8. Reduce to acres : 5 sq. mi.; 8 sq. mi.; 1 township.

9. If 1 sq. mi. is the unit of area, find the number which expresses the measure of 2560 A. Of 4 townships.

10. Into how many townships can a county be divided which contains 324 sq. mi. ?

11. What is the area of a square 6 ft. in length ?

12. What is the difference in area between two figures, one 6 in. sq. and the other 6 sq. in. ?

Illustrate by drawing.

UNITS OF VOLUME

143. A volume has three dimensions — length, breadth, and thickness.

144. The prime unit of volume is 1 cubic yard, which, like 1 cubic inch and 1 cubic foot, is derived from the corresponding unit of linear measure.

The measure of the volume of 1 cu. ft. = $12 \times 12 \times 12 = 1728$, the unit of volume being 1 cu. in.

The measure of the volume of 1 cu. yd. = $3 \times 3 \times 3 = 27$, the unit of volume being 1 cu. ft.

Cubic or Volume Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet   = 1 cubic yard (cu. yd.)

145. Firewood and rough stone are measured by the *cord* (cd.). The cord is a pile 8 ft. long, 4 ft. wide, and 4 ft. high. It contains 128 cu. ft. One cord foot (cd. ft.) is 1 ft. in length of the cord. Its volume is 16 cu. ft.

A cubic yard of earth is called a *load*.

How many loads of dirt are there in a pile 15 ft. long, 12 ft. wide, and 6 ft. deep ?

146. Mark off in one corner 1 cu. ft., 1 cu. yd., and 1 cd. Divide the cord into cord feet.

UNITS OF CAPACITY

147. The prime unit of capacity is 1 gallon.

LIQUID MEASURE

4 gills (gi.) = 1 pint (pt.)
2 pints   = 1 quart (qt.)
4 quarts   = 1 gallon (gal.)

148. The capacity of cisterns, reservoirs, and the like is often expressed in barrels (bbl.) of $31\frac{1}{2}$ gal. each, or in hogsheads (hhd.) of 63 gal. each.   A gallon contains 231 cu. in. Have a tin box made 11 in. long, 7 in. wide, and 3 in. deep, and note that 1 gal. of water will just fill it.

DRY MEASURE

2 pints (pt.) = 1 quart (qt.)
8 quarts   = 1 peck (pk.)
4 pecks   = 1 bushel (bu.)
One bushel contains 2150.42 cubic inches

149. Apothecaries' Fluid Measure is used by druggists in mixing medicines.

APOTHECARIES' FLUID MEASURE

60 minims ($\mathfrak{m}$) = 1 fluid dram (f $\mathfrak{z}$)
8 fluid drams = 1 fluid ounce (f $\mathfrak{z}$)
16 fluid ounces = 1 pint (O.)
8 pints   = 1 gallon (Cong.)
One minim is about equal to 1 drop

Exercise 127

1. What part of 1 gal. is 1 qt. ?   1 pt. ?   2 qt. 1 pt. ?

2. What is the number of cubic inches in 1 gal. ?   In 1 qt., liquid measure ?   In 1 pt., liquid measure ?

3. What part of 1 bu. is 1 qt. ?   1 pt. ?   3 pk. 4 qt. ?

4. What is the number of cubic inches in 1 bu. ?   In 1 qt., dry measure ?   In 1 pt., dry measure ?

5. How many more cubic inches are contained in 1 qt., dry measure, than in 1 qt., liquid measure?

6. Show that 1 bu. is nearly equal to 9.31 gal.

7. Find the number of cubic feet in 1 cd.

8. Find the number of cords of wood in a pile 30 ft. long, 6 ft. high, and 4 ft. wide.

9. Find the cost of a pile of wood 24 ft. long, $5\frac{1}{2}$ ft. high, and 4 ft. wide, at $6 a cord.

UNITS OF TIME

150. The prime unit of time is 1 day.

MEASURE OF TIME

60 seconds (sec.)	=	1 minute (min.)
60 minutes	=	1 hour (hr.)
24 hours	=	1 day (da.)
7 days	=	1 week (wk.)
365 days	=	1 common year (yr.)
366 days	=	1 leap year (l. yr.)
100 years	=	1 century (C.)

The year is divided into 12 calendar months:

January (Jan.)	31 da.	July	31 da.
February (Feb.)	28 or 29 da.	August (Aug.)	31 da.
March	31 da.	September (Sept.)	30 da.
April	30 da.	October (Oct.)	31 da.
May	31 da.	November (Nov.)	30 da.
June	30 da.	December (Dec.)	31 da.

In business transactions, 1 mo. is generally taken as equal to 30 da., and 1 yr. as equal to 360 da.

The following lines are useful in enabling one to remember the number of days in a month:

> "Thirty days hath September,
> April, June, and November."

A year is the period of the earth's revolution about the sun. It consists of 365 da. 5 hr. 48 min. 50 sec.

A common year lacks 11 min. 10 sec. of being 365 da. 6 hr., or $365\frac{1}{4}$ da. Hence, when we take 365 da. to a common year and 366 da. to a leap year,

we increase each year by 11 min. 10 sec. In 400 yr. this amounts to a little over 3 da. For that reason three out of four centennial years are counted as common years; *i.e.* the centennial years that do not divide equally by 400 have only 365 da.

Exercise 128

1. Express 9 hr. as a fraction of a week.

2. Express 12 sec. as a decimal of a minute.

3. Express 146 da. as a fraction of a year.

4. Express as a fraction of a month: 10 da., 15 da., 18 da.

5. Express as a decimal of a month: 21 da., 18 da., 27 da.

6. Find the number of days between Jan. 3 and Feb. 4; March 27 and April 30; Oct. 24 and Dec. 11.

7. How many days are there in November? January? December? April? February?

8. What part of a year each is: 1 mo. 10 da.? 2 mo. 12 da.? 7 mo. 6 da.? (1 mo. = 30 da.; 1 yr. = 360 da.)

Exercise 129

In the questions in the following exercise find the day on which the note is due.*

Find the date on which a note falls due, which I promise to pay:

1. Three months after March 3, 1900.

2. Four months after June 13, 1901.

3. Ninety days after May 13, 1900.

4. Sixty days after Sept. 16, 1899.

5. Ninety days after June 4, 1901.

Find the exact number of days between the day on which each of the following notes is discounted and the day on which it is due:

6. Day of discount, May 7, 1901; due June 6, 1901.

7. Day of discount, June 27, 1901; due Oct. 16, 1901.

---

* Add three days (called *days of grace*) to the given time if that is the custom in your state.

8. Day of discount, Sept. 4, 1901; due Oct. 30, 1901.

9. Day of discount, Dec. 23, 1901; due Feb. 20, 1902.

10. Day of discount, Jan. 15, 1901; due May 1, 1901.

CIRCULAR OR ANGULAR MEASURE

151. **Angular Measure** is used to measure arcs, angles, and in determining latitude, longitude, direction, the position of vessels at sea, and the like.

152. A **Circle** is a plane figure contained by one line called the **circumference**, all points of which are equally distant from a point within it called the **centre**.

One-half of the circumference is called the **semicircumference** and one-fourth a **quadrant**.
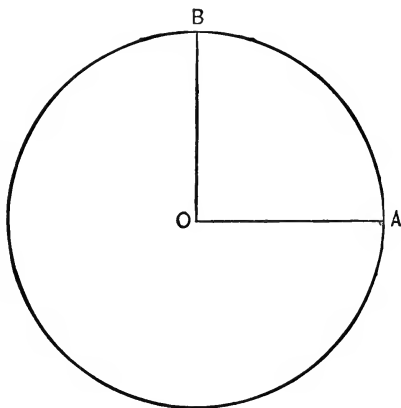
An **arc** is any portion of the circumference.

A line drawn through the centre and terminated at both extremities by the circumference is called the **diameter**.

The line drawn from the centre and terminated by the circumference is called the **radius**.

In the figure the line *OB* has revolved from *OA* through one-fourth of a *revolution*. The angle *AOB* is called a right angle, and contains 90°.

OA and *OB* are said to be *perpendicular* to each other.

Angular Measure

60 seconds ($''$) = 1 minute ($'$)
60 minutes   = 1 degree (°)
360 degrees   = 1 circumference (C.)

The circumference of the earth at the equator = 24,902 mi.
The length of a degree at the equator = 24,902 mi. ÷ 360 = 69.17 mi.

Exercise 130

1. What part of a revolution is 1 right angle ? 2 right angles ? 4 right angles ?

2. What part of a revolution is 60° ? 45° ? 225° ?

3. What is the length of the arc of 1° in a circle whose circumference is 360 yd. ?

4. If an arc of 3° is 6 ft. long, what is the length of the circumference of the circle ?

MISCELLANEOUS UNITS

153.

NUMBERS	PAPER
12 units = 1 dozen (doz.)	24 sheets = 1 quire
12 dozen = 1 gross	20 quires = 1 ream
12 gross = 1 great gross	2 reams = 1 bundle
20 units = 1 score	5 bundles = 1 bale

MISCELLANEOUS WEIGHTS

154.

A bushel of wheat	= 60 lb.
A bushel of beans	= 60 lb.
A bushel of clover seed	= 60 lb.
A bushel of shelled corn	= 56 lb.
A bushel of rye	= 56 lb.
A bushel of barley	= 48 lb.
A bushel of oats	= 32 lb.
A bushel of potatoes	= 60 lb.
A bushel of coarse salt (domestic)	= 56 lb.

These are the legal number of pounds per bushel in Michigan, Indiana, Illinois, Wisconsin, Iowa, Missouri, and New York.

On the Chicago Board of Trade seeds are sold by the cental.

A barrel of flour	= 196 lb.
A barrel of pork or beef	= 200 lb.
A cental of grain	= 100 lb.

Exercise 131

1. Pens are sold in boxes containing 1 gross. How many pens are there in a box ?

2. Lead pencils are sold in boxes containing $\frac{1}{2}$ gross. How many pencils are there in a box?

3. How many packages of lead pencils of 1 doz. each are there in a box?

4. Eggs are packed in crates holding 30 doz. How many eggs in a crate?

5. How many sheets of paper in 20 quires?

6. What articles of food weigh 60 lb. to the bushel?

7. How many bushels of wheat weigh as much as 10 bu. of barley?

8. What is the ratio of the weight of 1 bu. of barley to that of 1 bu. of oats?

155. A certain room is 8 yd. long. Here the unit of measurement is 1 yd., and the measure of the length of the room is the number 8.

The area of the floor of a room is 192 sq. yd. 6 sq. ft. Here the measure of the area is the sum of 192 units of 1 sq. yd. and 6 units of 1 sq. ft.

A pitcher holds $\frac{3}{4}$ of a gallon of water.

Here the measure of the capacity of the pitcher is $\frac{3}{4}$ and the unit of measurement is 1 gal.

Exercise 132

Name the measures of the following quantities, and the units of measurement:

1. The volume of a cistern which holds 450 cu. ft.

2. The volume of a cistern which holds 1200 gal.

3. The area of a field which contains $8\frac{1}{2}$ A.

4. The value of a house worth $2800. What are the measures of the value of the house with the following units: $5, $10, $50, $100?

5. The weight of 200 lb. of sugar. What are the measures of the weight with 10 lb., 1 cwt., and 1 T. as units?

6. The weight of a chest of tea containing 65 lb. If 5 lb. is used as the unit, what is the measure?

7. The weight of 8 oz. of gold. If 1 pwt. is the unit of measurement, what number expresses the measure? What number measures the weight when 1 lb. is the unit?

8. The weight of 40 gr. of quinine. What is the measure when 1 ℈ is the unit?

9. What are the measures of 1 lb. of gold, of lead, and of quinine, when the unit of measurement is 1 oz.? When 1 gr. is the unit?

10. The length of the circumference of a circle found to be 22 yd. long. What are the measures of the circumference when 1 ft. and 1 rd. are the measures?

11. The length between two points which measures 4 ch. What are the measures of the length when the units are 1 mi. and 1 link?

12. The area of a field which contains 80 sq. rd. What are the measures of the field when the units are 1 A. and 1 sq. yd.?

13. The capacity of a pitcher which contains $\frac{3}{4}$ of a gallon of water. What is the measure when the unit is 1 qt.?

14. The capacity of a basket which holds $6\frac{1}{2}$ qt. What are the measures, the units being 1 pt. and 1 pk.?

15. The time of a rainstorm, which lasted 2 hr. What are the measures, the units being 1 min. and 1 da.?

16. The weight of a silver cup which is 15 oz. 12 pwt. 12 gr.

156. The fundamental units used in the measurement of value are 1 cent, 1 dime, 1 dollar, and 1 eagle.

The value of a postage-stamp used to mail a letter to any part of the United States or Canada is measured by the number 2 and the unit 1¢.

The cost of a quart of berries worth 15¢ is measured by the number 3 and the unit 1 nickel, or by the number 1 and the unit 1 dime plus the number 1 and the unit the nickel.

It may also be measured by the number 15 and the unit 1¢. It may also be measured by the number 1 and the unit 1 quarter-dollar less the number 1 and the unit 1 dime.

The unit for measuring oil is 1 gal. That for buying spice by retail is 1 oz. Frequently a quantity is expressed with reference to two or more units. Thus, the length of a table being 1 yd. 2 ft. 6 in., the units are 1 yd., 1 ft., and 1 in.

Exercise 133

1. Name instances in which $1 is the unit of value; 1¢; 1 nickel; 1 dime. What unit of value is most commonly used?

2. Name instances in which the units of weight used are 1 oz.; 1 lb.; 1 cwt.; 1 T.

3. Name quantities whose weight is measured by these units: 1 gr.; 1 pwt.; 1 oz.; and 1 lb.

4. Name quantities which are measured by the unit 1 gr.; 1 ℈; 1 Ʒ; 1 ℥; 1 ℔.

5. What quantities are expressed in terms of these units: 1 in.? 1 ft.? 1 yd.? 1 rd.? 1 mi.?

6. Name things whose measurement is given in terms of the unit 1 pt.; 1 qt.; 1 gal.

7. Name articles whose measurement is expressed in terms of the unit 1 bu.; 1 pk.; 1 qt.

8. In measuring time give instances in which you use as the unit 1 century; 1 yr.; 1 mo.; 1 wk.; 1 da.; 1 hr.; 1 min.; 1 sec.

P

9. Name articles whose quantity is expressed in terms of the unit 1 doz.; 1 gross; 1 great gross; 1 score.

10. In measuring paper the following units are used: **1 quire**; 1 ream. Give instances when 1 quire is used as the unit, and also when 1 ream is used.

11. What unit of weight connects Avoirdupois, Troy, and Apothecaries' weight? What number expresses the measure of 1 lb. of each kind in terms of the common unit? Of 1 oz.?

12. What units are common to Apothecaries' and Troy weight and of equal value?

13. What units of length connect Surveyors' Long Measure with Linear Measure?

14. Name five units of area which are derived from corresponding units of length. Why is 1 A. chosen as a unit of area? Give instances in which 1 A. is used as the unit of area, and also when 1 sq. mi. is the unit.

15. Name a unit of volume larger than 1 cu. yd. Why have we no units of volume corresponding to the linear units 1 rd. and 1 mi.?

16. Find the number of cubic inches in 1 qt., dry measure, and find how much greater it is than 1 qt., liquid measure.

17. State the number of days in the years 1500, 1600, 1700, 1800, 1900, 2000.

18. The circumference of a circle is 1,296,000 in. in length. Find the length of 1°; of 1'; of 1".

19. Find the quantity measured by the number 4 and the unit 2 ft. 6 in. By the number 8 and the unit 2 gal. 3 qt.

20. How many square rods in 1 A.? How many acres in 1 sq. mi.? What part of a square mile is a farm of 40 A.? If this farm is in the form of a square, what is the length of one side? Of the perimeter?

REDUCTION

157. Reduction Descending is the process of reducing a quantity expressed in terms of a unit or units of measurement to a quantity expressed in terms of a smaller unit, or of smaller units of measurement.

Reduce   2 mi. 36 rd. 5 yd. 2 ft. to feet.

$$\begin{array}{r} 320 \\ \hline 640 \\ 36 \\ \hline 676 \text{ rd.} \\ 5\frac{1}{2} \\ \hline 338 \\ 3380 \\ 5 \\ \hline 3723 \text{ yd.} \\ 3 \\ \hline 11169 \\ 2 \\ \hline 11171 \text{ ft.} \end{array}$$

NOTE. — In this reduction we are to think of the operation as signifying 2×320 rd., or by the law of commutation as 320×2 rd., and *not* as representing 320×2 mi.

$$2 \text{ mi.} = 2 \times 320 \text{ rd.} = 640 \text{ rd.}$$

$$2 \text{ mi. } 36 \text{ rd.} = 640 \text{ rd.} + 36 \text{ rd.} = 676 \text{ rd.}$$

$$676 \text{ rd.} = 676 \times 5\frac{1}{2} \text{ yd.} = 3718 \text{ yd.}$$

$$2 \text{ mi. } 36 \text{ rd. } 5 \text{ yd.} = 3718 \text{ yd.} + 5 \text{ yd.} = 3723 \text{ yd.}$$

$$3723 \text{ yd.} = 3723 \times 3 \text{ ft.} = 11,169 \text{ ft.}$$

$$\therefore 2 \text{ mi. } 36 \text{ rd. } 5 \text{ yd. } 2 \text{ ft.} = 11,169 \text{ ft.} + 2 \text{ ft.} = 11,171 \text{ ft.}$$

Exercise 134

1. Reduce 5 gal. 3 qt. 1 pt. to pints.
2. Reduce 18 bu. 6 pk. 3 qt. to quarts.
3. Reduce £5 12*s.* 9*d.* to pence.
4. Reduce 8 oz. 15 pwt. 17 gr. to grains.
5. Reduce 7 T. 18 cwt. 14 lb. to pounds.
6. Reduce 15 da. 17 min. to minutes.
7. Reduce 18 rd. 4 yd. 2 ft. 6 in. to inches.
8. Reduce 5 oz. 5 dr. 2 sc. 16 gr. to grains.

9. Reduce 2 A. 4 sq. rd. 2 sq. yd. 8 sq. ft. to square feet.

10. Reduce 16 cu. ft. 1374 cu. in. to cubic inches.

11. Reduce 1 mi. 18 rd. 2 yd. 2 ft. 6 in. to inches.

12. State how to reduce a quantity from higher to lower denominations.

13. Find the number of acres in a township.

14. Find the number of cubic inches in a vessel containing 25 gal.

15. Reduce 1 lb. 7 oz. 14 pwt. 19 gr. to grains.

16. Reduce 7 T. 15 cwt. 56 lb. to pounds.

17. Reduce 17 ℔ 2 ʒ 2 ϶ to grains.

18. Reduce 3 cu. yd. 1001 cu. in. to cubic inches.

19. Reduce 760 bu. 3 pk. to quarts.

20. Reduce 6 reams 19 quires 18 sheets to sheets.

21. Reduce 5 da. 17 hr. to seconds; and 1 wk. 23 hr. 59 sec. to seconds.

22. Reduce 5 A. 80 sq. rd. 28 sq. yd. to square yards.

158. Reduction Ascending is the process of reducing a quantity expressed in terms of a unit, or of units, to a quantity expressed in terms of a larger unit, or of larger units.

(1) Reduce 242,337 in. to higher denominations.

```
12 | 242337
 3 |  20194 ft. 9 in.
   |   6731 yd. 1 ft.
   |      2
11 |  13462 half yd.
320|   1223 rd. 9 half yd., i.e. 4 yd. 1 ft. 6 in.
   |      3 mi. 263 rd.
```

∴ 242,337 in. = 3 mi. 263 rd. 4 yd. 1 ft. 9 in. }
             1 ft. 6 in. }

    = 3 mi. 263 rd. 5 yd. 0 ft. 3 in.

242,337 in. = 20,194 ft. 9 in.

20,194 ft. 9 in. = 6731 yd. 1 ft. 9 in.

6731 yd. 1 ft. 9 in. = 1223 rd. $4\frac{1}{2}$ yd. 1 ft. 9 in.

1223 rd. $4\frac{1}{2}$ yd. 1 ft. 9 in. = 3 mi. 263 rd. $4\frac{1}{2}$ yd. 1 ft. 9 in.

= 3 mi. 263 rd. 5 yd. 0 ft. 3 in.

∴ 242,337 in. = 3 mi. 263 rd. 5 yd. 3 in.

(2) To prove this answer correct, reduce 3 mi. 263 rd. 5 yd. 3 in. to inches, by the method of the preceding exercise.

159. We reduce a quantity expressed in terms of a smaller unit to larger units in order to get a more definite idea of its value.

Thus we form no definite idea of a distance between two points when we are told that it is 242,337 in.; but we have a definite idea of the same distance when we are told that it is 3 mi. 263 rd. 5 yd. 3 in.

Exercise 135

Reduce to higher denominations, and prove every third answer correct:

1. 678 pt.
2. 4622 pt. of dry measure.
3. 483,197 sec.
4. 4728 cu. ft. to cords.
5. 18,420 lb. of wheat to bushels.
6. 21,489d.
7. 93,742 oz.

8. State how to reduce a quantity expressed in terms of a lower unit to higher units.

9. 5420 gr.
10. 141,728 gr.
11. 57,893 cu. in.
12. 56,735d.
13. 38,297 oz.
14. 364,428 in.
15. 273,460 sq. yd.
16. 6,188,724 sq. in.
17. 429,678 in.

18. 73,940 $\mathfrak{m}$.

19. 89,673 gr., Apothecaries' weight.

20. 7493 units.

21. Reduce 37,921 in. to chains, rods, etc.

22. Reduce 121,838 A. to townships, etc.

COMPOUND ADDITION AND SUBTRACTION

160. Add:

mi.	rd.	yd.	ft.	in.
2	27	1	2	8
1	146	2	1	6
8	91	2	0	4
7	152	1	2	9
19	97	$2\frac{1}{2}$	1	3
		$\frac{1}{2}$ =	1	6
19	97	2	2	9

The sum of the inches column is 27 in., or 2 ft. 3 in.

The sum of the feet column, increased by 2 ft., is 7 ft., or 2 yd. 1 ft.

The sum of the yards column, increased by 2 yd., is 8 yd., or 1 rd. $2\frac{1}{2}$ yd.

The sum of the rods column, increased by 1 rd., is 417 rd., or 1 mi. 97 rd.

The sum of the miles column, increased by 1 mi., is 19 mi.

Changing $\frac{1}{2}$ yd. to 1 ft. 6 in. and adding, we have the sum = 19 mi. 97 rd. 2 yd. 2 ft. 9 in.

As in the problems in addition in Chapter IV, we are required in the preceding question to find the whole quantity measured by the four given parts, of which the first is 2 mi. 27 rd. 1 yd. 2 ft. 8 in. What are the other three measured parts?

In this question, how would the work of writing and adding be diminished if our units of length were arranged according to the *decimal* system?

161. Subtract 53 lb. 5 oz. 18 pwt. from 72 lb. 4 oz. 7 pwt.

lb.	oz.	pwt.
72	4	7
53	5	18
18	10	9

Since we cannot take 18 pwt. from 7 pwt., take 1 oz. or 20 pwt. from 4 oz. and add it to the 7 pwt., making 27 pwt. 18 pwt. from 27 pwt. leaves 9 pwt. Since we cannot take 5 oz. from 3 oz., take 1 lb. or 12 oz. from 72 lb. and add it to the 3 oz., making 15 oz. 5 oz. from 15 oz. leaves 10 oz. 53 lb. from 71 lb. leaves 18 lb. Hence the difference = 18 lb. 10 oz. 9 pwt.

As in the problems in subtraction in Chapter V, we are given in the preceding question the whole quantity measured by 72 lb. 4 oz. 7 pwt., and one part, viz. 53 lb. 5 oz. 18 pwt., and are required to find the part measured by their difference.

Exercise 136

Add:

	bu.	pk.	qt.	pt.
1.	3	5	6	1
	8	4	1	0
	7	3	5	1
	9	4	3	1

	lb.	oz.	pwt.
2.	18	11	16
	16	9	22
	23	8	6
	17	6	13

	£	s.	d.
3.	5	17	10
	36	0	11
	7	3	4
	73	19	8
	30	14	5

	T.	cwt.	lb.
4.	16	17	74
	13	10	20
	17	15	19
	84	0	87
	11	11	36

	℥	ʒ	℈	gr.
5.	22	3	2	19
	56	0	1	10
	3	2	2	11
	15	6	1	9
	79	4	1	10

	cu. yd.	cu. ft.	cu. in.
6.	3	23	171
	17	17	31
	28	26	1000
	34	23	1101

7. State how to add compound quantities.

Subtract:

	lb.	oz.	pwt.
8.	144	8	14
	106	11	16

	yd.	ft.	in.
9.	15	1	5
	13	2	7

	℔	℥	ʒ	℈
10.	144	9	4	1
	129	0	7	3

	lb.	oz.	pwt.	gr.
11.	5836	0	0	0
	4976	7	13	19

	cu. yd.	cu. ft.	cu. in.
12.	37	18	857
	35	24	1280

13. State how to subtract one compound quantity from another.

14. State in what respects addition and subtraction of compound quantities are the same as addition and subtraction of numbers, and state how they differ.

COMPOUND MULTIPLICATION AND DIVISION

162. Multiply 5 wk. 6 da. 18 hr. by 11.

We are here required to find the whole quantity measured by the unit 5 wk. 6 da. 18 hr. and the number 11.

wk.	da.	hr.
5	6	18
		11
65	4	6

We multiply 18 hr. by 11 and obtain the product 198 hr., or 8 da. 6 hr. Then we multiply 6 da. by 11 and obtain the product 66 da., which, increased by 8 da., is 74 da., or 10 wk. 4 da.　Multiplying 5 wk. by 11, and adding 10 wk., we have 65 wk.　Hence the product is 65 wk. 6 da. 18 hr.

What is the ratio of 65 wk. 4 da. 6 hr. to 5 wk. 6 da. 18 hr. ?

163. (1) Divide 88 rd. 3 yd. 1 ft. by 34.

```
            rd. yd. ft.   rd. yd. ft.
     34) 88   3   1  (2   3   1
         68
         ──
         20 rd.
          5½
         ──────
         110 yd.
           3
         ──────
         113 yd.
         102
         ──────
          11 yd.
           3
         ──────
          33 ft.
           1
         ──────
          34 ft.
          34
```

Hence the quotient or unit of measure is 2 rd. 3 yd. 1 ft.

The remainder on dividing 88 rd. by 34 is 20 rd.　20 rd. 3 yd. = 113 yd. The remainder on dividing 113 yd. by 34 is 11 yd.　11 yd. 1 ft. = 34 ft.

On dividing 34 ft. by 34, there is no remainder.　Hence the quotient is 2 rd. 3 yd. 1 ft.　What part of the dividend is the quotient?

(2) Divide 73 gal. 1 pt. by 16 gal. 1 qt.

We are here required to find the number which is the ratio of the quantity 73 gal. 1 pt. to the unit 16 gal. 1 qt.

$$73 \text{ gal. 1 pt.} = 585 \text{ pt.} \; ; \; 16 \text{ gal. 1 qt.} = 130 \text{ pt.}$$
$$585 \text{ pt.} \div 130 \text{ pt.} = 4\tfrac{1}{2}.$$
$$\therefore \text{ the quotient or ratio is } 4\tfrac{1}{2}.$$

Exercise 137

1. Multiply 7 gal. 3 qt. 1 pt. by 9.
2. Multiply 8 da. 12 hr. 25 min. by 8.
3. Divide £ 199 6s. 8d. by 13.
4. Divide 459 lb. 4 oz. 5 pwt. 22 gr. by 29.
5. Multiply 86 lb. 7 oz. 16 pwt. 11 gr. by 8.
6. Multiply 5 wk. 6 da. 18 hr. 14 min. by 11.
7. Divide 1738 cu. yd. 382 cu. in. by 302.
8. Divide 684 da. 8 hr. 9 min. by 47.
9. Multiply 70 yd. 2 ft. 10 in. by 7.
10. Divide 1 mi. 54 rd. 1 ft. 2 in. by 29.
11. Multiply 2 hr. 8 min. 9 sec. by 15.
12. Divide 13° 26' by 15.

13. (a) State how to multiply a compound quantity by a given number. (b) State how to divide a compound quantity by a given number.

14. State in what respects multiplication and division of compound quantities are like multiplication and division of numbers, and how they differ.

FRACTIONS OF SIMPLE AND COMPOUND QUANTITIES

164. (1) Find the value of $\tfrac{5}{6}$ of a mile.

$$\tfrac{5}{6} \text{ mi.} = \tfrac{5}{6} \text{ of } 320 \text{ rd.} = 266\tfrac{2}{3} \text{ rd.}$$
$$\tfrac{2}{3} \text{ rd.} = \tfrac{2}{3} \text{ of } 5\tfrac{1}{2} \text{ yd.} = 3\tfrac{2}{3} \text{ yd.}$$
$$\tfrac{2}{3} \text{ yd.} = \tfrac{2}{3} \text{ of } 3 \text{ ft.} = 2 \text{ ft.}$$
$$\therefore \tfrac{5}{6} \text{ of a mile} = 266 \text{ rd. } 3 \text{ yd. } 2 \text{ ft.}$$

(2) Find the value of $\frac{5}{9}$ bu. $- \frac{5}{7}$ pk.

$$\frac{5}{9} \text{ bu.} = \frac{5}{9} \text{ of 4 pk.} = 2\frac{2}{9} \text{ pk.}$$
$$2\frac{2}{9} \text{ pk.} - \frac{5}{7} \text{ pk.} \qquad = 1\frac{32}{63} \text{ pk.}$$
$$\frac{32}{63} \text{ pk.} = \frac{32}{63} \text{ of 8 qt.} = 4\frac{4}{63} \text{ qt.}$$
$$\therefore \ \frac{5}{9} \text{ bu.} - \frac{5}{7} \text{ pk.} = 1 \text{ pk.} \ 4\frac{4}{63} \text{ qt.}$$

Exercise 138

Find the value of:

1. $\frac{7}{8}$ of £1; $\frac{2}{3}$ of £2.

2. $\frac{3}{10}$ of a day; $\frac{5}{11}$ of a mile.

3. $\frac{5}{8}$ of 3 T.; $7\frac{2}{5}$ lb. Avoir.

4. $\frac{5}{9}$ bu. $- \frac{5}{7}$ pk.; $\frac{3}{8}$ lb. Troy $+ \frac{5}{6}$ lb. Troy $- \frac{3}{4}$ oz. Troy.

5. $\frac{2}{3}$ A.; $\frac{5}{9}$ A. $- \frac{2}{3}$ sq. rd.; $\frac{5}{9}$ sq. mi.

6. $\frac{3}{4}$ of 5 lb. 8 oz. 6 pwt.

7. $\frac{4}{7}$ of 2 mi. 38 rd. 4 yd. 2 ft. 2 in.

8. State how to express a fraction of a unit of measure in terms of smaller units.

Express .854 of an acre in lower denominations.

```
.854 A.
  160
51240
 854
136.640 sq. rd.
   30¼
   16
  1920
19.36 sq. yd.
   9
 3.24 sq. ft.
  144
   96
  96
  24
34.56 sq. in.
```

$\therefore$ .854 A. = 136 sq. rd. 19 sq. yd. 3 sq. ft. 34.56 sq. in.
Adapt note, § 157, to this problem.

Exercise 139

Find the value of:

1. .84 of a day.
2. .045 of a mile.
3. .6 of a pound Troy.

4. 5.923 mi. − 75.18 rd.
5. £75.43 − 16.76s.
6. 4.7 A. − 2.93 sq. rd.

165. Reduce 213 rd. 5 ft. 6 in. to the fraction of 3 mi.

$$213 \text{ rd. } 5 \text{ ft. } 6 \text{ in.} = 42{,}240 \text{ in.}$$

$$3 \text{ mi.} = 3 \times 63{,}360 \text{ in.} = 190{,}080.$$

$$\therefore 213 \text{ rd. } 5 \text{ ft. } 6 \text{ in.} = \tfrac{42240}{190080}, \text{ or } \tfrac{2}{9} \text{ of } 3 \text{ mi.}$$

The G. C. M. of 42,240 and 190,080 is 21,120, which divides the numerator twice and the denominator 9 times.

Exercise 140

1. Reduce £1 7s. 6d. to the fraction of £2.

2. Reduce 266 rd. 3 yd. 2 ft. to the fraction of a mile.

3. Reduce 14 hr. 24 min. to the fraction of a day.

4. Reduce 4 mo. 3 da. to the fraction of a year. (30 da. to a month, and 360 da. to a year.)

5. Reduce 10 mo. 24 da. to the fraction of a year.

6. Reduce $1\frac{1}{2}$ in. to the fraction of $1\frac{1}{2}$ yd.

7. Reduce $\frac{8}{7}$ lb. Avoirdupois to the fraction of 5 lb. Troy.

8. Express 1 yd. 2 ft. 6 in. as a fraction of 2 yd. 1 ft. 6 in.

166. Express 58 rd. 2 yd. 7.2 in. as a decimal of a mile.

12	7.2 in.
3	.6 ft.
5.5	2.2 yd.
320	58.4 rd.
	.1825 mi.

$$7.2 \text{ in.} = .6 \text{ ft.} = .2 \text{ yd.}$$

$$2 \text{ yd.} + .2 \text{ yd.} = 2.2 \text{ yd.} = .4 \text{ rd.}$$

$$58.4 \text{ rd.} = .1825 \text{ mi.}$$

$$\therefore 58 \text{ rd. } 2 \text{ yd. } 7.2 \text{ in.} = .1825 \text{ mi.}$$

Prove this answer correct by reducing .1825 mi. to lower denominations.

Exercise 141

1. Reduce 8 oz. 15.2 pwt. to the decimal of a pound.

2. Reduce 21 hr. 57 min. 36 sec. to the decimal of a day.

3. Reduce 147 rd. 1 yd. 3.6 in. to the decimal of a mile.

4. 25 sq. mi. 128 A. is what decimal of a township?

5. Reduce 67 sq. rd. 6 sq. yd. 64.8 sq. in. to the decimal of an acre.

Board Measure

167. Boards, planks, joists, etc., are sold by the board foot. The board foot is equal in volume to one square foot of board, one inch thick.

Thus a board 18 ft. long, 14 in. wide, and 1 in. thick or less contains $18 \times \frac{14}{12}$, or 21 ft., board measure.

To find the number of feet, board measure, in lumber more than 1 inch thick, we find the number of square feet in the surface of the board and multiply this result by the number of inches that the lumber is thick.

Thus a board 15 ft. long, 8 in. wide, and $2\frac{1}{2}$ in. thick contains $15 \times \frac{8}{12} \times \frac{5}{2}$, or 25 board feet.

Exercise 142

How many feet, board measure, in:

1. A board 20 ft. long, 9 in. wide, and 1 in. thick? $\frac{3}{4}$ in. thick?

2. A board 18 ft. long, 8 in. wide, and $2\frac{1}{2}$ in. thick?

3. A scantling 16 ft. long, 3 in. wide, and 4 in. thick?

4. Twenty scantlings, 24 ft. long, 5 in. wide, and 7 in. thick?

5. A stick of timber 33 ft. long and 14 in. square?

6. One cubic foot?

7. Find the number of board feet in 2 doz. of each of the following scantlings:

(1) $2 \times 4 \times 10$ ft.	(4) $2 \times 4 \times 20$ ft.	(7) $3 \times 10 \times 16$ ft.
(2) $2 \times 6 \times 14$ ft.	(5) $2 \times 8 \times 18$ ft.	(8) $4 \times 6 \times 18$ ft.
(3) $2 \times 10 \times 24$ ft.	(6) $2 \times 12 \times 22$ ft.	(9) $6 \times 6 \times 10$ ft.

8. What is the cost of 25 joists each 6 in. by 4 in. by 15 ft. at $22 per thousand?

9. What is the cost of 24 joists each 5 in. by 7 in. by 10 ft. at $21 per thousand?

10. How much will it cost to enclose a rectangular lot 50 ft. wide and 100 ft. deep with a tight board fence 6 ft. high with boards that cost $18 per thousand?

11. Find the cost of the lumber needed for a tight board sidewalk 100 ft. long, 4 ft. wide, 2 in. thick, if sold in 16 ft. lengths at $16 per thousand.

Longitude and Time

168. Turn to your geography and find several meridian lines. Find the prime meridian which passes through Greenwich, England.

The imaginary lines drawn on the earth's surface from pole to pole are called *meridians*. The meridian passing through Greenwich, a town near London, England, having the royal observatory, is called the *prime* or *standard* meridian.

Places west of the prime meridian are in west longitude, and places east of the prime meridian are in east longitude. Thus, Washington is 77° 7′ west longitude, and Paris 2° 20′ east longitude.

169. Find from the maps in your geographies to the nearest degree the longitude of these cities: New York, Pittsburg, Richmond, Atlanta, Chicago, Denver, Salt Lake City, San Francisco.

Find also the longitude of Rome, Stockholm, Athens, Constantinople, St. Petersburg, and Moscow.

170. The difference in longitude between Philadelphia, which is 75° 9′ west longitude, and Portland, which is 70° 15′ west longitude, is 4° 54′.

The difference between the longitude of Philadelphia and that of Paris, which is 2° 20′ east longitude, is 77° 29′, and is obtained by finding the sum of the longitudes.

171. Find on the map of the United States and name the meridians that denote Eastern time, Central time, Mountain time, and Pacific time. How many degrees are there between these meridian lines? What is the difference in time between places situated on these meridians?

172. As the sun rises in the east, it is sunrise in New York earlier than in Chicago; consequently at any time during the day the clock time in New York is later than in Chicago. Similarly, clock time in San Francisco, which is west of Chicago, is earlier than in the latter city.

173. Since the sun appears to move in a circle about the earth, *i.e.* through 360° in 24 hr., we have the following:

In 24 hr. the sun passes through 360°.
In 1 hr. the sun passes through 15°.
In 1 min. the sun passes through $\frac{1}{60}$ of 15° = $\frac{1}{4}$° = 15′.
In 1 sec. the sun passes through $\frac{1}{60}$ of 15′ = $\frac{1}{4}$′ = 15″.

Hence, to reduce longitude expressed in time to longitude expressed in degrees, we multiply by 15, and to reduce longitude expressed in degrees to longitude expressed in time, we divide by 15.

174. Make the multiplication table of 15 and memorize it, so as to be able to work questions in longitude and time by short multiplication and division.

Exercise 143

1. What is the difference in longitude between two places whose difference in time is 1 hr.? 2 hr.? 4 hr.? 2 min.? 3 min.? 1 sec.? 3 sec.?

2. What is the difference in longitude between two places whose difference in time is 1 hr. 2 min. ? 1 hr. 3 min. 2 sec. ?

3. What is the difference in time between two places whose difference in longitude is 30° ? 75° ? 120′ ? 90′ ? 135″ ? 105″?

4. What is the difference in time between two places whose difference in longitude is 15° 45′ 30″ ? 75° 15′ 45″ ?

5. Find the difference in longitude between the following places, and illustrate your answers by diagrams:

Washington 77° west longitude and Helena 112° west longitude.
Washington 77° west longitude and Hamburg 10° east longitude.
Cairo 32° east longitude and Hamburg 10° east longitude.

6. What is the difference in time between two places:

   (1) One 64° west longitude, the other 34° east longitude?

   (2) One 64° west longitude, the other 26° east longitude?

   (3) One 64° east longitude, the other 34° east longitude?

7. When it is 6 A.M. at San Francisco, what time is it at a place 45° east of San Francisco? 30° east? 15° 45′ east? 30° 15′ 45″ east?

8. When it is 11 A.M. at Chicago, what time is it at a place 60° west of Chicago? 30° west? 45′ west? 15° 45′ ?

175. (1) Find the difference in time between St. Louis 90° 19′ 26″ west longitude and Sacramento 121° 25′ 41″ west longitude.

$$\begin{array}{r r r}121° & 25′ & 41′ \\ 90 & 19 & 26 \\ \hline 15)31° & 6′ & 15″ \qquad \text{difference in longitude} \end{array}$$

      2 hr. 4 min. 25 sec. difference in time

A difference in longitude of 31° gives a difference of 31 ÷ 15, or 2 hr. of time, with a remainder 1°. A difference in longitude of 1° 6′, or 66′, gives a difference of 66 ÷ 15, or 4 min. of time, with a remainder 6′. A difference of 6′ 15″, or 375″, gives a difference of 375 ÷ 15, or 25 sec. of time.

(2) Berlin is 13° 23′ 53″ east longitude and Boston is 71° 4′ 9″ west longitude. When it is 1.15 p.m. at Boston, what time is it at Berlin ?

$$
\begin{array}{r r r}
13° & 23′ & 53″ \\
71 & 4 & 9 \\
\hline
15)84° & 28′ & 2″ \quad \text{difference in longitude}
\end{array}
$$

5 hr. 37 min. $52\frac{2}{15}$ sec. difference in time
1 hr. 15 min. \qquad time in Boston
6 hr. 52 min. $52\frac{2}{15}$ sec. time in Berlin

∴ it is 52 min. $52\frac{2}{15}$ sec. after 6 p.m., or 7 min. $7\frac{13}{15}$ sec. to 7 p.m.

Exercise 144

Find the difference in time between the following cities :

1. Brooklyn 73° 58′ W. and Omaha 95° 28′ W.

2. St. Paul 93° 3′ 45″ W. and Cleveland 81° 39′ W.

3. Indianapolis 86° 6′ 57″ W. and San Francisco 122° 26′ 12″ W.

4. Cincinnati 84° 28′ 36″ W. and Glasgow 4° 17′ 6″ W.

5. Detroit 83° 5′ 7″ W. and Vienna 16° 22′ 22″ E.

6. Pillsbury 79° 55′ 43″ W. and Amsterdam 4° 52′ 13″ E.

7. Newark 74° 9′ 12″ W. and Rome 12° 27′ 58″ E.

8. When it is 11 a.m. at Cleveland, what o'clock is it at St. Paul ?

9. What time is it at Indianapolis at the opening of school at 9 a.m. in San Francisco ?

10. When it is 7 a.m. at Cincinnati, what time is it at Glasgow ?

11. When it is 8 a.m. at Omaha, what time is it at Brooklyn ?

12. A man travels until his watch is 1 hr. 5 min. 16 sec. slow. Does he travel east or west, and how many degrees has he gone ?

13. A vessel sailed from a port directly on a line of latitude a certain distance, and then due north to port, where the captain found that his chronometer was 40 min. slow. In which direction did he sail at first, and how many degrees ?

14. What is the difference in longitude between two places whose difference in time is:

> (*a*) 2 hr. 33 min. 18 sec. ?
>
> (*b*) 4 hr. 27 min. 46 sec. ?
>
> (*c*) 6 hr. 12 min. 29 sec. ?

15. Buffalo is 78° 57′ 48″ W. and Constantinople is 28° 59′ 3″ E. What time is it in Constantinople when it is 20 min. after 6 A.M., July 6, in Buffalo ?

16. What time is it in Buffalo when it is 20 min. after 6 A.M., July 6, in Constantinople ?

17. Given the longitude of two places, state how to find the time in the place east at a given time in the place west.

ROOFING, PAPERING, ETC.

176. A square of roofing is 100 sq. ft.

177. Shingles are 16 in. long and are estimated to average 4 in. in width.

178. Wall paper is sold by the double roll 16 yd. long and 18 in. wide, or by the single roll 8 yd. long and 18 in. wide.

179. Bricks are usually 8 in. × 4 in. × 2 in. In estimating the amount of work or material, masons measure the length of walls on the outside, thus counting the corners twice.

Exercise 145

1. Find the areas of these rectangles:

24 ft. by 16 ft.	40 ft. 6 in. by 24 ft.
18 ft. by 12 ft.	32 ft. 8 in. by 18 ft.

2. Find the cost of the roofing for a barn, each side of the roof being 40 ft. long and 25 ft. wide, at $ 5.13 a square.

Q

3. Find the cost of the roofing for a building, each side being
(1) 46 ft. 8 in. long and 24 ft. wide, at $ 5 a square.

(2) 37 ft. 6 in. long and 26 ft. 8 in. wide, at $ 3 a square.

4. How many shingles that average 4 in. in width and are laid

(1) 4 in. to the weather will cover a square ?

(2) $4\frac{1}{2}$ in. to the weather will cover a square ?

(3) 6 in. to the weather will cover a square ?

5. How many thousand shingles, laid 4 in. to the weather, are
needed to cover both sides of a roof 60 ft. long and 25 ft. wide ?
What is the cost at $ 3.30 a thousand ?

6. A roof is covered with shingles laid $4\frac{1}{2}$ in. to the weather.
Find their cost at $ 2.90 a thousand, the roof being 90 ft. long
and 30 ft. wide.

7. How many yards of carpet, $\frac{3}{4}$ yd. wide, will be required
for a room 16 ft. long and 12 ft. wide, the carpet running length-
wise ?   Find its cost at 75 ¢ a yard.

8. Find the cost of the hemp carpet, $\frac{8}{9}$ yd. wide, required for
a room 18 ft. long and 16 ft. wide at 18 ¢ a yard, the carpet run-
ning lengthwise.

9. Find the cost of the carpet, $\frac{3}{4}$ yd. wide, required for a room
24 ft. long and 20 ft. wide, the carpet running lengthwise and
costing $ .96 a yard, there being a waste in matching of $4\frac{1}{2}$ in. on
each breadth except the first.

10. A double roll of wall paper is 16 yd. long and 18 in. wide.
How many square feet does it contain ?

11. A room is 16 ft. long, 12 ft. wide, and 9 ft. high.   How
many double rolls of wall paper are required to paper the four
walls ?   Find the cost at 23 ¢ a double roll.

12. A room is 18 ft. long, 15 ft. wide, and 10 ft. high and con-
tains two windows and one door.   How many double rolls of wall
paper are required for the four walls, an allowance of 20 sq. ft.
being made for each opening ?

13. How many bricks 8 in. × 4 in. × 2 in. will make 1 cu. ft. ?

Note. — Including mortar, allow 22 bricks for 1 cu. ft. of wall.

14. What is the volume of a solid wall 24 ft. × 3 ft. × 9 in. ?

15. How many bricks will be required to build a solid wall 36 ft. long, 4 ft. high, and 2 bricks thick ?

Exercise 146

1. Find the length measured by the number 4 and the unit 1 ft. 6 in.

2. Find values of the quantity measured by the number 6 according as the unit is £ 2 5s. or 6 oz. 10 pwt. 16 gr.

3. Find the value of a pile of cordwood 16′ long by 6′ high at $ 4 a cord.

4. Find the value of a pile of cordwood 13′ 4″ long by 3′ 9″ high at $ 4.50 a cord.

5. An English sovereign weighs 123.274 gr. How many sovereigns will weigh 61637 gr. ?

6. If a sovereign weighs 123.274 gr., how many sovereigns will weigh 21 lb. 4 oz. 16 pwt. 10 gr. ?

7. A race course is 170 ft. shorter than ¾ mi. How many feet long is it ?

8. A railway bridge over the Des Moines River is 40 ft. more than ½ mi. long. How many feet long is it ?

9. An ocean steamer sailed across the Atlantic, a distance of 2780 knots, in 6 da. 2 hr. Find the average number of knots per hour.

10. The steamer *Kaiser* made an ocean voyage of 3044 knots in 5 da. 18 hr. Find the average rate per hour.

11. How many feet in 1 mi. ?

A railway station in Peru is 16,635 ft. above the sea. This is how many yards more than 3 mi. ?

12. How many square yards in 1 A. ?　Square feet ?

The Boers of the Transvaal were allotted 40,000 sq. ft. at the Paris Exposition.　This is how many square feet less than 1 A. ?

13. Wire fencing costs 6 ¢ per yard.; find what must be paid for enclosing a field 305 yd. long and 156 yd. wide, there being 4 rows of wire.

14. Convert £6 16s. sterling into dollars and cents, £1 being worth $4.8665.

15. Find the selling price of one bale of cotton (1 bale = 500 lb.) at $4\frac{3}{4}d$. a pound.　Reduce the result to pounds, shillings, and pence.

16. A box 3 ft. long and 2 ft. wide contains 12 cu. ft.　Find its depth.

17. A ravine 400 ft. long and 80 ft. deep is filled by dumping into it 128,000 cu. yd. of earth.　Find its width.

18. A race horse has a record of 1 min. 15 sec. for 6 furlongs. At the same rate how long would he take to go 1 mi. ?

19. A bicyclist rode 39 mi. in 3 hr. 15 min.　Find the rate in miles per hour.

20. Find the number of days, hours, and minutes in $\frac{5}{73}$ yr. $+ \frac{1}{28}$ wk. $+ \frac{5}{12}$ hr.

21. Make out the following account neatly, accurately, and in proper form.　All fractions are to be retained.

John Wilson bought from you to-day :

$7\frac{1}{2}$ lb. cheese @ $12\frac{1}{2}$ ¢ per pound;

$6\frac{1}{4}$ lb. butter @ 23¢ per pound;

$2\frac{1}{2}$ lb. tea　　@ 55 ¢ per pound;

27 lb. sugar　@ $ 1 per 18 lb.

22. A yard measure is $\frac{1}{8}$ of an inch too long.　What is the actual distance between two points which is found by this measure to be 288 yd. ?

23. Express 1 lb. Troy as the fraction of 1 lb. Avoirdupois; express 1 lb. Avoirdupois as the fraction of 1 lb. Troy.

24. Which is the heavier, a pound of gold or a pound of feathers, and an ounce of gold or an ounce of feathers? By how much in each case?

25. How many silver spoons, each weighing 2 oz. 16 pwt., could be made out of a bar of silver, the weight of which is 50 oz. 8 pwt.?

26. A man bought a quantity of tea supposed to be done up in packages of 1 lb. each, for which he was to pay $64; on weighing them, however, it was found that each package was 1 oz. too light. How much should he pay for the tea?

27. A balloon passes over 114 mi. in 285 min. Show that this is at the rate of 2 mi. in 5 min.

28. What part of an hour is 1 min. 15 sec.? A horse ran ¾ mi. in 1 min. 15 sec. Find the rate in miles per hour.

29. The winner of the automobile race at the Paris Exposition made 351 mi. in 19 hr. 9 min. Find the rate in miles per hour.

30. In a 100-mi. bicycle race Charles Andrews was given a handicap of 2 hr. 45 min. over R. H. Smith. Charles Andrews finished the distance in 7 hr. 58 min. 30 sec. and R. H. Smith in 5 hr. 57 min. 40 sec. By how much did the former win?

31. Find the number of tons of steel rails required for a railway track 170 mi. long, weighing 90 lb. to the yard.

32. A lot 150 ft. long and 100 ft. wide is to be surrounded by a close board fence 6 ft. high. What will the boards cost at $12.50 per thousand feet?

33. If a room is 12 ft. square, what must its height be in order that the area of the walls may amount to 60 sq. yd.?

34. Find the value of a rectangular field 330 yd. by 160 yd. @ $36.50 per acre.

35. Find the volume of a rectangular block $3'\ 9'' \times 2'\ 4'' \times 1'\ 3''$.

36. Express as a fraction of an acre the sum of the following: $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{15}{16}$ of 1 A.; $\frac{2}{3}$ of $\frac{13}{21}$ of $\frac{63}{65}$ of 100 sq. rd.; and $\frac{16}{27}$ of $2\frac{1}{8}$ times 600 sq. yd.

37. The Manufacturers and Liberal Arts Building of the Columbian Fair was in the form of a rectangle and covered an area of 30 A. 76 sq. rd. 19 sq. yd. 7 sq. ft. The building was 787 ft. wide. How many feet in length was it?

38. A 200-acre farm is sown with grain as follows: Peas, 25 A. 126 sq. rd. 10 sq. yd.; oats, 46 A. 134 sq. rd. 15 sq. yd.; wheat, 75 A. 125 sq. rd. 25 sq. yd. The buildings, garden, and orchard occupy 12 A., and the rest is pasture. How many acres of pasture are there?

39. If a road is 4 rd. wide, how many miles of it will make 10 A.?

40. A map is drawn to a scale of half an inch to a mile. How many acres are represented by a square inch on the map?

41. After drawing off 124 gal. of water from a cistern, $\frac{3}{11}$ of the water still remained. How many gallons did the cistern at first contain? How many gallons were left in it?

42. Some Atlantic liners consume 198 T. of coal per day. They average 8 da. out and 8 back. For fear of accidents they carry a supply for 4 da. extra. How many cubic yards of the hold of such a steamer will be occupied with coal for her round trip if each ton is 33 cu. ft.?

43. A pile of wood 12 ft. long, 4 ft. wide, and 6 ft. high was sold for $13.50.

(a) What was the price per cord?

(b) At $4 per cord, what would the load be worth?

44. Find the value of a pile of tan bark 180 ft. long, 48 ft. wide, and 16 ft. high at $2.25 per cord.

45. Find the amount of the following bill, retaining all fractions:

$3\frac{3}{4}$ lb. tea	@ $80\cent$;
300 lb. sugar	@ $4\frac{3}{4}\cent$;
45 yd. print	@ $11\frac{1}{2}\cent$;
$2\frac{1}{4}$ gal. syrup	@ $65\cent$;
$12\frac{1}{2}$ yd. towelling	@ $12\frac{1}{2}\cent$;
$\frac{3}{4}$ doz. knives and forks	@ $2.50;
27 lb. cheese	@ $15\cent$;
1 lb. 10 oz. lemon peel	@ $32\cent$ per pound.

46. Make a drawing to show how many yards a train 80 yd. long must go to cross a bridge 140 yd. long. This is what part of 1 mi.?

47. A train 80 yd. long crossed a bridge 140 yd. long in $22\frac{1}{2}$ sec. Find the average speed of the train while crossing.

48. Find the weight of a bar 3 yd. 1 ft. 9 in. long, of which a yard weighs 15 lb.

49. Find the cost price of lead per hundredweight, if the sale of 48 cwt. for $218.70 gives a profit of $\frac{1}{8}$ of the original price.

50. Find the expense of fencing a railway (both sides) 73 mi. in length, at the rate of $5.50 per rod.

51. If a wheel makes 260 revolutions in passing over 1170 yd., what is its circumference?

52. A block of stone is 4 ft. long, 2 ft. 6 in. broad, and 1 ft. 3 in. thick; it weighs 27 cwt. Find the weight of 50 cu. in. of the stone.

53. A rectangular lot 45 ft. front by 99 ft. deep was sold for $3150. What was the price per foot frontage, and what the price per acre, at the rate of the selling price of the lot?

54. If I buy 147 gallons of molasses at $19\cent$ a gallon, and use 33 gallons of it, at how much must I sell the remainder per gallon so as to receive as much as the whole cost?

55. When 1 oz. of gold costs $19.45, what is the cost of .04 lb. ?

56. A grocer receives $9.60 for a bill of goods weighed on scales that gave only 15¼ oz. to the pound. How many cents' worth did he cheat his customer ?

57. If a cow gives 12 qt. 1 pt. of milk every day, and 1 lb. 8 oz. of butter can be made from 25 qt. of milk, how many pounds of butter can be made in one week from the milk of 16 cows ?

58. Find the expense of sodding a plot of ground, which is 40 yd. long and 100 ft. wide, with sods each a yard in length and a foot in breadth; the sods when laid costing 75¢ per hundred.

59. Make out the following bill neatly and accurately. John Smith, a merchant of Chicago, sold to William Jones, on June 15, 1895 :

> 5 lb. 8 oz. of butter   @ 16¢ per pound ;
> 2 lb. 10 oz. of tea   @ 3¢ an ounce ;
> 4 doz. lemons   @ 4¢ for 3 lemons ;
> 8 lb. coffee   @ 37½¢ per pound ;
> 1 bu. 3 pk. chestnuts   @ 10¢ per quart ;
> 11 doz. penholders   @ 1½¢ each.

60. Find cost of digging a cellar 48 ft. long, 30 ft. wide, and 6 ft. deep, at 20¢ per cubic yard, and flooring it with Portland cement at 10¢ per square yard.

61. How many bushels of potatoes can be sold out of a garden in which there are 160 rows of potatoes, in each row 240 hills, and on an average 10 potatoes in each hill, if 128 potatoes make 1 bu. ?

62. Farmer B. sold to a merchant the following articles to apply on an overdue account of $54.45 :

> 1680 lb. of hay   @ $15 per ton ;
> 3¾ cd. of wood   @ $4.80 per cord ;
> 4 bbl. of apples   @ $2.75 per barrel ;
> 350 lb. of flour   @ $2.50 per hundredweight ;
> 30 lb. 10 oz. butter   @ 16¢ per pound.

Make out the account neatly, showing the balance and to whom due.

63. The times when game may be shot in Illinois are : wild turkeys, from Sept. 1 to Jan. 15 inclusive; quail, from Nov. 1 to Dec. 20; turtle doves, from Sept. 1 to Dec. 1; water fowl, from Sept. 1 to April 15.

Find the number of days each year for which the above game are protected.

64. The price of crushed stone was advanced from $1.15 to $1.50 a cubic yard.   Find the increase in the cost of laying a road-bed of crushed stone 1 mi. long, 12 ft. wide, and 6 in. deep.

65. Name the quantities that are measured by the following numbers: 1760, 1728, 5280, 5760, 7000, 231, 7.48, 3.1416.

CHAPTER XIV

PERCENTAGE

180. The expressions $\$\frac{5}{100}$ and $\$.05$ denote that the quantity $\$1$ is conceived as made up of 100 equal parts or units, and that 5 of these parts or units have been taken to measure the quantity denoted by $\frac{5}{100}$ or .05.

The phrase *per cent* means *hundredths*. Thus the fraction $\frac{5}{100}$ and the decimal .05 are also written 5 per cent or 5%.

Hence $\frac{5}{100}$, .05, and 5% of any quantity are equal.

181. (1) Express $\frac{1}{3}$ as hundredths and also as per cent.

$$\frac{1}{3} = .33\frac{1}{3} \text{ or } \frac{33\frac{1}{3}}{100} = 33\frac{1}{3}\%.$$

(2) A horse dealer who had 600 horses sold 480 of them. What per cent of his horses did he sell?

We are here given the *measured* quantity or 480 horses to compare with the quantity 600 horses, and we are required to find the per cent which is the *number*.

The number sold $= \frac{480}{600}$ or $\frac{4}{5}$ or 80% of the whole number.

182. The term *per cent* is used constantly in business. The merchant gains 20%, meaning that he gains $\$20$ on every $\$100$ he has invested in goods. The insurance company charges 2% for insuring furniture, meaning that $\$2$ is charged on every $\$100$ worth of furniture insured. A man borrows money at 5%, meaning that he is to pay $\$5$ interest on every $\$100$ borrowed. The commission merchant charges 2% of the buying or selling price. The broker charges $\frac{1}{8}\%$ for buying stocks, and so on.

Exercise 147

Express as hundredths and also as per cent:

1. $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{4}{4}$; $\frac{1}{5}$; $\frac{2}{5}$; $\frac{4}{5}$; $\frac{5}{5}$; $\frac{1}{10}$; $\frac{3}{10}$; $\frac{7}{10}$; $\frac{1}{20}$; $\frac{3}{20}$; $\frac{9}{20}$; $\frac{17}{20}$; $\frac{1}{25}$; $\frac{11}{25}$; $\frac{23}{25}$.

2. $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{6}$; $\frac{5}{6}$; $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{1}{12}$; $\frac{1}{15}$; $\frac{1}{16}$; $\frac{15}{16}$; $\frac{17}{19}$; $\frac{14}{27}$; $\frac{41}{45}$.

3. Out of a class of 25 pupils 5 are absent. What part of the class is absent? How many hundredths? What per cent of the class?

4. A merchant paid $3 for hats which he sold for $4. What fraction of the cost price did he gain? How many hundredths? What per cent of the cost?

5. A person bought a house for $5000 and afterward sold it for $4000. The loss was what fraction of the cost? How many hundredths? What per cent?

6. A fruit dealer bought strawberries for $1.75 a crate and sold them for $2.25 a crate. What per cent did he gain?

7. How do you find the gain per cent when you are given the cost price and the selling price of an article?

8. A man bought a horse for $234 and afterward sold it for $273. What per cent of the cost did he gain?

9. The population of a town of 32,000 inhabitants increases 1120 in one year. What is the per cent of increase?

183. Express 25% and $37\frac{1}{2}\%$ as fractions in their lowest terms.

$$25\% = \tfrac{25}{100} = \tfrac{1}{4}.$$

$$37\frac{1}{2}\% = \frac{37\frac{1}{2}}{100} = \frac{75}{200} = \frac{3}{8}.$$

Exercise 148

What fractions in their lowest terms are equivalent to the following:

1. 1%; 4%; 5%; 10%; 20%; 25%; 30%; 40%; 80%; 90%; 100%?

2. 60%; 75%; 35%; 24%; 70%; 16%; 85%?

3. $12\frac{1}{2}\%$; $37\frac{1}{2}\%$; $62\frac{1}{2}\%$; $87\frac{1}{2}\%$?

4. $6\frac{1}{4}\%$; $8\frac{1}{3}\%$; $6\frac{2}{3}\%$; $16\frac{2}{3}\%$; $83\frac{1}{3}\%$?

5. $11\frac{1}{9}\%$; $14\frac{2}{7}\%$; $9\frac{1}{11}\%$?

6. $10\frac{5}{7}\%$; $5\frac{5}{9}\%$; $1\frac{12}{13}\%$?

7. 100%; 120%; 125%; 175%; 250%; 325%?

8. A horse which cost \$ 120 was sold at a gain of 25%. The gain is equal to what part of the cost? How much was gained? What was the selling price?

9. Cloth which cost $60\cent$ per yard was sold at a loss of $16\frac{2}{3}\%$. The loss was what fraction of the cost? What was the loss on each yard? What was the selling price per yard?

10. An article costing \$ 4.20 was sold at a gain of $8\frac{1}{3}\%$. Find the gain. Find the selling price.

11. How do you find the gain on an article when you are given the cost price and the gain per cent? How do you find the selling price?

12. Tea is bought for $84\cent$ per pound and sold at an advance of $14\frac{2}{7}\%$. What was the selling price of each pound?

13. A drover sold 400 sheep at a gain of 10%. He gained the cost price of how many sheep?

184. The following results should be memorized so that the fractions or the per cent can be given rapidly in any order:

$20\% = \frac{1}{5}$	$33\frac{1}{3}\% = \frac{1}{3}$	$37\frac{1}{2}\% = \frac{3}{8}$	$100\% = 1$
$40\% = \frac{2}{5}$	$66\frac{2}{3}\% = \frac{2}{3}$	$50\% = \frac{1}{2}$	$6\frac{1}{4}\% = \frac{1}{16}$
$60\% = \frac{3}{5}$	$100\% = 1$	$62\frac{1}{2}\% = \frac{5}{8}$	$6\frac{2}{3}\% = \frac{1}{15}$
$80\% = \frac{4}{5}$	$12\frac{1}{2}\% = \frac{1}{8}$	$75\% = \frac{3}{4}$	$8\frac{1}{3}\% = \frac{1}{12}$
$100\% = 1$	$25\% = \frac{1}{4}$	$87\frac{1}{2}\% = \frac{7}{8}$	$16\frac{2}{3}\% = \frac{1}{6}$

NOTE. — The expression $20\% = \frac{1}{5}$ signifies that 20% of a quantity $= \frac{1}{5}$ of it. Thus, 20% of 80 A. $= \frac{1}{5}$ of 80 A.

Exercise 149

Read the following decimals as per cents:

1. .25; .16⅔; .40; .75; .03⅓; 1.20; 1.25; 2.50; 3.16.

2. .15; .37½; .45⅓; .05⅓; .00⅓; 2.40; .00¼; .06¼; .06⅔.

185. I sold an article that cost $840 at a gain of 16⅔%. Find the gain.

The gain = 16⅔% or ⅙ of $840 = $140.

Exercise 150

1. I sold an article that cost $720 at a gain of 8⅓%. What was the gain? The selling price?

2. I sold an article that cost $465 at a loss of 20%. What was the loss? The selling price?

3. I sold an article that cost $885 at a gain of 6⅔%. What was the gain? The selling price?

4. I sold an article that cost $1275 at a loss of 32%. What was the loss? The selling price?

5. How do you find the gain when given the cost price and the gain per cent? How do you find the loss when given the cost price and the loss per cent?

6. I sold an article that cost $468 at a gain of 8⅓%. Find the selling price.

7. I sold an article that cost $345 at a loss of 33⅓%. Find the selling price.

8. When given the cost price and the gain or loss per cent, how do you find the selling price?

9. The gain on selling an article that cost $450 was 7%. Find the selling price.

10. Find the selling price of an article which cost $600 and was sold at a loss of 5%.

186. A speculator bought a house for $2349 and sold it at a gain of 17%. Find the selling price.

In this question the selling price is the sum of the cost, which is known, and the gain, which is unknown. The gain is measured by the number 17% or .17, and the cost price $2349.

$$
\begin{array}{r}
\$2349 \text{ cost} \\
.17 \text{ number} \\
\hline
16443 \\
2349 \\
\hline
\$399.33 \text{ gain}
\end{array}
$$

The gain = 17 % of $2349 = $399.33.

∴ the selling price = $2349 + $399.33 = $2748.33.

Exercise 151

1. Write as decimals: 17%; 13%; 37%; $23\frac{1}{2}$%; 146%; 346%; 6%; 8%; $8\frac{1}{2}$%; $1\frac{1}{2}$%; $\frac{1}{2}$%; $\frac{1}{4}$%.

2. Find 34% of $893; 19% of 643.

3. Find 27% of 6594 bu. of wheat; 31% of 1954.

4. If 39% of a cargo of flour, consisting of 8492 bbl., was damaged, how many barrels were damaged?

5. A farmer who sold his crop of wheat in 1899 for $967.20, received 13% less the next year. How much less did he receive for his crop in 1899 than in 1900?

6. A grain dealer invested $6459 in wheat, and 23% of that amount in oats. How much did he invest in oats?

7. What does a bill for $1896 become after a reduction of 3%?

8. What is the selling price of an article costing $18, and sold at a loss of 9%?

9. What is the selling price of an article costing $7, and sold at a gain of 7%?

187. Express $\frac{4}{5}\%$ as a decimal and also as a common fraction in its lowest terms.

$$\frac{4}{5}\% = .00\frac{4}{5}.$$

$$\frac{4}{5}\% = \frac{\frac{4}{5}}{100} = \frac{4}{500} = \frac{1}{125}.$$

$$\therefore \ \frac{4}{5}\% = .00\frac{4}{5} = \frac{1}{125}.$$

Exercise 152

Express as decimals and also as common fractions in their lowest terms:

1. $\frac{1}{2}\%$; $\frac{1}{5}\%$; $\frac{3}{5}\%$; $\frac{3}{4}\%$; $\frac{2}{5}\%$; $\frac{4}{4}\%$; $\frac{5}{6}\%$; $\frac{1}{7}\%$.

2. $\frac{3}{7}\%$; $\frac{6}{7}\%$; $\frac{1}{8}\%$; $\frac{5}{8}\%$; $\frac{3}{8}\%$; $\frac{1}{9}\%$; $\frac{8}{9}\%$; $\frac{4}{11}\%$.

3. $\frac{1}{10}\%$; $\frac{7}{10}\%$; $\frac{2}{11}\%$; $\frac{16}{25}\%$; $\frac{9}{17}\%$; $\frac{12}{13}\%$; $\frac{4}{15}\%$; $\frac{16}{15}\%$.

4. What part of 1% is $\frac{1}{2}\%$? $\frac{3}{5}\%$? $\frac{3}{8}\%$? $\frac{7}{10}\%$?

What is 1% of $\$800$? $\frac{1}{2}\%$? $\frac{3}{4}\%$? $\frac{7}{8}\%$?

5. If $\frac{1}{4}\%$ is charged for sending money from Chicago to New York, what is charged for sending $\$1200$?

6. If $\frac{1}{8}\%$ is charged for sending money to St. Louis, find how much is charged when $\$1632$ is sent.

7. If $\frac{1}{8}\%$ is charged as commission for buying stock, what is the commission on buying $\$2400$ stock ?

8. If $\frac{1}{8}\%$ is charged for selling stock, find the commission charged for selling $\$1600$ stock.

9. 6% per annum is what per cent for 1 month ? $\frac{2}{3}\%$ a month is what per cent per annum ?

10. What is the cost of insuring 550 bbl. of flour, worth $\$4$ per barrel, the cost of insurance being $\frac{1}{2}\%$ of the value of the flour ?

188. (1) Express $102\frac{1}{2}\%$ as a decimal.

$$102\frac{1}{2}\% = \frac{102\frac{1}{2}}{100} = 1.02\frac{1}{2} = 1.025.$$

(2) I send my agent $5100, which is 102 % of the money which he invested for me in cotton. What does my agent pay for cotton ?

102 % or 1.02 of the cost of the cotton = $ 5100.

∴ the cost of the cotton = $ 5100 ÷ 1.02 = $ 5000.

Exercise 153

1. Express as decimals: 103 % ; 104 % ; 101 % ; 102 %.

2. Express as decimals: 97 % ; 96 % ; 99 % ; 98 %.

3. Express as decimals: $103\frac{1}{2}$ % ; $104\frac{1}{4}$ % ; $102\frac{3}{4}$ % ; $101\frac{1}{2}$ %.

4. Express as decimals: $97\frac{1}{2}$ % ; $96\frac{3}{4}$ % ; $99\frac{1}{4}$ % ; $98\frac{3}{5}$ %.

5. A real estate broker in St. Louis received $ 5047, which was 103 % of the sum he was to invest in real estate. What sum did he invest in real estate ?

6. My agent sent me $ 3395, which was 97 % of the selling price of some railway stock. What did the stock sell for ?

7. My agent on selling a quantity of wheat sent me $98\frac{1}{2}$ % of the proceeds. If I received $ 3546, what did the wheat sell for ?

8. I sent my agent $4545.30, which was $104\frac{1}{4}$ % of the sum he was to invest in buying silk. What did he pay for the silk ?

189. (1) A house is sold for $16,400, and 25 % of the purchase money is paid down, the balance to remain on mortgage. How much remains on mortgage ?

In this problem we are given the measured whole, *i.e.* the selling price, and the number or 25 % of it. We are required to find the balance which is the difference between the selling price and the sum paid.

The sum paid = 25 % or $\frac{1}{4}$ of $ 16,400 = $4100.

∴ the balance = $ 16,400 − $ 4100 = $ 12,300.

(2) On Jan. 10 a merchant buys goods invoiced at $876.40, on the following terms : 4 months, or less 6% if paid within 10 days.   What sum will pay the debt on Jan. 15 ?

Since Jan. 15 is less than 10 days after Jan. 10, the sum due will be 6% less than $876.40.

The discount = 6% or .06 of $876.40 = $52.584.

∴ the sum due = $876.40 − $52.58 = $823.82.

Exercise 154

1. A maltster malts 7200 bu. of barley, which in the process increases $12\frac{1}{2}\%$.   How many bushels of malt has he ?

2. Certain books are bought at $1.75 each.   At what must they be sold to gain 12% ?

3. A merchant asked 30 % advance on goods which cost $120, but finally took 25 % less than the price asked.   What did he sell them for ?

4. A merchant bought apples at 60¢ per bushel, and sold them at a gain of 25 %.   Find the selling price per bushel.   How many bushels did he sell if he received all together $37.50 ?

5. Bought $64 worth of apples at 80¢ per bushel, part of which being damaged and rendered worthless, I sold the remainder at an advance of 50%, receiving $76.80.   How many bushels were damaged ?

6. If 10% of an army of 23,400 men were slain in battle, and 5% of the remainder were mortally wounded, find the sum of the killed and mortally wounded.

7. The population of a town of 64,000 inhabitants increases at the rate of $2\frac{1}{2}\%$ in each year.   Find its population 1, 2, and 3 years hence.

8. The population of a city is a million; it increases $1\frac{1}{2}\%$ for 3 years successively.   Find the population at the end of 3 years.

9. A lawyer collected $287.50, and charged 5% for his services.   How much did he retain and how much did he pay over ?

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10. The cost price of a book is $1.60, the expense of sale 5% upon the cost price, and the profit 25% upon the whole outlay. Find the selling price of the book.

11. The cattle on a certain stock farm increase at the rate of $18\frac{3}{4}$% per annum. If there are 4096 cattle in 1899, how many will there be in 1901?

12. A man bought a store and contents for $4720; he sold the same for $12\frac{1}{2}$% less than he gave, and then lost 15% of the selling price in bad debts. Find his entire loss.

13. A person having bought goods for $40 sells half of them at a gain of 5%. For how much must he sell the remainder so as to gain 20% on the whole?

14. A grocer mixes two kinds of tea which cost him 38¢ and 42¢ per pound respectively in equal quantities. What must be the selling price of the mixture in order that he may gain 15% on his outlay?

15. A grain dealer expended $2150 in the purchase of wheat, one-half as much again in the purchase of barley, and twice as much in the purchase of corn; he sold the wheat at a profit of 6%, the barley at a loss of 5%, and the corn at a gain of 2%. Find his gain on the whole transaction.

16. A person gave $150 for one horse and $225 for another. He sold the first horse at a gain of 20%, and the second at a loss of 20%. Find the selling price of each horse and the gain or loss on the whole transaction.

17. A sells goods to B which cost him $465, at a gain of 6%, B sells them to C at a loss of 3%, and C sells them to D, gaining 10%. What did D give for the goods?

18. A man having bought a lot of goods for $450 sells $\frac{1}{3}$ at a loss of 5%, $\frac{1}{5}$ at a gain of 7%, and the remainder at a gain of 2%. Find the total gain.

19. A merchant began business with a capital of $30,000. He gained $16\frac{2}{3}$% the first year, which he added to his capital, and

$12\frac{1}{2}\%$ the second year, which he added to his capital. In the third year he lost 20%. Find his capital at the end of the third year.

20. Sugar being composed of 49.856% of oxygen, 43.265% of carbon, and the remainder hydrogen, find how many pounds of each of these materials there are in 1 T. of sugar.

21. A merchant buys a bill of dry goods, April 16, amounting to $6377.84, on the following terms: 4 mo., or less 5% if paid within 30 da. How much would settle the account on May 15?

22. Water is composed of 88.9% of oxygen and 11.1% of hydrogen. How many pounds are there of each in 1 cu. ft. of water? (A cubic foot of water weighs 1000 oz.)

190. I sold an article which cost $64 at a gain of $24. Find my gain per cent.

The gain per cent $= \frac{24}{64}$ or $\frac{3}{8}$ or $37\frac{1}{2}\%$ of the cost.

Exercise 155

1. I sold an article which cost $75 at a gain of $25. Find the gain per cent.

2. A farmer had 200 A. and sold 50 A. Find what per cent of his farm he sold.

3. I paid $225 for a horse, and sold it at a profit of $45. Find my gain per cent.

4. I lost $36 on selling an article which cost $108. Find the loss per cent.

5. What two things do you compare to find the gain per cent? The loss per cent?

6. A suburbanite earns $125 a month, and pays $5 a month for a monthly railway ticket. What per cent of his salary does his ticket cost?

191. (1) A merchant sold 60 yd. of cloth from a web containing 150 yd. What per cent of the web did he sell?

We are here given the measured part and the measured whole, **and we are** required to find the number expressing the ratio of the part to the whole.

The quantity sold = $\frac{60}{150}$ or $\frac{2}{5}$ or 40 % of the web.

(2) An article which cost $3.60 was sold for $4.32. Find the gain per cent.

The gain = $4.32 − $3.60 = 72 cents.

∴ the gain = $\frac{72}{360}$ or $\frac{1}{5}$ or 20 % of the cost.

Exercise 156

1. The cost price of an article is $64, and the gain on selling $16. Find the gain per cent.

2. Tea is bought at 84 ¢ per pound and sold at 98 ¢. Find the gain per cent.

3. Out of 48 eggs 6 were broken. What per cent of the whole number was broken?

4. The cost price of an article was $56, and the selling price $49. Find the loss per cent.

5. What per cent is 1 in. of 1 ft.? 1 ft. of 1 yd.? 1 yd. of 1 rd.? 1 rd. of 1 mi.?

6. What per cent is 1 min. of 1 hr.? 1 hr. of 1 da.? 1 da. of 1 wk.? 1 wk. of 1 yr.?

7. What per cent is 1 pt. of 1 qt.? 1 qt. of 1 gal.?

8. What per cent of 1 pk. is 1 qt.? Of 1 bu. is 1 pk.?

9. A merchant by selling 1 lb. of butter gains the cost price of 1 oz. What is his gain per cent?

10. 1 lb. Troy is what per cent of 1 lb. Avoirdupois? 1 lb. Avoirdupois is what per cent of 1 lb. Troy?

11. The volume of 1 gal. is what per cent of 1 cu. ft.? 1 cu. ft. is what per cent of 1 gal.?

12. An area containing 1 sq. yd. is increased by 4 sq. ft. Find the per cent of increase.

13. If £1 is worth $4.866, what per cent of £1 is $1?

Exercise 157

1. A paymaster receives $150,000 from the treasury, but fails to account for $2250. What is the percentage of loss to the government?

2. A city of 16,000 inhabitants increases in a given time to 20,000. Find the increase per cent.

3. $640 increased by a certain per cent of itself equals $720. Required the rate per cent.

4. A house worth $3500 rents for $420. For what per cent of its value does it rent?

5. If a tradesman gains $1.32 on an article which he sells for $5.28, what is his gain per cent?

6. An article which cost 84 ¢ is sold for 93 ¢. Find the gain per cent.

7. A city gained 2467 in population in 5 years. If its population was 14,802 five years ago, what was the gain per cent?

8. In a certain year the number of graduates of a school was 70. Ten years later it was 210. Find the rate per cent of increase.

9. A tea merchant mixes 40 lb. of tea at 45 ¢ per pound with 50 lb. at 27 ¢ per pound, and sells the mixture at 42 ¢ per pound. What per cent profit does he make?

10. Paid $80 freight on goods that cost $1120. What must they be sold for to make a profit of 20% on the full cost?

11. A grocer uses for a 1-pound weight one which weighs only 15¾ oz. What does he gain per cent by his dishonesty?

12. I bought 500 sheep at $4 a head; their food cost me $1.50 a head; I then sold them at $6 a head. Find my gain per cent.

13. A man's income is derived from the proceeds of $4550 at a certain rate per cent, and $5420 at 1% more than the former rate. His whole income being $453, find the rates.

14. Coffee is bought in 50-pound bags for $16 and sold for 36 ¢ a pound. Find the rate of profit per cent.

192. The gain $ 36 is $22\frac{2}{9}\%$ of the cost ; find the cost.

The gain $= 22\frac{2}{9}\%$ or $\frac{2}{9}$ of the cost.

$\frac{2}{9}$ of the cost $= \$ 36$

$\therefore$ the cost $= \$ 36 \times \frac{9}{2} = \$ 162.$

Exercise 158

1. The gain $ 25 is $12\frac{1}{2}\%$ of the cost.   Find the cost.

2. The gain $ 36 is $37\frac{1}{2}\%$ of the cost.   Find the cost.

3. $ 18 is 6% of my month's salary.   What is my salary ?

4. 8% of my rent is equal to $ 32.   What rent do I pay ?

5. The gain $ 60 is 12% of the cost.   Find the cost.

6. The selling price $ 3810 is 127% of the cost.   Find the cost.

7. I sold an article for $ 92, gaining 15% of the cost.   Find the cost.

8. The selling price $ 5640 is 94% of the cost.   Find the cost.

9. I sold an article for $ 720, losing 4% of the cost.   Find the cost.

193. A trader sold a horse at an advance of 12%, gaining $ 18.   Find the cost price of the horse.

12% or $\frac{3}{25}$ of the cost $= \$ 18.$

$\frac{1}{25}$ of the cost $= \$6.$

$\frac{25}{25}$ of the cost $= \$ 150.$

$\therefore$ the cost $= \$ 150.$

Exercise 159

1. A quantity of sugar was sold at an advance of $12\frac{1}{2}\%$.   If the gain was $ 17, what was the cost ?

2. Cloth was sold at a loss of $37\frac{1}{2}\%$.   If the loss was 36 ¢ a yard, what was the cost per yard ?

3. Forty-five per cent of a piece of cloth was sold.   If 135 yd. were sold, how many yards were in the piece at first ?   How many remained unsold ?

4. 3% more is gained by selling a horse for $333 than by selling him for $324; find his original price.

5. A man bought a horse which he sold at a loss of 8%. If he had received $24 more, he would have gained 7%. What did the horse cost him?

6. A clerk pays 16% of his salary each year for board. If his board costs him $208 a year, what is his salary?

7. A man sold a field consisting of 15 A., which was $6\frac{1}{4}$% of his farm. How many acres were in the farm at first?

8. 25% of my money is invested in bank stock and the remainder in business; what per cent of my money is invested in business? My bank stock is worth what part of my business capital? If I have $4800 invested in business, what is the value of my bank stock?

9. Twenty per cent of my money is invested in business, and the remainder, which is $12,800, in real estate. How much have I invested in business?

10. I invested 25% of my money in business, and put $6\frac{2}{3}$% of the remainder in the bank. If I put $600 in the bank, how much money did I have at first?

11. Twenty-eight per cent of a sum of money was invested in business, and $12\frac{1}{2}$% of the remainder in real estate. If the sum invested in business exceeds that invested in real estate by $1900, find the amount of money I had at first.

194. (1) A man invests $77\frac{1}{2}$% of his capital in bank stock, and has $29,367 left. What is his capital?

The amount left = 100 % − $77\frac{1}{2}$ % or $22\frac{1}{2}$ % of his capital.

$22\frac{1}{2}$ % or $\frac{9}{40}$ of his capital = $29,367.

$\frac{1}{40}$ of his capital = $3263.

$\frac{40}{40}$ of his capital = $130,520.

∴ his capital = $130,520.

(2) A profit of 17 % is made by selling an article at an advance of $ 24.50. What would have been the selling price if the loss had been 8 % ?

$$17 \% \text{ of the cost} = \$ 24.50.$$
$$1 \% \text{ of the cost} = \$ 1.4412.$$
$$92 \% \text{ of the cost} = \$ 132.59.$$
$$\therefore \text{ the second selling price} = \$ 132.59.$$

Exercise 160

1. A man invests 42% of his capital in real estate, and has $ 53,070 left. What is his capital ?

2. A bankrupt's assets are $ 23,625, and he pays $62\frac{1}{2}\%$ of his debts. How much does he owe ?

3. A merchant loses $6\frac{1}{2}\%$ of the cost price by selling an article at a loss of $ 27.30. Find the cost price, and also at what he must sell it to gain $7\frac{1}{2}\%$.

4. By selling a house at a loss of $ 150, a real estate dealer loses $6\frac{2}{3}\%$ of the cost. Find the cost and also the gain per cent if it had been sold for $ 2625.

5. I sold a lot at a gain of $8\frac{1}{3}\%$, thereby gaining $ 113. What should I have sold it for to gain 9% ?

6. Coals are 20% cheaper this year than last. If the price were to rise $ 1 per ton, they would still be 50 ¢ per ton cheaper than last year. Find the price last year.

7. A person asked for a lot of land 40% more than it cost him, but finally reduced his price 15%, gaining on the whole $ 380. How much did the land cost him ?

8. A merchant sold $\frac{3}{4}$ of a quantity of cloth at a gain of 20%, and the remainder at cost. His gain was what per cent of the cost ? If he gained $ 7.29, what was the cost of the goods ?

9. A merchant sold $\frac{2}{3}$ of a quantity of tea at a gain of 12%, and the remainder at a gain of 9%, gaining all together $ 2.75. Find the cost of the tea.

10. A speculator gained 20% on $\frac{3}{4}$ of his investment, and lost 24% on the remainder. All together he made $270. Find the amount of his investment.

11. Ten per cent of an army were slain on the field of battle, and 5 per cent of the remainder were mortally wounded. The difference between the killed and mortally wounded was 1100. How many men went into battle?

195. (1) A horse was sold for $117, which was $8\frac{1}{3}\%$ more than it cost. Find the cost price.

$$\text{The gain} = 8\frac{1}{3}\% \text{ or } \tfrac{1}{12} \text{ of the cost.}$$
$$\text{The selling price} = 1\tfrac{1}{12} \text{ of the cost.}$$
$$1\tfrac{1}{12} \text{ of the cost} = \$117.$$
$$\therefore \text{ the cost} = \$117 \times \tfrac{12}{13} = \$108.$$

(2) A horse was sold for $154, which was $8\frac{1}{3}\%$ less than it cost. Find the cost price.

$$\text{The loss} = 8\frac{1}{3}\%, \text{ or } \tfrac{1}{12} \text{ of the cost.}$$
$$\text{The selling price} = 1\tfrac{1}{12} \text{ of the cost.}$$
$$1\tfrac{1}{12} \text{ of the cost} = \$154.$$
$$\therefore \text{ the cost} = \$154 \times \tfrac{12}{11} = \$168.$$

Exercise 161

1. A house and lot were sold for $3600, which was 20% more than they cost. Find the cost price.

2. A house and lot were sold for $4200, which was 25% less than they cost. Find the cost price.

3. A speculator gained 7% by selling wheat for $2140. Find the cost price.

4. Eggs are sold at the rate of $15\cancel{c}$ per dozen, a profit of 25% being made. What is the cost price per dozen?

5. I sold a book for $42\cancel{c}$, gaining $16\frac{2}{3}\%$. Find the cost price. How much would be gained by selling at a gain of 25%? What would then be the selling price?

6. By selling hats at 60¢ each, a merchant gains $33\frac{1}{3}\%$. Find the cost price. What would have been the actual loss, and what the loss per cent, if they had been sold at 36¢ each?

7. I sold a lot of land for $600, thereby gaining 20%. Find the cost price.

8. I sold a lot of land for $600, thereby losing 20%. Find the cost price.

9. What is the cost of both lots in questions 7 and 8? What is their selling price? How much is the cost of both greater than their selling price?

10. A dealer sold two bicycles for $45 each, losing 25% on one and gaining 25% on the other. How much did he lose on the whole transaction?

196. (1) If a debt, after a reduction of 3%, becomes $1008.80, what would it become after a reduction of 4%?

After a reduction of 3%, the amount owed = 97% of the original debt, and after a reduction of 4% it becomes 96% of the original debt.

$$97\% \text{ of the debt} = \$1008.80.$$

$$1\% \text{ of the debt} = \$10.40.$$

$$96\% \text{ of the debt} = \$998.40.$$

$$\therefore \text{ after a reduction of } 4\% \text{ the debt} = \$998.40.$$

(2) The population of a city increases 2% yearly. It now has 132,651 inhabitants. How many had it 1, 2, and 3 years ago?

The population now = 102%, or 1.02 of that 1 year ago.

$$1.02 \text{ of the population 1 year ago} = 132,651.$$

$$\therefore \text{ the population 1 year ago} = 132,651 \div 1.02 = 130,050.$$

$$\therefore \text{ the population 2 years ago} = 130,050 \div 1.02 = 127,500.$$

$$\therefore \text{ the population 3 years ago} = 127,500 \div 1.02 = 125,000.$$

Prove these answers correct.

Exercise 162

1. A horse was sold for $658, which was $16\frac{2}{3}\%$ more than its cost. How much did it cost?

2. A speculator gained 3% by selling wheat for $6437.50. Find the cost price.

3. A merchant, after a business of five years, found his capital increased to $28,000, showing a gain of 60% on his original capital. Find that capital.

4. Eggs are sold at the rate of 5 for 6¢, a profit of 20% being made. Find the price at which they are bought.

5. In 1901 a city has a population of 28,000 inhabitants. If its population increased $17\frac{11}{17}\%$ in the two years previous, what was it in 1899? If its population decreased $17\frac{11}{17}\%$ in the two years previous, what was it in 1899?

6. By selling an article for $2.64 a merchant loses 12%. What was the cost, and for what must he sell it to gain $16\frac{2}{3}\%$?

7. A merchant sells tea at 75¢ per pound, thereby losing 5%. What was the cost, and at what price per pound must it be sold to gain $4\frac{1}{2}\%$?

8. Flour is sold for $4.80 per barrel, at a loss of 20%. What selling price would give 20% gain?

9. By selling an article for $23, 8% is lost. What per cent is gained if it is sold for $31?

10. I sold goods at $21.60 per hundredweight, thereby gaining $14\frac{2}{7}\%$. Find the cost per pound.

11. A farmer sold his crop of wheat in 1871 for 8% more than he obtained for his crop of the preceding year; he received for both crops $2600; how much did he get for his crop of 1870?

12. I sold two houses, receiving $2400 for each. On the first I gained 25%, and on the second lost 25%. Find my loss on both transactions.

13. I sold a lot of land for $600, thereby gaining 20%; a second for $600, losing 20%. Find my loss on both transactions.

PROFIT AND LOSS

197. The **Profit** is the amount by which the selling price exceeds the buying price.

The *rate of profit* is usually expressed as a certain per cent of the cost price.

The **Loss** is the amount by which the selling price falls short of the cost price.

The *rate of loss* is usually expressed as a certain per cent of the cost price.

198. (1) At a forced sale a bankrupt's house was sold for $8000, which was 20% less than its real value. If the house had been sold for $12,000, what per cent of its real value would it have brought?

$$80\% \text{ of the value of the house} = \$\ 8,000.$$
$$1\% \text{ of the value of the house} = \qquad 100.$$
$$100\% \text{ of the value of the house} = \ 10,000.$$

$$\therefore \text{ the second selling price} = \tfrac{12000}{10000} \text{ of the value}$$
$$= 120\% \text{ of the value.}$$

(2) The manufacturer of an article makes a profit of 25%, the wholesale dealer makes a profit of 20%, and the retail dealer makes a profit of 30%. What is the cost to the manufacturer of an article that retails at $15.60?

$$\begin{aligned}
&\text{Let the cost to the manufacturer} &&= 100 \text{ units of money.}\\
&\text{The selling price of the manufacturer} &&= 125 \text{ units of money.}\\
&\text{The gain of the wholesale dealer} &&= 25 \text{ units of money.}\\
&\text{The selling price of the wholesale dealer} &&= 150 \text{ units of money.}\\
&\text{The gain of the retail dealer} &&= 45 \text{ units of money.}\\
&\text{The selling price of the retail dealer} &&= 195 \text{ units of money.}
\end{aligned}$$

$$195 \text{ units of money} = \$15.60.$$
$$1 \text{ unit of money} = \qquad .08.$$
$$100 \text{ units of money} = \qquad 8.00.$$
$$\therefore \text{ the prime cost} = \qquad 8.00.$$

GENERAL STATEMENT OF SOLUTION

(3) Represent the cost to the manufacturer by 100 units of money, and then find the number of units representing respectively the selling prices of the manufacturer, the wholesale dealer, and the retail dealer. Put the number of units of money which represent the retail price equal to $15.60 and find the value of 100 units of money, which is the cost of manufacturing.

QUESTION IN PROOF

(4) The manufacturer of an article makes a profit of 25%, the wholesale dealer a profit of 20%, and the retail dealer a profit of 30%. What is the retail price of an article which cost the manufacturer $8?

PROOF

The manufacturer's gain $\quad = 25\%$ of $8 = $2.

The manufacturer's selling price $\quad = $10.

The wholesale dealer's gain $\quad = 20\%$ of $10 = $2.

The wholesale dealer's selling price $= $12.

The retail dealer's gain $\quad = 30\%$ of $12 = $3.00.

The retail dealer's selling price $\quad = $15.60.

$\therefore$ $8 is the correct answer to the previous question.

MAKING QUESTIONS

(5) Make a question in which you are given the selling price and the gain per cent, and are required to find the cost price.

MAKING

Let the cost of a house $= $6250.

Let the gain per cent on selling $= 37\frac{1}{2}\%$.

The gain $= 37\frac{1}{2}\%$ of $6250 = $2343.75.

The selling price $= $6250 + $2343.75 = $8593.75.

Problems

A house was sold at $37\frac{1}{2}\%$ above cost. If the selling price was $8593.75, find the cost price.

Other questions may also be written down from the same making, thus:

A house which cost $6250 was sold at a gain of $37\frac{1}{2}\%$. Find the selling price.

A house which cost $6250 was sold for $8593.75. Find the gain per cent.

A house which cost $6250 was sold at a gain of $2343.75. Find the gain per cent.

In the following exercise state in general terms how to solve each question. Prove some of your answers correct, framing the question in proof. Make questions similar to problems in the exercise.

Exercise 163

1. A lot of dry goods was sold at an advance of 18%. If the gain was $436.50, what was the cost?

2. I made a mixture of wine consisting of 1 gal. at $50\cancel{c}$, 3 at $90\cancel{c}$, 4 at $1.20, and 12 at $40\cancel{c}$. I sell the mixture at $80\cancel{c}$ a gallon. Find my gain per cent.

3. A merchant's price is 25% above cost. If he allows a customer a discount of 12% on his bill, what per cent profit does he make?

4. Cloth when sold at a loss of $16\frac{2}{3}\%$ brings $2.50 a yard. What would be the gain or loss per cent if sold at $4 a yard?

5. Eggs are bought at $25\cancel{c}$ a dozen, and sold at the rate of 8 for $20\cancel{c}$. Find the rate of profit.

6. A merchant sells goods to a customer at a profit of 60%, but the buyer becomes bankrupt and pays only 70 cents on the dollar. What per cent does the merchant gain or lose on the sale?

7. A man bought a horse which he sold again at a loss of 10%. If he had received $45 more for him he would have gained $12\frac{1}{2}$%. Find the cost of the horse.

8. A bookseller sold a book at 17% below cost, but had he charged 48 cents more for it, he would have gained 7%. Find the cost of the book to the bookseller, and the price at which he sold it.

9. A tradesman bought goods for $1200 and sold $\frac{1}{3}$ of them at a loss of 10%. For how much must he sell the remainder to gain 20% on the whole?

10. A man bought a house and lot for $4750. After spending $1122 on repairs and improvements, and paying $128 for taxes and other expenses, he sold the property for $6400. What rate per cent of profit did his investment yield him?

11. By selling cloth at $1.20 per yard, a tradesman lost $6\frac{1}{4}$% on his outlay. At what price must he sell it to gain $12\frac{1}{2}$%?

12. If a manufacturer sells an article of which the first cost is $400, to a wholesale dealer at 10% profit, the wholesale dealer to the retailer at 15% profit, and the retailer to the consumer at 30% profit, what sum is paid by the consumer as profits in addition to the first cost of the article?

13. A grocer sold, at 51¢ per pound, a portion of a stock of tea, incurring a loss of 15% and a total loss of $18 on the quantity sold. How many pounds did he sell?

14. A merchant marks his goods so that he may allow a discount of 5%, and still make a profit of 15%. Find the marked price of broadcloth that cost him $3.80 a yard.

15. A person sold two horses at $160 each, losing 20% on one and gaining 20% on the other. Did he gain or lose on the whole transaction, and how much?

16. A speculator paid $1400 for two lots, the price of one of them being 40% that of the other. He sold the cheaper lot at a gain of 50%, and the dearer one at a loss of 30%. Find his gain or loss per cent on the whole transaction.

COMMERCIAL OR TRADE DISCOUNT

199. Commercial discount is an allowance made by merchants upon their catalogue prices.

The commercial discount is reckoned at a certain rate per cent.

Sometimes several discounts are allowed to a purchaser.

In such a case, the first discount is to be deducted, and then the second discount is to be reckoned upon the remainder and then deducted, and so on for each successive discount.

200. What is the net amount of a bill for $720 subject to discounts of 20% and 6%? Find a single discount equivalent to these successive discounts.

$$\text{The first discount} = 20\% \text{ of } \$720 = \$144.$$

$$\text{The first remainder} = \$720 - \$144 = \$576.$$

$$\text{The second discount} = 6\% \text{ of } \$576 = \$34.56.$$

$$\therefore \text{ the net amount} = \$576 - \$34.56 = \$541.44.$$

Again, the single equivalent discount = $720 − $541.44 = $178.56.

$\therefore$ the rate of a single discount = $178.56 ÷ $720 = .248 = 24.8%.

Why is the single discount 24.8% less than the sum of the two discounts?

Exercise 164

1. An invoice was $650, trade discounts 20% and 8% off. Find the cost of the goods.

2. What is the net amount of a bill of goods, the list price of which is $245, trade discounts 18% and 5% off for cash?

3. What is the difference on an invoice of $540, between 40% direct discount, and discounts of 25% and 15%?

4. A dealer buys a book, list price 80¢, at a discount of 25%; he sells the book for 80¢. What per cent is the profit?

5. What is the net amount of a bill of $480, discounts being $12\frac{1}{2}\%$ and 8%? Find a single discount equivalent to these successive discounts.

6. A man bought goods at discounts of 20% and 5%. The list price was $400. Find what he paid for the goods.

7. A dealer bought goods at 15% and 12% off. The list price of the goods was $250. Find what he paid for the goods.

8. Find the net cash amount of a bill for $1266, subject to discounts of $33\frac{1}{3}\%$, 10%, and 5%, for cash.

9. Find the difference between a single discount of 40%, and successive discounts of 30% and 10%.

10. Find the net amount of a bill of $250, discounts being 30, 16, and 6.

11. Find the net cash amount of a bill of $256, discounts being 25%, $12\frac{1}{2}\%$, 5%. Find a single discount equivalent to these three successive discounts.

12. In the following examples, in which the list prices and the rates of discount are given, find the cost prices:

List Price						Rates of Discount		
a.	$125.00	.	.	.	.	15%,	10%	
b.	112.50	.	.	.	.	30%,	25%	
c.	3147.00	.	.	.	.	45%,	20%,	5%
d.	796.00	.	.	.	.	35%,	$12\frac{1}{2}\%$,	8%
e.	2378.00	.	.	.	.	$33\frac{1}{3}\%$,	15%,	6%
f.	432.75	.	.	.	.	50%,	10%,	5%

13. A merchant who receives successive discounts of 20%, 15%, and 10%, on a bill of $750, sells at an advance of $33\frac{1}{3}\%$. What does he sell his goods for?

14. What is the difference between discounting a bill of $3000 at 40%, and then taking a discount off the remainder of 5% for cash, and discounting the whole at 45%?

s

15. A merchant buys goods at 40 and 20 off the list price and sells them at 30 and 10 off the list price. What is his gain per cent?

16. An invoice of crockery, amounting to $1500, was sold Jan. 3, at 90 days, subject to 40% and 10% discount, with an additional discount of 6% if paid within 20 days. How much will be required to pay the bill on Jan. 21?

COMMISSION AND BROKERAGE

201. A **Commission Merchant** is one who buys or sells goods for other persons by their authority. Commission merchants are usually placed in possession of the goods bought.

A **Broker** is a person who, in the name of his principal, effects contracts to buy or sell.

The broker is not in general placed in possession of the goods bought or sold.

The title Broker is also applied to persons who deal in stocks, bonds, bills of exchange, promissory notes, etc., and to mercantile agents, who transact the business for a ship in port.

Commission is the charge made by an agent for transacting business.

In buying, the commission is reckoned on the *cost* price; in selling, the commission is reckoned on the *selling* price.

202. (1) A commission merchant sold 270 bbl. of flour at $6 a barrel, and received 5% commission. What was his commission? How much did he remit to his employer?

The selling price = 270 × $6 = $1620.

∴ the commission = 5% of $1620 = $81.

∴ the amount remitted = $1620 − $81 = $1539.

(2) A commission of $242.58 was charged for selling $3772 worth of goods. What was the rate of commission?

$$\text{The commission} = \frac{242.58}{3772} \text{ of the selling price}$$

$$= .0643 \text{ of the selling price.}$$

∴ the rate of commission = 6.43 %.

(3) A grain dealer charged $3\frac{1}{2}$% for selling a quantity of wheat, and received for his commission $218.40. For how much did he sell the wheat?

The commission = $3\frac{1}{2}$% or .035 of the selling price.

.035 of the selling price = $218.40.

∴ the selling price = 218.40 ÷ .035 = $6240.

(4) If $512.50 include the price paid for certain goods, and $2\frac{1}{2}$% commission to the agent, how much money does the agent expend in purchasing the goods?

Let the cost price of the goods = 100 units of money.

Then the commission = $2\frac{1}{2}$ units of money.

The amount sent to the commission merchant = $102\frac{1}{2}$ units of money.

$102\frac{1}{2}$ units = $512.50.

1 unit = 512.50 ÷ 102.5 = $5.

100 units = $500.

∴ the cost of the goods = $500.

As in Exercise 163 give the general statements of solutions, prove answers, and make questions. Do this also in each of the following exercises:

Exercise 165

1. A commission merchant sold 480 bbl. of flour at $3.50 a barrel on a commission of 2%. What was his commission? How much did he remit to his employer?

2. My agent sold coffee to the amount of $850 on a commission of 3%. Find his commission and also the amount remitted to his employer.

3. An agent sold 210 bu. of oats at 60¢ a bushel, and charged $3.78 for doing so. Find his rate of commission.

4. On a debt of $1725 a creditor receives a dividend of 60%, on which he allows his attorney 5%. He receives a further dividend of 20%, on which he allows his attorney 5%. What is the net amount that he receives?

5. If a commission of $212.94 is paid for buying $6552 worth of goods, find the rate per cent of commission.

6. An agent received $40.62½ for selling a house for $1625. Find his rate per cent of commission.

7. An agent, who is paid a commission on what he invests, received $4896, and invests $4800. Find his rate per cent of commission.

8. An agent received $56 for selling grain on a commission of 4%. Find value of grain sold.

9. A commission merchant charged 2½% for selling a quantity of pork, and received for his commission $64.82. Find the selling price of the pork.

10. The owner of a house offered an agent $500 commission, if the agent could sell the house for $10,500. What rate per cent commission was the owner offering? Had the owner offered 5% commission, what would have been the commission on $10,500?

11. I bought a bicycle for $70, which was ⅙ of my commission at 3½% for selling a quantity of land. For how much was the land sold?

12. A real estate dealer sold land for 100 units of money, on a commission of 4%. How many units of money did he keep for his commission? How many units of money did he send his employer? If his employer received $2880, what did the land sell for? What was the agent's commission?

13. An agent remits $4850 to his employer after taking out his commission of 3%. Find the selling price.

14. My agent sent me as my share of the selling price of flour $2038.40. If the flour sold for $3.25 a barrel, and the agent's commission was 2%, how many barrels did he sell?

15. My agent bought a quantity of goods for me on a commission of 2%. If the cost of the goods was 100 units of money, how many units of money did his commission equal? How many units did I have to send him to cover the cost of the goods and his commission?

16. A merchant in Buffalo sends a commission merchant in New York $3120, instructing him to purchase goods, reserving his commission at 4%. Find his commission.

17. A merchant sent $3238.30 to New Orleans to be expended in cotton. The broker in New Orleans charged 6% commission. What sum was paid for the cotton?

18. Sent to a commission merchant in Chicago $2080.80 to invest in flour, his commission being 2% on the amount expended. How many barrels of flour could be purchased at $4.25 a barrel?

19. A real estate broker sold a house on $3\frac{1}{2}$% commission, and sent to the owner $6176. What was the broker's commission, and what sum did he receive for the house?

20. I send $5250 to a commission merchant in St. Louis, who charges 5% for investing, with instructions to purchase certain goods, deducting his commission from the amount of money sent him. Find his commission.

INSURANCE

203. Insurance is a contract by which a person whose property is insured receives security against loss by fire or accident in consideration of a sum of money paid to the insurance company.

The **Premium** is the sum paid for insurance. It is always a certain per cent of the *sum insured*.

The **Policy** is the written contract of insurance.

204. A factory valued at $35,000 was insured for $\frac{3}{5}$ of its value, the rate of insurance being $\frac{5}{8}\%$ for one year. What was the amount of the premium?

$$\text{The premium} = \tfrac{5}{8}\% \text{ of } \tfrac{3}{5} \text{ of } \$35,000.$$
$$= \frac{5}{800} \times \frac{3}{5} \times \frac{\$35000}{1} = \$131.25.$$

Exercise 166

1. A store worth $4500 was insured for $\frac{2}{3}$ its value at $1\frac{1}{5}\%$. Find the premium.

2. A store worth $4800 is insured for $\frac{3}{4}$ of its value at $1.55 per $100. Find the premium.

3. Find the premiums paid to insure property against loss by fire for the following amounts, at the given rates:

(*a*) $2500 at $1.15 per $100; $3600 at $1.20 per $100.

(*b*) $8400 at $1.45 per $100; $7600 at $1.25 per $100.

(*c*) $9600 at $1.60 per $100; $8500 at $1.50 per $100.

4. The rate of insuring an hotel, insured for $75,000, was advanced from $1.75 to $2.32 per $100. Find the increase in the premium.

5. Find the increase in the premiums on the following properties, the old and new rates and the amounts of insurance being given:

Amount Insured	Old Rate	New Rate	Amount Insured	Old Rate	New Rate
$25,000	$1.50	$2.22	$85,000	$1.25	$1.97
90,000	1.25	1.81	45,000	1.40	1.66
87,500	1.25	1.60	72,500	1.25	1.63

6. Find the entire premium for insuring a house for 3 yr. for $3000 at $1.75 per $100, and its contents for $1200 at the same rate.

7. A house is worth $4500 and its contents $1800. Find the entire premium paid for insuring both at $\frac{2}{3}$ of their value, the rate being $1\frac{3}{4}\%$.

8. A fire insurance company charged $262.50 for insuring a house for $17,500. What was the rate per cent of insurance?

9. A merchant's stock was worth $120,000. He insured it at $\frac{2}{3}$ its value, paying $600 premium. What was the rate per cent of insurance? What was the rate in cents per $100?

10. A shipment of goods is insured for $7500 at $\frac{3}{4}\%$. Find the premium.

11. For what sum was a house insured if the premium paid was $17.50 and the rate of insurance $\frac{7}{8}\%$?

12. For what sum was a shop insured if the rate of insurance was 65 ¢ per $100 and the premium paid was $81.25?

13. An insurance company took a risk at $2\frac{1}{4}\%$, and reinsured $\frac{3}{5}$ of the risk at 2%. The premium received exceeded the premium paid by $42. Find the amount of the risk.

14. What will be the cost of insuring a cargo of 24,000 bu. of wheat valued at 72 ¢ per bushel, the insurance covering $\frac{3}{4}$ of the value of the cargo, the premium rate being $1\frac{1}{8}\%$?

15. A merchant's stock was insured for $42,000, $\frac{1}{2}$ of this amount being at $\frac{7}{8}\%$, $\frac{2}{3}$ of the remainder at $\frac{3}{4}\%$, and the remainder at $\frac{5}{8}\%$. Find the total amount of premium paid.

16. A merchant insured his stock for $33,000 for one year at $\frac{7}{8}\%$. Six months thereafter the policy was cancelled at the request of the insured. Find the amount of premium returned, the short rate for six months being $\frac{5}{8}\%$.

17. A warehouse valued at $62,500 was insured for $\frac{3}{5}$ of its value. The rate of insurance was $1\frac{1}{4}\%$. What was the amount paid for the insurance?

18. A factory and the machinery therein is insured for $65,000; ⅔ of this sum is at ¾% premium and the remainder is at ⅞%. What is the entire premium?

19. A fire insurance company received $350 for insuring a factory at 1¼% premium. What was the amount of insurance?

20. A building and contents are insured as follows: $12,000 in the first, $8000 in the second, and $5000 in the third insurance company. Were a loss to the extent of $3500 to occur through fire, what portion of the loss should each company bear?

21. Merchandise valued at $63,000 was insured in the first insurance company for $15,000, in the second for $12,000, and in the third for $8000. If the merchandise is damaged by fire to the extent of $10,500, how much of the damage should each company pay?

22. A steamboat worth $60,000 is insured in three companies; in two to the amount of $15,000 each, and in the third to the amount of $20,000. For what sum would each company be liable if the vessel were to sustain damage to the extent of $6600?

TAXES AND DUTIES

205. A **Tax** is a sum of money assessed on persons or property for public purposes.

The tax on property is reckoned at a certain *rate per cent* of the assessed value of the property.

Direct taxes are levied by the state, county, township, city, or the school district.

Some states levy a tax upon each voter, independent of the property he owns. Such a tax is called a **Poll-tax**, and as a rule does not exceed $2 a year.

Indirect taxes, called **Duties**, are levied by the general government on imported goods.

An **Ad Valorem Duty** is reckoned at a certain *rate per cent*

of the cost of the goods in the country from which they have been imported.

A **Specific Duty** is a fixed charge on the quantity of goods without reference to their cost, as a specific tax of one cent a pound.

206. The people of a school section wish to build a new schoolhouse, which will cost $2850. The taxable property of the section is assessed at $190,000; what will be the rate of taxation, and what will be the tax on property assessed at $7500 ?

The tax on $190,000 = $2850.

∴ the rate of taxation = $2850 ÷ $190,000 = .015 or $1\frac{1}{2}$%.

∴ the tax on $7500 = $1\frac{1}{2}$% of $7500 = $112.50.

Exercise 167

1. State expenses which the government meets by taxation; the county; the township; the village or city; the school district.

2. A village levies a tax of $12,000 and pays the tax collector 2% and $150 for collecting the taxes. Find the net amount of the taxes.

3. A village levied a tax of $15,000 and paid the tax collector 2% and $200 for collecting the taxes. If $250 taxes were noncollectible, find the net amount of the taxes.

4. What is the tax on property assessed at $6400, the rate of taxation being $1\frac{1}{2}$% ?

5. Find the tax on:

       (a) $8000 at 6 mills on a dollar.

       (b) $4500 at 8 mills on a dollar.

       (c) $2800 at $5\frac{1}{2}$ mills on a dollar.

       (d) $4800 at 9.2 mills on a dollar.

       (e) $3750 at 6.4 mills on a dollar.

       (f) $7250 at 8.6 mills on a dollar.

6. A man owns a farm of 80 A., worth $60 an acre. What is his tax at 8 mills on a dollar, his property being assessed at $\frac{2}{3}$ of its value?

7. A man whose property is assessed for $7500 pays the following taxes: state $1\frac{1}{4}$ mills, town .6 mill, and school $1\frac{1}{2}$ mills. Find his total tax.

8. In a school section a tax of $4000 is to be raised. If the assessed valuation of the property is $250,000, what will be the tax on the dollar, and what is A's tax, whose property is valued at $1800?

9. What is the assessed value of property taxed $37.50, at the rate of 15 mills on the dollar?

10. What is the assessed value of property taxed $37.80, at the rate of $4\frac{1}{2}$ mills on the dollar?

11. A man whose property is taxed 16 mills on a dollar pays a tax of $134.40. Find the assessed value of his property.

12. A village makes the following tax levy at the rate of 3.4 mills on a dollar: police $2200, roads $2500, treasurer $600, attorney $200, clerk $300, tax collector $300, miscellaneous $2400. Find the assessed value of the property in the village.

13. The municipal rates being reduced from $19\frac{3}{8}$ mills to $17\frac{1}{8}$ mills on the dollar, my taxes are lowered by $4.05. For how much am I assessed?

14. In a certain section a schoolhouse is to be built at an expense of $8400, to be defrayed by a tax upon property valued at $700,000. What is the rate of taxation to cover both the cost of the schoolhouse and the collector's commission of $350?

15. The assessed valuation of a town is $972,250, and the town has 320 polls paying $1.50 each; what is the rate of taxation when the tax levy is $19,925? What tax must a person pay whose property is assessed for $7500, and who pays for one poll?

16. If the assessed value of a town is $1,260,000, and the town has 420 polls paying $1.25 each, what is the rate of taxation on property when the tax levy is $15,645? What does A pay, whose property is assessed at $8500 and who pays one poll?

17. A house assessed at $2200 was rented for $23 a month, the tenant to pay taxes and water-rates. The taxes were 17 mills on the dollar, and the water-rates were $5 per quarter year. How much all together did the tenant pay per year for the house? If the property had cost the landlord $2500, what rate per cent per year was he receiving on his investment?

18. What is the duty on 600 boxes of cigars costing $7.50 a box, the duty on cigars being 35%?

19. What is the duty on 800 yd. cloth at 12s. a yard, the duty being 25%? (1 £ = $4.86.)

20. What is the duty on 600 yd. of cloth invoiced at 6 francs per yard, the duty being 30%? (1 franc = 19.3 ¢.)

Miscellaneous Exercise 168

1. I bought an article for $3.60 and sold it for $4.20. What is my gain per cent?

2. I sold goods for $3360 and gained 12%. What was the cost price?

3. If 425 yd. of silk are sold for $1657.50, and 20% profit is made, what did it cost per yard?

4. By selling goods for $1088, I lost 16%. How much per cent should I have lost or gained if I had sold them for $1344?

5. A tradesman's prices are 25% above cost price. If he allows a customer 8% on his bill, what profit does he make?

6. 8% is gained by selling a piece of ground for $8251.20; what would be gained per cent by selling it for $8404?

7. Find the brokerage on $1324 at ¼%.

8. Find the brokerage on $375 at 5%.

9. What amount of money was invested, when the broker's charges at $1\frac{1}{4}\%$ amounted to $150?

10. My agent has purchased real estate, on my account, to the amount of $19,384. What is his commission at $1\frac{1}{2}\%$?

11. The price of flour per barrel at different times during the year 1900 was:

April 26 . . . . $3.80	June 18 . . . . $4.20
April 30 . . . . 3.90	June 19 . . . . 4.40
June 8 . . . . 4.00	June 21 . . . . 4.65
June 9 . . . . 4.10	June 22 . . . . 4.75

Find the rate per cent, at each date, above or below the price on June 8.

12. The price of cotton advanced from 6.3 ¢ to 9.3 ¢ per pound. Find rate per cent increase in price.

13. What is the premium of insurance on the contents, insured for $1500 at $2\frac{1}{2}\%$?

14. What is the premium for insuring a cargo for $16,450, at $3\frac{1}{4}\%$?

15. A person at the age of 40 insures his life in each of two offices for $4500, the premiums being at the rate of $3\frac{1}{8}$ and $3\frac{1}{2}\%$ respectively. Find his annual payment.

16. A trader gets 600 bbl. of flour insured for 80% of its cost, at $2\frac{1}{4}\%$, paying $37.80 premium. At what price per barrel did he purchase the flour?

17. A shipment of dry goods was insured at $1\frac{3}{5}\%$ to cover $\frac{7}{8}$ of its value. The premium was $28. What were the goods worth?

18. A man who owns $12,750 worth of property pays a tax of $216.75. Find the rate on the dollar.

19. A certain town has property assessed at $520,000, and levies a tax of $7800. What should B pay, whose property is assessed at $2500?

20. A town has levied a tax of $7690, which sum includes the amount voted for building a town hall and the collector's fees, at 3%. What was expended on the town hall?

21. What is the rate per cent of commission when I receive $ 5 for selling goods to the value of $ 125 ?

22. I sold a quantity of goods for $ 273.68, on a commission of $2\frac{3}{8}\%$. Find my commission.

23. A and B insure their houses against fire, and A has to pay $ 7.50 more than B, who pays $ 28.75. Find the amount for which their houses are insured, the rate of insurance being $\frac{5}{8}\%$.

24. A merchant bought goods amounting to $ 7460 subject to 25 and 5 off, $ 3730 subject to 30 off, and $ 1492 subject to 20 and 10 off. Find the net cost of the goods.

25. If, in the preceding example, the invoice clerk were to bill all the goods subject to 30 off, what would be the error in their net cost?

26. A man bought a house and lot for $ 2250. After spending $ 630 on repairs and improvements, and paying $ 30 for taxes and other expenses, he sold the property for $ 3880. What rate per cent of profit did his investment yield him?

27. In an examination A obtained 78% of the full number of marks, beating B by 16% of the full number. If A received 975 marks, how many did B receive?

28. By selling a certain book for $ 3.96, I would lose 12% of the cost. What advance on this proposed selling price would give a profit of 12% of the cost?

29. Goods are sold at a loss of 20% on the cost. By what percentage of itself should the selling price be advanced to yield a profit of 20% on the cost?

30. A man having bought a certain quantity of goods for $ 150, sells $\frac{1}{3}$ of them at a loss of 4%. By what increase per cent must he raise that selling price that by selling the whole at that increased rate he may gain 4% on his entire outlay?

31. The cost of freight and insurance on a certain quantity of goods was 15%, and that of duty 10% on the original outlay.

The goods were sold at a loss of 5%, but had they brought $3 more there would have been a gain of 1%. How much did they cost?

32. A man began business with a certain capital; he gained 20% the first year, which he added to his capital, and $37\frac{1}{2}$% the second year, which he added to his capital; in the third year he lost 40%; had he received $600 more for the goods sold the last year, he would have cleared in the three years 2% of his original capital. Find the capital with which he commenced business.

33. A merchant bought 400 lb. of tea and 1600 lb. of coffee, the cost of the latter per pound being $16\frac{2}{3}$% that of the former; he sold the tea at a profit of $33\frac{1}{8}$%, and the coffee at a loss of 20%, gaining, however, on the whole $60. Find his buying prices and his selling prices.

34. A sells potatoes for $1 per bushel and gains 25%. Afterward he sold some of the same lot of potatoes to the amount of $36 and gained 50%. How many bushels were there in the last lot and at what rate per bushel did he sell them?

35. A person marks his goods so that he may allow a discount of 4%, and still make a profit of 15%. What must be the marked price of an article that cost him $4.80?

36. A manufacturer who employed men at $1.60 a day found that he could save 15% by employing women. What wages were paid the latter, supposing a woman could do $\frac{3}{4}$ as much as a man in the same time?

37. A merchant buys goods; the cost of freight is 8%, and that of insurance 12% on the original outlay; he is obliged to sell them at a loss of 7%; but if he had received $5.10 more for them he would have gained $1\frac{1}{2}$%. Find the original outlay.

38. A merchant sells 50 yd. of broadcloth at a gain of 15%, and 75 yd., which cost the same per yard, at a gain of 10%, and finds that if he had sold the whole at a uniform gain of $12\frac{1}{2}$%, he would have received $2.25 more than he actually did receive. What was the cost price per yard?

39. A man buys goods for a certain sum, and marks $\frac{1}{3}$ of them at a profit of 24%, and $\frac{2}{3}$ of them at a profit of 36%; but had he marked $\frac{2}{3}$ of them at 24% gain, and $\frac{1}{3}$ at 36% gain, he would have realized $240 less than before. Find the cost of the goods.

40. A wheat buyer sold $\frac{1}{4}$ of his wheat at a certain gain per cent, $\frac{1}{3}$ of it at a gain of twice the former rate per cent, and the remainder at a gain per cent of 3 times the first gain. If the gain on the entire stock was 26%, what did he gain on each part? If he gained 5% on the first part, what was the entire gain per cent?

41. A merchant wishes to mark some goods which cost $1.20 per yard, so that after making a reduction of 20% off the marked prices, he may yet gain 10%. At what price per yard must he mark the goods?

42. Sold goods to a certain amount on a commission of 5%, and having remitted the net proceeds to the owner, received for prompt payment $\frac{1}{2}$%, which amounted to 24.22\frac{1}{2}$. What was the selling price of the goods?

43. What single discount is equivalent to successive discounts of 20% and 10%?

44. Show that successive discounts of specified rates may be taken off a list price in any order without affecting the net price. Thus 20 and 10 off is equivalent to 10 and 20 off, so also 30 and 10 and 5 off, 10 and 30 and 5 off, and 5 and 30 and 10 off are all equivalent.

45. An agent sold 6 mowing-machines at $120 each, and 12 at $140 each. After deducting his commission he remitted $2280 to his employer. What was the rate of commission?

CHAPTER XV

INTEREST

207. Interest is money paid for the *use* of money.

The **Principal** is the sum loaned.

The **Amount** is the sum of the principal and interest.

The **Rate of Interest** is always expressed as a rate per cent of the principal.

The unit of time is 1 year.

208. (1) What is the interest on $638 for 1 yr. at 6%?

$$\begin{array}{r} \$638 \text{ principal} \\ .06 \text{ rate per unit} \\ \hline \$38.28 \text{ interest for 1 yr.} \end{array}$$

∴ the interest on $638 for 1 yr. at 6% = 6% of $638 = $38.28.

(2) Find the interest on and the amount of $473.28 for 81 da. at 7%.

$$\begin{array}{r} \$473.28 \text{ principal} \\ .07 \text{ rate} \\ \hline \$33.1296 \text{ interest for 1 yr.} \end{array}$$

$$\begin{array}{r} \$33.13 \text{ interest for 1 yr.} \\ 9 \\ \hline 40)\$298.17 \\ \hline 7.45 \text{ interest for 81 da.} \\ 473.28 \\ \hline \$480.73 \text{ amount} \end{array}$$

The interest for 1 yr. = 7% of $473.28 = $33.13.

The interest for 81 da. = $\frac{81}{360}$ or $\frac{9}{40}$ of $33.13 = $7.45.

The amount = $473.28 + 7.45 = $480.73.

272

(3) Find the amount of $385.35, from July 7, 1895, to Oct. 13, 1895, at $7\frac{1}{2}\%$.

$$\begin{array}{lcc} & \text{mo.} & \text{da.} \\ \text{Oct. } 13 = & 10 & 13 \\ \text{July } 7 = & 7 & 7 \\ \hline \text{The time} = & 3 & 6 = 3\frac{1}{5} \text{ mo.} = \frac{4}{15} \text{ yr.} \end{array}$$

The rate $= \frac{4}{15} \times \frac{15}{2}\% = 2\%.$

The interest $= 2\%$ of $385.35 = \$6.71.$

The amount $= \$385.35 + \$6.71 = \$392.06.$

209. Six Per Cent Method.

The interest at 6% for 1 yr. $= .06$ of the principal.

The interest at 6% for 1 mo. $= \frac{1}{12}$ of .06 or .005 of the principal.

The interest at 6% for 1 da. $= \frac{1}{30}$ of .005 or $.000\frac{1}{6}$ of the principal.

Find the interest on $435 for 9 mo. 24 da. at 6%.

The interest for 9 mo. $= 9 \times .005 = .045$

The interest for 24 da. $= 24 \times .000\frac{1}{6} = .004$

The interest for 9 mo. 24 da. $=$ $\qquad$ .049

$\therefore$ the interest $= .049 \times \$435 = \$21.315.$

To find the interest at any other rate per cent, divide the interest at 6% by 6 and multiply by the given rate per cent.

The interest at $7\frac{1}{2}\%$ may be found by increasing the interest at 6% by $\frac{1\frac{1}{2}}{6}$ or $\frac{1}{4}$; that at $5\frac{1}{2}\%$ by diminishing the interest at 6% by $\frac{\frac{1}{2}}{6}$ or $\frac{1}{12}$.

By what fraction must the interest at 6% be increased in order to give the interest at each of the following rates : $7\%, 8\%, 9\%, 6\frac{1}{4}\%, 6.2\%$?

By what fraction must the interest at 6% be diminished in order to give the interest at each of the following rates : $5\%, 4\%, 3\%, 4\frac{1}{2}\%, 4\frac{1}{4}\%, 5\frac{3}{4}\%$?

Exercise 169

Find the interest on :

1. $449 for 1 yr. at 5%.
2. $757 for 1 yr. at 4%.
3. $643.17 for 1 yr. at 7%.
4. $725 for 4 mo. at 8%.
5. $587.50 for 5 mo. at 6%.
6. $628.90 for 9 mo. at $4\frac{1}{2}\%$.
7. $323.75 for 60 da. at 8%.
8. $958.50 for 90 da. at $4\frac{1}{4}\%$.

T

9. $2865 for 33 da. at 6%.   12. $225.90 for 63 da. at 7%.

10. $312.80 for 93 da. at 6%.   13. $390.50 for 93 da. at 6%.

11. $612.94 for 33 da. at 7½%.   14. $8396.40 for 123 da. at 8%.

15. $4087.50 for 1 mo. 3 da. at 9%.

16. $1465.53 for 3 mo. 3 da. at 5%.

17. $1350 for 3 mo. 21 da. at 7%.

18. $295.36 for 57 da. at 6.2%.

19. $1200 from May 7 to June 6 at 7%.

20. $975.65 from Sept. 16 to Dec. 8 at 6¼%.

21. $450 from Sept. 4 to Oct. 27 at 7%.

22. $79.50 from Dec. 23 to Feb. 20 of the next year at 7½%.

23. $586.67 from Jan. 15, 1901, to May 1, 1901, at 8%.

24. State how to find the simple interest when the principal, rate per cent, and time are given.

25. Name the terms in problems in Profit and Loss, Commission and Insurance, which correspond to principal and rate per cent of interest.

26. Find the relation between the interest, principal, and amount, when the time is 3 mo., and rate 8%; time 120 da., rate 9%.

27. Find the amount of $473.28 for 3 mo. at ½% per month.

28. Find the amount of $885.85 for 1 mo. 15 da. at 5%.

29. Find the amount of $628.25 for 185 da. at 4½%.

30. Find the amount of $935.68 for 66 da. at 6½%.

31. Find the amount of $147.50 for 93 da. at 7%.

32. Find the amount of $250 from July 9 to Aug. 18 at 8%.

33. Find the amount of $2394 from May 8 to Sept. 21 at 4%.

34. Find the amount of $5246 from March 1 to Aug. 3 at 5%.

35. Find the amount of $230.80 from Jan. 4, 1901, to June 23, 1901, at 6%.

36. Find the amount of $657.60 from Aug. 9 to Dec. 5 at 8%.

37. A person loaned $ 480 for 2 mo. and 13 da. at 9 %. What interest did he receive?

38. On March 20, a merchant sold goods to the value of $ 1168, and received a note, due June 8, next, for that sum with interest at 7 % per annum. For what amount was the note drawn?

39. A debt of $ 175 became due on June 13, after which date interest was charged at the rate of 8%. What must be paid to settle the debt Sept. 14?

40. A owes $ 15,000 bearing interest at 5% per annum; he pays at the end of each year for interest and part payment of principal $ 2500. Find the amount of his debt at the end of the third year.

41. A man engaged in business with a capital of $ 10,920 is making 12½% per annum on his capital, but on account of ill health he quits the business and loans his money at 5%. How much is his income diminished?

42. $ 420.   CHICAGO, June 4, 1901.

Sixty days from date I promise to pay Samuel Jones, or order, four hundred and twenty dollars, with interest at six per cent, value received.

RICHARD WALSH.

What is the amount of this note at maturity?

43. A merchant borrows $ 1600 for 1 yr. at 7%. Find what he owes at the end of the year. In case he pays only $ 12 interest, how much will he owe at the beginning of the next year? What will he owe at the end of the year?

44. If I borrowed $ 1200 Jan. 1, 1900, at 6%, what would I owe Jan. 1, 1901? If I kept the money until Jan. 1, 1902, what would I then owe?

EXACT INTEREST

210. In order to find the *exact interest* we must reckon 365 da. to a year. Exact interest is used by the United States Government and sometimes in business transactions.

211. The exact interest at 5 % for 1 da. is $\frac{5}{365}$, or $\frac{1}{73}$ of the principal.

The common interest is $\frac{5}{360}$, or $\frac{1}{72}$ of the principal. Therefore the exact interest is $\frac{1}{73} \div \frac{1}{72}$ or $\frac{72}{73}$ of the common interest. Hence the exact interest is equal to the common interest diminished by $\frac{1}{73}$ of itself.

212. Find the exact interest on $4250 from May 12 to Oct. 3 at 7%.

The number of days from May 12 to Oct. 3 = 19 + 30 + 31 + 31 + 30 + 3 = 144.

The interest on $4250 at 7 % for 1 yr. = $297.50.

The interest on $4250 at 7 % for 144 da. = $\frac{144}{365}$ of $297.50 = $117.37.

Exercise 170

Find the exact interest on:

1. $2450 for 146 da. at 6%.
2. $3475 for 292 da. at 7%.
3. $1560 for 60 da. at 5%.
4. $629 for 113 da. at 6%.
5. $1400 from July 6 to Dec. 4 at $5\frac{1}{2}$%.
6. $1850 from March 1 to Aug. 6 at $6\frac{1}{2}$%.
7. $2500 from May 1 to Sept. 24 at 5%.
8. $2480 from Aug. 9 to Sept. 18 at 6%.

BANK DISCOUNT

213. A merchant, who desires to obtain a loan of $800 for 90 da., makes a note and takes it to the bank, which deducts the interest on $800 for 90 da. at a certain rate per cent, which varies from time to time. This bank gives him the *proceeds*, and collects the $800 at the end of 90 da.

In those states that have not abolished days of grace, three days, known as *days of grace*, are added to the specified time to find when the payment is due. In these states the bank discount in the above instance would be reckoned for

93 da. The note would then be due in 93 da. Days of grace have been abolished in all but the following states (Jan., 1901):

Alabama	Kentucky	New Mexico Territory
Arizona Territory	Louisiana	North Carolina
Arkansas	Michigan	Oklahoma Territory
Georgia	Minnesota	South Carolina
Indiana	Mississippi	South Dakota
Indian Territory	Missouri	Texas
Iowa	Nebraska	Wyoming
Kansas	Nevada	Canada

Make a list of those states that have abolished days of grace.

214. Bank Discount is, therefore, simple interest collected *in advance* upon the sum due on a note at its maturity.

Nearly all notes specify the place of payment. In case the place of payment is not specified in the note, it is to be paid at the *business office* of the maker of the note.

215. $450.75.   Chicago, July 3, 1901.

Sixty days after date I promise to pay to the order of James Smith, four hundred fifty and $\frac{75}{100}$ dollars at the First National Bank. Value received.   Horace Ward.

Discounted July 3, at 6 %. Find proceeds.

The discount = the interest on $450.75 at 6 % for 60 da. = $4.51.

The proceeds = $450.75 − $4.51 = $446.24.

In those states that have not abolished days of grace use 63 da. The discount will then be $4.73 and the proceeds $446.02.

216. The **Day of Maturity** is the day on which the note becomes legally due.

The **Proceeds of a Note** is the sum of money received for it when discounted.

It is found by subtracting the discount from the value of the note at maturity.

The **Time to run** is the time between the day on which the note is discounted and the day of maturity.

Exercise 171

1. $ 600.   CHICAGO, July 6, 1898.

Thirty days after date I promise to pay to George Boies, or order, six hundred dollars, value received.   ROBERT BROWN.

Discounted at 7 %, July 6, 1898.   Find proceeds.

	Face of Note	Date of Note	Time	Rate of Discount	
2.	$ 312.80;	May 13, 1899;	90 da.;	6 %.	Find proceeds.
3.	$ 225.90;	June 14, 1896;	2 mo.;	7 %.	Find proceeds.
4.	$ 100.00;	Feb. 12, 1898;	30 da.;	5 %.	Find proceeds.

5. State how to find the proceeds of any note discounted at once.

6. Write the notes corresponding to examples 2 and 3.

7. $ 390$\frac{50}{100}$.   SPRINGFIELD, ILL., May 1, 1900.

Three months after date I promise to pay to the order of Thomas A. Stuart, three hundred ninety and $\frac{50}{100}$ dollars. Value received.   JAMES HENDERSON.

Discounted May 1, 1900, at 6 %. Find proceeds.

217. (1) $ 712.65.   CHICAGO, July 6, 1899.

Sixty days from date I promise to pay George Wilson, or order, seven hundred twelve and $\frac{65}{100}$ dollars, for value received.   SAMUEL JONES.

Discounted at 7 %, Aug. 6, 1899.

In the above note, find the *day of maturity*, the *time to run*, the *discount*, and the *proceeds*.

The day of maturity = 60 da. after July 6 = Sept. 4, 1899.
The time to run = the number of days between Aug. 6 and Sept. 4.
= 29 da. = $\frac{29}{360}$ yr.
The discount = the interest on $712.65 for 29 da. at 7 % = $4.02.
The proceeds = $712.65 − $4.02 = $708.63.
In those states that have not abolished days of grace the results are:
Sept. 7, 32 da., $4.43, $708.22.

(2) $450.76.   NEW YORK, May 5, 1901.

Three months after date, for value received, I promise to pay Thomas King, or order, four hundred fifty and $\frac{76}{100}$ dollars, at the First National Bank, with interest at 6 %.

ARTHUR HILL.

Discounted July 1, 1901, at 8 %.

The day of maturity = 3 mo. after May 5 = Aug. 5, 1901.
The amount of the note, Aug. 5, 1901 = the amount of $450.76 for 3 mo.
at 6 % = $457.52.
The time to run = the number of days between July 1 and Aug. 5 = 35 da.
The Discount = the interest on $457.52 for 35 da. at 8 % = $3.56.
The proceeds = $457.52 − $3.56 = $453.96.
In those states that have not abolished days of grace the results are :
Aug. 8, $457.75, 38 da., $3.87, $453.88.

In the following exercise find the *day of maturity*, the *time to run*, the *discount*, and the *proceeds*. If the state in which you live has not abolished days of grace add them, otherwise, not.

Exercise 172

1. $2400.   CLEVELAND, O., March 3, 1901.

Three months after date I promise to pay Ralph Barker, or order, twenty-four hundred dollars, value received.

ROBERT PETERSON.

Discounted at 7 %, May 7.

2. A note for $572.80, drawn on June 13 and payable 4 mo. after date, was discounted at 7% on June 27.

3. $2400.   CLEVELAND, O., March 3, 1898.

Three months after date I promise to pay Ralph Barker, or order, twenty-four hundred dollars, for value received, with interest at 6%.   ROBERT PETERSON.

Discounted at 7%, May 7.

4. State business transactions which may have preceded the giving of the notes in examples 1, 2, and 3.

5. On July 7, James Monroe bought a farm from John Harris, paying $2000 cash and giving his note, without interest, for $1200, payable in 60 da.   Write the note.

	Face of Note	Date of Note	Time	Date of Disc.	Rate of Disc.
6.	$312.80;	May 13, 1899;	90 da.;	May 13;	$6\frac{1}{2}\%$.
7.	$975.65;	Sept. 5, 1899;	3 mo.;	Sept. 16;	7%.
8.	$450.00;	Aug. 28, 1901;	60 da.;	Sept. 4;	7%.
9.	$79.50;	Dec. 17, 1899;	2 mo.;	Dec. 23;	$7\frac{1}{2}\%$.
10.	$586.67;	Dec. 28, 1901;	4 mo.;	Jan. 15, 1902;	8%.

11. $2480.   BUFFALO, N. Y., Nov. 19, 1900.

Six months after date I promise to pay Alfred Jameson, or order, two thousand four hundred and eighty dollars, value received, with interest at 5%.   WILLIAM O'CONNOR.

Discounted at 6%, Jan. 4, 1901.

12. State how to find the proceeds of a note, not bearing interest, when discounted.   What change is to be made in the solution when the note bears interest?

13. $2065.76.   NEW ORLEANS, June 4, 1899.

Ninety days after date I promise to pay to the order of Edgar Johnston two thousand sixty-five and $\frac{76}{100}$ dollars, for value received, with interest at 6%.   ALEXANDER GRANT.

Discounted at 8%, July 4, 1899.

COMPOUND INTEREST

218. **Compound Interest** is interest which is found for stated periods and added at the end of each period to the principal, the sum of the principal and interest becoming the new principal.

The unit of time is 1 year, although the interest may be compounded annually, semiannually, quarterly, and so on.

219. If $5000 deposited at a savings bank draws interest at 4%, semiannually, the interest due at the end of the first half year will be 2% of $5000, or $100.

If this $100 is not drawn, it is placed to the credit of the depositor, who has now $5100 on deposit.

The interest for the second half year is 2% of $5100, or $102.

If this is not drawn, it is placed to the credit of the depositor, making his deposit $5202.

The interest for the third half year is 2% of $5202, or $104.04.

If this is not drawn, it is placed to the credit of the depositor, making his deposit $5306.04 at the end of 1 yr. 6 mo.

Thus $5000 at 4% interest, compounded semiannually, will in 1 yr. 6 mo. amount to $5306.04; and the compound interest for that time will be $5306.04 − $5000 = $306.04.

$$
\begin{array}{rl}
\$\,5000 & \text{original principal} \\
.02 & \\
\hline
100.00 & \text{first interest} \\
5000 & \\
\hline
\$\,5100 & \text{amount at the end of the first period} \\
.02 & \\
\hline
102.00 & \text{second interest} \\
5100 & \\
\hline
\$\,5202 & \text{amount at the end of the second period} \\
.02 & \\
\hline
104.04 & \text{third interest} \\
5202 & \\
\hline
\$\,5306.04 & \text{amount at the end of the third period}
\end{array}
$$

220. Find the compound interest on $5000 for 1 yr. 10 mo. 15 da. at 4%, payable semiannually.

As in the last paragraph, find the amount of $5000 for 1 yr. 6 mo., and then complete the work thus:

The rate per cent for 4 mo. 15 da. or $\frac{3}{8}$ yr. = $\frac{3}{8} \times 4\% = 1\frac{1}{2}\%$.

$5306.04 amount at the end of the third period

$$
\begin{array}{r}
.01\frac{1}{2} \\
\hline
265302 \\
530604 \\
\hline
79.5906 \text{ fourth interest} \\
5306.04 \\
\hline
\$5385.63 \text{ amount at the end of the fourth period}
\end{array}
$$

∴ the compound interest = $5385.63 − $5000 = $385.63.

Or thus, find the interest on $5306.04 for 1 yr. at 4%, then take $\frac{3}{8}$ of it.

Exercise 173

Find the amount and the compound interest of:

1. $800 for 3 yr. at 5%, compounded annually.

2. $425 for 4 yr. at 4%, compounded annually.

3. $250 for 2 yr. at 6%, compounded semiannually.

4. Find the amount and also the compound interest on **$1000** for 3 yr. at 5%.

5. How do you find the amount of a sum of money for 2 yr. at 6% interest, payable semiannually?

6. Find the amount of $360 for 2 yr. at 6%, interest payable semiannually.

7. Find the amount of $650 for 1 yr. 3 mo., interest payable quarterly at 4% per annum.

8. Find the compound interest on $8240 for 1 yr. 6 mo. at 5%, payable semiannually.

9. State how to find the amount of a sum of money at compound interest, for a given time and rate.

10. Find the amount and also the compound interest on $2500 for 1 yr. 10 mo. 15 da. at 6%, payable semiannually.

11. Find the difference between the interest on $1050 for 1 yr. at 4%, and 1 yr. at 4% compounded quarterly.

12. Find the amount of $2000 in 2 yr. at 6%, compounded annually.

13. A man deposits in the savings bank $1500, on which the interest at 3% per annum is to be added to the principal every 6 mo. How much money has the man in the bank at the end of 2 yr.?

14. What will be the amount, compound interest, of $2400 for $1\frac{1}{2}$ yr. at 6% per annum, paid half-yearly?

EXCHANGE *

221. If A of Chicago owes B of St. Paul a sum of money, he can discharge the debt in any one of several ways. He can buy a *post-office order* at the Chicago post-office payable to B at the post-office in St. Paul; he can buy an *express order* at the office of an express company, payable to B at any office of the same company; or he can buy a *draft* at a bank payable to B at a bank in St. Paul.

Give some reasons why it is better to discharge a debt by means of a post-office order, express order, or draft than by sending the money in a registered letter or by express or check.

222. The following are the rates charged for express orders to any part of the United States or Canada:

Rates for orders not over

$2.50	3¢	$40.00	15¢
5.00	5¢	50.00	18¢
10.00	8¢	60.00	18¢
20.00	10¢	75.00	23¢
30.00	15¢	100.00	28¢

* For Stocks and Bonds, see Chapter XXI.

Over $100 at above rates.

Single express orders are not issued for more than $50, and for larger amounts additional orders are issued.

223. The fees charged for post-office orders to any part of the United States, Porto Rico, and the Philippines are the same as for express orders up to $50. The fee on an order not exceeding $60 is 20¢, $75 is 25¢, $100 is 30¢. Single post-office orders are not issued for more than $100, and for larger amounts additional orders are issued.

Exercise 174

1. What is the cost of an express order for $25? $44? $73? $78?

2. What is the cost of a post-office order for $80? $32? $95? $1.50?

3. What is the cost of an express order for $75? $100?

4. What is the cost of a post-office order for $75? $100?

5. What is the cost of a draft for $75? $100? $150? $240? $325? $180? The charge in each case is $\frac{1}{4}\%$ and the least charge 25¢.

6. By which of the three methods given in questions 3, 4, and 5 is it cheaper to send money in sums greater than $75? In sums less than $75, if 25¢ is the smallest charge for a draft?

7. What is the cost of a draft for $87.50? $120? $175? $287.50? $192.80? The charge in each case is $\frac{1}{4}\%$, and the least charge is 25¢.

224. Exchange is generally conducted through bankers, who issue *drafts* directing a second bank to pay a specified sum of money to the order of the person named in the draft.

A **Time Draft** is one payable at a specified time after sight or date.

If A in Chicago owes B in St. Paul a sum of money, B may send a draft to A for the amount. If A accepts the draft, he writes the word "accepted" with the date across the face and signs his name.

Exchange is the system of paying debts to persons in distant places without actually sending the money, by means of money orders and drafts.

225. (1) Find the cost of a draft on New York for $600, when exchange is $\frac{1}{4}\%$ premium.

$$\text{The premium} = \tfrac{1}{4}\% \text{ of } \$600 = \$1.50.$$

$$\therefore \text{ the cost} = \$600 + \$1.50 = \$601.50.$$

(2) Find the cost of a draft on New Orleans for $1200, payable 60 da. after date, exchange being $\frac{1}{4}\%$ discount, and interest 6%.

$$\text{The discount} = \tfrac{1}{4}\% \text{ of } \$1200 = \$3.00.$$

$$\text{The discount for 63 da.} = 6\% \text{ of } \$1200 \text{ for 63 da.} = \$12.60.$$

$$\therefore \text{ the cost} = \$1200 - \$3.00 - \$12.60 = \$1184.40.$$

Show that if exchange had been $\frac{1}{4}\%$ premium, the cost would have been $1190.40. Why 63 days? See § 213.

Exercise 175

1. Find the cost of a draft for $900 at $\frac{1}{4}\%$ premium.

2. Find the cost of a draft for $1600 at $\frac{1}{8}\%$ discount.

3. Find the cost of a draft for $4500 at $\frac{5}{8}\%$ discount.

4. Find the cost of a draft for $2800 at $\frac{3}{8}$ of 1% premium.

5. Find the cost of a draft for $1000, payable in 60 da., exchange being $\frac{1}{4}\%$ premium, and interest 6%.

6. Find the cost of a draft for $360, payable in 30 da., exchange being $\frac{1}{4}\%$ discount, and interest 5%.

7. Find the cost of a draft for $1250, payable in 60 da., exchange being $\frac{1}{4}\%$ premium, and interest $4\frac{1}{2}\%$.

8. Find the cost of a draft for $1800, payable in 30 da., when exchange is at par, and interest 4%.

9. Find the cost of a bill of exchange on London for £600, when exchange is quoted at $4.88.

10. Find the cost of a 60-da. draft on Liverpool for £750, exchange at 60 da. being $4.86.

11. What is the cost of a bill of exchange in Paris for 2400 francs at $5.16\frac{1}{4}$ francs per $1?

12. What is the cost of a bill of exchange on Berlin, for 2400 marks, the rate of exchange being $95\frac{1}{5}$ ¢ for 4 marks?

13. August 21, 1899, wheat was reported 6 d. per bushel higher in London than on the previous day. Find in dollars and cents the increase in price on 100 bu. (£1 = $4.86\frac{2}{3}$).

14. August 22, 1899, wheat was reported 15 centimes per bushel higher in Paris than on the previous day. Find in dollars and cents the increase in price on 100 bu. (1 franc = 19.4 ¢).

15. The White Star Liner Oceanic was open to the public at Belfast, Ireland, August 19, 1899, the charge being 2 s. 6 d. This is how many cents? (1 s. = 25¢).

CHAPTER XVI

RATIO AND PROPORTION

226. If two quantities are expressed in terms of the same unit, their **Ratio** is the quotient obtained by dividing the number measuring the first quantity by the number measuring the second quantity.

Thus the ratio of $\$3$ to $\$5 = \frac{3}{5}$, or, as it is frequently written, $3:5$.

The first term of a ratio is called the **Antecedent**, and the second the **Consequent**.

Since a ratio may be expressed as a fraction, both terms of a ratio may be multiplied or divided by the same number without changing its value.

Thus $8:12 = 2:3$, dividing each term by 4.

$3:4 = 15:20$, multiplying each term by 5.

$3\frac{3}{4}:4\frac{1}{6} = 9:10$, multiplying each term by 12 and dividing by 5.

227. Reduce the following to equivalent ratios by multiplication or division, and write your results as in the preceding paragraph :

$6:8$	$15:25$	$24:18$	$14:21$
$2:3$	$5:4$	$5:7$	$8:9$
$\frac{2}{3}:\frac{3}{4}$	$\frac{5}{6}:\frac{2}{3}$	$1\frac{1}{2}:1\frac{1}{5}$	$2\frac{1}{4}:3\frac{3}{5}$

228. A **Proportion** * is the equality of two or more ratios.

Thus $6:8 = 9:12$ is a proportion.   Each of the two ratios is equal to $3:4$ or $\frac{3}{4}$.

* For Compound Proportion, see Chapter XXI.

229. The first and fourth terms of a proportion are called the **Extremes**, and the second and third terms the **Means**.

230. In the following proportions show that *the product of the means is equal to the product of the extremes.*

$$6 : 8 = 9 : 12 \qquad\qquad 7 : 14 = 3 : 6$$
$$10 : 15 = 8 : 12 \qquad\qquad \tfrac{2}{3} : \tfrac{3}{4} = 8 : 9$$
$$6 : 9 = 4 : 6 \qquad\qquad 2\tfrac{1}{2} : 3\tfrac{3}{4} = 4 : 6$$

231. (1) Find the value of x in the proportion $10 : 15 = x : 36$.

$$15\,x = 360 ;$$
$$x = 24.$$

Show by § 230 that 24 is the correct answer.

(2) Find the value of x in the proportion $\tfrac{3}{4} : 6 = \tfrac{4}{5} : x$.

$$\frac{3\,x}{4} = \frac{24}{5};$$
$$15\,x = 96.$$

Multiplying each side of the equation by 20,

$$x = 6\tfrac{2}{5}.$$

Exercise 176

Find the value of x in the following proportions:

1. $6 : 8 = x : 12.$
2. $9 : 6 = 18 : x.$
3. $8 : x = 12 : 15.$
4. $x : 14 = 24 : 16.$
5. $x : 5 = 9 : 15.$
6. $\tfrac{2}{3} : x = 2 : 6.$
7. $8 : x = \tfrac{3}{4} : 6.$
8. $x : 9 = \tfrac{2}{3} : \tfrac{4}{5}.$
9. $\tfrac{3}{4} : \tfrac{5}{6} = x : 10.$
10. $2\tfrac{1}{2} : 2\tfrac{1}{4} = 10 : x.$

232. If the antecedent and consequent of a ratio are interchanged, the resulting ratio is called the *reciprocal* of the given ratio.

Thus, the reciprocal of the ratio $3:4$ is the ratio $4:3$.

Write the reciprocals of the following ratios: $2:3$; $4:7$; $6:5$; $21:35$; $4:x$; $x:8$.

233. (1) If 15 bbl. of flour cost \$63, what will 35 bbl. cost?

Let $x =$ the *number* of dollars in the cost of 35 bbl.

Then,
$$15:35 = 63:x;$$
$$15\,x = 35 \times 63;$$
$$x = \frac{35 \times 63}{15} = 147.$$

$\therefore$ 35 bbl. will cost \$147.

(2) If 56 men can do a piece of work in 21 da., how long will it take 24 men to do it?

Let $x =$ the *number* of days 24 men will take.

Then,
$$56:24 = x:21.$$
$$24\,x = 21 \times 56;$$
$$x = \frac{21 \times 56}{24} = 49.$$

$\therefore$ 24 men will take 49 days.

In this solution why use the reciprocal ratio $x:21$ instead of the ratio $21:x$?

In the following exercise be careful to note whether you should use the direct or the reciprocal ratio as one-half of the proportion.

Exercise 177

1. Solve the examples in Exercise 66, from 4 through 20, using x to represent the unknown quantity.

2. If 6 articles cost \$14.30, how much will 13 cost at the same rate?

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3. If 25 lb. of tea cost $ 16, how many pounds can be bought for $ 56 ?

4. If a loaf of bread costs 11 ¢ when flour is $ 6 a barrel, find its cost when flour is $ 7½ a barrel.

5. A bankrupt owes $ 3000; his assets are $ 1740. What sum will a creditor receive whose claim is $ 350 ?

6. The expense of carpeting a room was $ 100; if the breadth of the room had been 4 ft. greater, the expense would have been $ 120. Find the breadth.

7. If a man working 9¾ hr. per day finishes a piece of work in 6 da., in what time would he have finished it if he had worked 8⅛ hr. per day ?

8. A garrison of 1500 men has provisions for 13 mo.; how long will their provisions last if it is increased to 2200 men ?

9. If 4 men *or* 6 women can do a piece of work in 20 da., how long will it take 3 men *and* 15 women to do the same work ?

10. A creditor receives $ 1.50 for every $ 4 of what was due to him, and thereby loses $ 301.05. What was the sum due ?

11. In a certain business one partner, whose share is $\frac{3}{11}$ of the whole, receives from it a profit of $ 859.20. What share is owned by another, whose profit is $ 1969 ?

12. A person contracts to do a piece of work in 30 da., and employs 15 men upon it; the work is half finished in 24 da. How many additional workmen must be then introduced in order to perform the contract ?

13. The profits of a garden for 2 yr. were $ 1456; the profits of the second year being $\frac{27}{25}$ of those of the first. Find the profits of each year.

14. If 10 men can do a piece of work in 12 da., how soon after beginning must they be joined by 3 more so as to finish the work in 10 da. ?

15. If $120 gain $5.81 in 126 da., find the gain in 360 da.

16. A bankrupt who is paying $37\frac{1}{2}$ ¢ on the dollar divides among his creditors $6300. What do his debts amount to?

17. If 3 men or 5 boys can do a piece of work in 18 da., in how many days will 3 men and 5 boys do a piece of work 3 times as great?

18. If 3 men can do as much work in a day as 4 boys, how long will it take 64 boys to finish a piece of work of which 12 men have done $\frac{1}{4}$ in 16 da.?

19. If a debt after a deduction of 3 % becomes $1008.80, what would it have become after a deduction of 4 % had been made?

20. Six sheets of paper measuring 8 in. by 10 in. weigh an ounce. Find the weight of 120 sheets of the same kind of paper, each sheet measuring 6 in. by 9 in.

21. A person walks from his house to his office at the rate of 4 mi. per hour; but finding he has forgotten something, returns at the rate of 5 mi. per hour. Compare the time spent in going with that spent in returning.

22. One train travels $8\frac{1}{2}$ mi. in 20 min., and a second train 9 mi. in 15 min. Compare their rates per hour.

23. A man can row 6 mi. an hour in still water. Compare his rate of rowing down a stream which flows at the rate of $2\frac{1}{2}$ mi. an hour with his rate of rowing up.

PROPORTIONAL PARTS

234. (1) Divide $720 in parts proportional to 4, 5, and 6.

The total number of parts $= 4 + 5 + 6 = 15$.

$\therefore$ the first part $= \frac{4}{15}$ of $720 = $192,

the second part $= \frac{5}{15}$ of $720 = $240,

the third part $= \frac{6}{15}$ of $720 = $288.

Or thus:

Let x = the number of dollars in the value of one share.

Then
$$4x + 5x + 6x = 720;$$
$$15x = 720;$$
$$x = 48.$$

$$4x = 192, \qquad 5x = 240, \qquad 6x = 288.$$

$$\therefore \text{ the parts are } \$192, \$240, \$288.$$

(2) Divide 316 lb. into parts proportional to $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{8}$.

Multiplying $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{1}{8}$ by their L. C. M. 120, we have the parts proportional to 40, 24, and 15.

The total number of parts = $40 + 24 + 15 = 79$.

$\therefore$ the parts are respectively $\frac{40}{79}$, $\frac{24}{79}$, and $\frac{15}{79}$ of 316 lb. = 160, 96, and 60 lb.

PROOF. — Dividing 160, 96, and 60 by 480, the denominator which reduces 160 to $\frac{1}{3}$, we have $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{8}$, which proves the results found to be correct.

Or thus:

Let x = the number of pounds in one share.

Then
$$\frac{x}{3} + \frac{x}{5} + \frac{x}{8} = 316;$$

or multiplying by 120,
$$40x + 24x + 15x = 37920;$$
$$79x = 37920;$$
$$x = 480.$$

$$\frac{x}{3} = 160, \qquad \frac{x}{5} = 96, \qquad \frac{x}{8} = 60.$$

$$\therefore \text{ the parts are 160 lb., 96 lb., 60 lb.}$$

Exercise 178

1. Divide 1331 into parts proportional to 2, 4, 5.

2. Divide $73.50 into parts proportional to $\frac{1}{4}$, $\frac{2}{5}$, $\frac{1}{6}$.

3. Divide 19 T. 1104 lb. into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

4. Divide $1064 into parts proportional to 2, $2\frac{1}{4}$, $2\frac{2}{5}$.

5. Divide 180 lb. into parts proportional to 3.3, .7, .5.

6. Divide $4500 between two persons in proportion to their ages, which are 21 and 24 yr.

7. Two men receive $15 for doing a certain piece of work. Now one man had worked only 3 da., while the other had worked 5 da. on the job. If the money is to be divided in proportion to the lengths of time the men worked, how much should each receive?

8. Divide 4472 into parts which shall be to each other in the ratio of 3, 5, 7, 11.

9. Divide $84.42 into two parts which shall be to each other as 5 : 16.

10. A company of militia consisting of 72 men is to be raised from three towns which contain respectively 1500, 7000, and 9500 men. How much must each town provide?

11. Sugar is composed of 49.856 parts oxygen, 43.625 carbon, and 6.879 hydrogen. How many pounds of each are there in 1300 lb. of sugar?

12. Gunpowder is composed of nitre, charcoal, and sulphur in the proportion of 33, 7, and 5.

(1) How many pounds of sulphur are there in 180 lb. of powder?

(2) How many pounds of powder can be made with 30 lb. of sulphur?

(3) How much nitre and sulphur must be mixed with 112 lb. of charcoal to form gunpowder?

13. A man divides $3300 amongst his three sons, whose ages are 16, 19, and 25 yr., in sums proportional to their ages; 2 yr. afterwards he similarly divides an equal sum, and again after 3 yr. more. How much does each receive in all?

14. Two persons travelling together agree to pay expenses in the ratio of 7 to 5. The first (who contributes the greater sum) pays on the whole $103.40, the second $63.40. What must one pay the other to settle their expenses according to agreement?

15. Divide $480 among A, B, C, and D, so that B may receive as much as A; C as much as A and B together; and D as much as A, B, and C together.

PARTNERSHIP *

235. In **Simple Partnership** the capital of each partner is supposed to be invested for the same time.

236. A, B, and C engage in business. A furnishes $7500, B $5000, and C $4500. If they gain $2380, what is each one's share?

Dividing their capitals by $500, we have their capitals, and therefore their gains proportional to 15, 10, and 9.

The total number of parts $= 15 + 10 + 9 = 34$.

∴ their respective gains are $\frac{15}{34}$, $\frac{10}{34}$, and $\frac{9}{34}$ of $2380 = $1050, $700, and $630.

Or thus:

Let $x =$ the number of dollars gain on $100 capital.

Then $$75\,x + 50\,x + 45\,x = 2380 ;$$

$$170\,x = 2380 ;$$

$$x = 14.$$

$$75\,x = 1050, \qquad 50\,x = 700, \qquad 45\,x = 630.$$

∴ their respective gains are $1050, $700, $630.

Exercise 179

1. Two merchants, A and B, form a joint capital. A puts in $1200 and B $1800. They gain $400. How ought the gain to be divided between them?

2. A bankrupt owes three creditors, A, B, and C, $175, $210, and $265, respectively; his property is worth $422.50. What ought each to receive?

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* For Compound Partnership, see Chapter XXI.

3. A, B, and C entered into partnership. A puts in $6000, B $4000, and C $2000. They gained $2250. What is each one's share of the gain?

4. Two men purchase a house for $3600, the first contributing $1600 and the second $2000. If it rents so as to pay 12% on its value, what share of the rent should each receive?

5. Two persons have gained in trade $3456; one put in $10,560 and the other $8640. What is each person's share of the profits?

6. R. Stuart and G. Armstrong enter into partnership. Stuart contributes $4500 to the partnership and Armstrong contributes $7500. Their net gain at the end of the year is $1750. How much of this sum should each partner receive?

7. Three partners invest respectively $7800, $5750, and $9450 in business. At the end of the first year they find their net gain to be $3156. What is the amount of each partner's share of this gain?

8. A, B, and C form a partnership with a capital of $20,000. A contributes $5000, B $7000, and C the remainder. They gain 20% of the total capital. Find each man's share of the profits.

9. T. Allan and E. Jamieson engage in business with a joint capital of $19,200, and agree to share gains and losses in proportion to their investments. At the end of a year Allan receives a dividend of $1100, and Jamieson a dividend of $1300. What was the amount of the investment of each?

10. D. Rowan, F. Galbraith, and J. Munro enter into partnership. They gain $7500, of which Rowan receives $2100, Galbraith $3100, and Munro the balance. How much did Rowan and Galbraith respectively invest if the amount of Munro's investment was $18,000?

11. A, B, and C pay $37.80 as rent for a pasture. A puts in 5 horses, B 12 cows, and C 60 sheep. If 1 horse eats as much as 2 cows, and 1 cow as much as 3 sheep, what rent should each pay?

CHAPTER XVII

SQUARE ROOT *

237. (1) Find the square root of 17.3056.

$$
\begin{array}{r|l}
 & 17.30'56 \ \underline{|\ 4.16} \\
 & 16 \\
81 & 130 \\
 & 81 \\
826 & 4956 \\
 & 4956 \\
\hline
\end{array}
$$

To prove 4.16 the right answer, square 4.16, and the result will be found to be 17.3056.

(2) Extract the square root of 35 to three decimal places.

$$
\begin{array}{r|l}
 & 35 \ \underline{|\ 5.916} \\
 & 25 \\
109 & 1000 \\
 & 981 \\
1181 & 1900 \\
 & 1181 \\
11826 & 71900 \\
 & 70956 \\
\hline
\end{array}
$$

(3) Extract the square roots of $\frac{25}{49}$, $\frac{35}{49}$, $\frac{5}{8}$.

$$\sqrt{\frac{25}{49}} = \frac{\sqrt{25}}{\sqrt{49}} = \frac{5}{7}.$$

$$\sqrt{\frac{35}{49}} = \frac{\sqrt{35}}{\sqrt{49}} = \frac{5.916}{7} = .845.$$

$$\sqrt{\tfrac{5}{8}} = \sqrt{.625} = .7905.$$

Which denominator is not a perfect square ? Why reduce $\frac{5}{8}$ to a decimal before extracting the square root ?

* Review Chapter IX. For Cube Root, see Chapter **XXI**.

Exercise 180

Find the square root of:

1. 40.96; 65.61; 2.1025.

2. 167.9616; 28.8369; 57648.01.

3. .042849; .00139876; .00203401.

4. 5774409; 5.774409.

5. 10.3041; 2321.3124; .0050367409.

6. 2; 20; .4; 1000 to four decimal places.

7. $\frac{144}{289}$; $\frac{324}{361}$; $6\frac{1}{4}$.

8. $20\frac{1}{4}$; $1\frac{56}{169}$; $\frac{1}{3}$; $\frac{2209}{9801}$.

9. $\frac{3}{8}$; $\frac{5}{9}$; $\frac{7}{11}$.

CHAPTER XVIII

MENSURATION

238. The rectangle has been treated of in preceding paragraphs.

Exercise 181

1. What is the area of a rectangle 15 ft. long and 12 ft. wide?

2. What is the area of a rectangular field 15 chains long and 6 chains wide? How many acres does it contain?

3. How many acres in a field 16 chains long and 10 chains wide?

4. How many acres in a field 32 rd. long and 25 rd. wide?

5. A rectangular flower bed 6 ft. long and 4 ft. wide is surrounded by a walk 1 ft. 6 in. wide. How many square feet in the walk? (Make a drawing to represent the bed and walk, on the scale of 1 in. to 1 ft.)

6. A garden 50 ft. long and 40 ft. wide is surrounded by a walk 3 ft. wide. How many square feet in the walk?

7. Make problems similar to examples 5 and 6.

8. Find the value of a field 60 rd. long and 40 rd. wide, at $75 an acre.

9. A town lot 4 rd. wide and 6 rd. deep sold for $1200. What is that per acre?

10. A rectangular room is 18 ft. long and 12 ft. wide. How much smaller is it than a square room of equal perimeter?

11. A rectangular room is 16 ft. long and 9 ft. wide. Find the length of the side of a square room of equal area.

12. A square field contains $2\frac{1}{2}$ A. Find the length of a side of the field in chains.

13. Find the cost of painting a surface 19 ft. 6 in. by 83 ft. 4 in. at 5¢ a square foot.

14. A square field contains exactly 8 A. Determine the length of a side of the field in chains and links.

15. The area of a chess-board marked in 8 rows of 8 squares each is 100 sq. in. Find the length of a side of a square.

16. On a certain map an area of 16 sq. mi. is represented by 9 sq. in. What part of an inch represents a mile?

17. On a certain map it is found that an area of 25 sq. mi. is represented by an area of 6.25 sq. in. Give the scale of the map in miles to the inch.

18. A rectangle measures 18' by 30'. Find the difference between its area and that of a square of equal perimeter.

19. Two rectangular fields are of equal area. One field measures 15 ch. by 20 ch.; the other is square. Find the length of a side of the latter field, correct to the nearest link.

20. How many stalks of wheat could grow on 1 sq. yd. of ground, allowing each stalk a rectangular space of 2'' by 3''? How many on 1 A.?

21. How many pieces of turf 3' 6'' by 1' 3'' will be required to sod a rectangular lawn 28' by 60'?
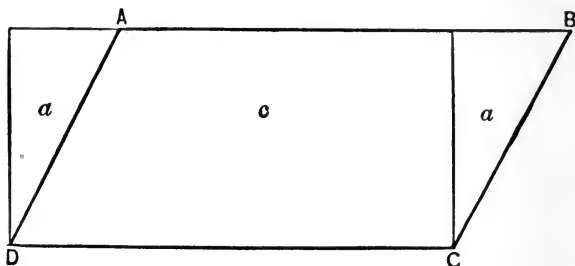
22. Sidewalks 4 ft. wide are laid on both sides of a street 440 yd. long. Find the cost of the sidewalks at $1.35 per square yard for the pavement and 75¢ per lineal yard for curbing.

23. A board containing 6 sq. ft. is 9 in. wide. Find its length.

24. What length must be cut off a board, which is $7\frac{1}{2}$ in. broad, so that the area may contain 3 sq. ft.?

239. A **Quadrilateral** is a plane figure having four sides.

A **Parallelogram** is a quadrilateral whose opposite sides are parallel.
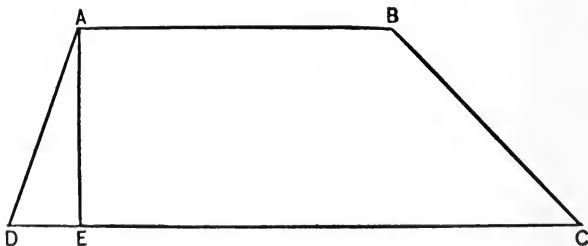

240. To find the area of a parallelogram:

Let perpendiculars be drawn from C and D perpendicular to AB. Then it is evident that the triangles marked a are equal. Adding to each the quadrilateral marked c, it is evident that the parallelogram $ABCD$ is equal to the rectangle upon the base CD.

Hence, to find the measure of the area of a parallelogram, multiply the measure of its base by the measure of its altitude.

$$a = bh.$$

241. A **Trapezoid** is a quadrilateral two of whose sides are parallel.

The parallel sides are called **Bases** and the perpendicular distance between the two bases is called the **Altitude.**

Thus, in the trapezoid, AB and CD are the bases, and AE the altitude.

242. To find the area of a trapezoid:

Let *ABCD* be a trapezoid, and let perpendiculars be drawn through *E* and *F*, the middle points of *AD* and *BC*, to *AB* and *CD*. Then it is evident that the triangles *a* and *a'* are equal, and also *c* and *c'*.

To *a'* and *c'* and also to *a* and *c* add the figure *ABFNOE*, and we have the trapezoid equal to the rectangle *LMNO*.

Again,
$$EF = AB + AL + BM,$$
$$EF = CD - DO - NC.$$

Adding,
$$2\ EF = AB + CD,$$
$$\text{since } AL = DO, \text{ and } BM = NC,$$
$$i.e.\ EF = \tfrac{1}{2}(AB + CD).$$
$$\therefore\ ON = \tfrac{1}{2}(AB + CD).$$

Therefore, to measure the area of the trapezoid, we multiply the measure of *ON*, *i.e.* of $\tfrac{1}{2}(AB + CD)$, by that of the altitude.

Hence the area of a trapezoid is found by multiplying the measure of one-half the sum of its parallel sides by the measure of its altitude.
$$a = \tfrac{1}{2}(b + b')h.$$

243. Find the area of a trapezoid whose parallel sides are respectively 8 in. and 4 in. long and altitude 6 in.

The sum of the bases = 8 in. + 4 in. = 12 in.

Half the sum of the bases = 6 in.

The altitude = 4 in.

∴ the area = 6 × 4 sq. in. = 24 sq. in.

Draw this trapezoid and the equivalent rectangle. Show by measurement that the base of the rectangle is one-half of (8 + 4) in., or 6 in. long.

Exercise 182

1. Find the area of a parallelogram whose base is 6 in. and altitude 3 in. Draw this parallelogram and the equivalent rectangle.

2. Find the areas of the following parallelograms:

Base	Altitude	Base	Altitude
16 in.	9 in.	15 ch.	8 ch.
12 ft.	10 ft.	20 rd.	16 rd.
18 yd.	15 yd.	14 ch.	15 ch.

3. Find the number of acres in each of these parallelograms:

Base	Altitude	Base	Altitude
90 rd.	80 rd.	60 rd.	18 rd.
36 rd.	24 rd.	15 ch.	9 ch.
25 ch.	6 ch.	63 ch.	21 ch.

4. Find the area of a trapezoid whose bases are 24 in. and 20 in. and altitude 12 in.

Draw this trapezoid and the equivalent rectangle on the scale of 4 in. to 1 in.

5. Find the area of each of these trapezoids:

Bases		Altitude	Bases		Altitude
20 in.	16 in.	12 in.	8 rd.	12 rd.	6 rd.
15 ft.	10 ft.	6 ft.	6 rd.	4 rd.	16½ yd.
27 ft.	14 ft.	9 ft.	16 yd.	12 yd.	4 rd.

6. Find the number of acres in each of these trapezoids:

Bases		Altitude	Bases		Altitude
100 rd.	60 rd.	25 rd.	17 ch.	8 ch.	6 ch.
75 rd.	45 rd.	36 rd.	142 yd.	100 yd.	80 yd.
25 ch.	15 ch.	9 ch.	16 ch.	13 ch.	14 ch.

7. Find the side of a square equal in area to a trapezoid, the bases of which are 120 ft. and 60 ft. and the altitude 40 ft.

8. The length of the base of a parallelogram is 45 ft.; the length of the perpendicular on the base from the opposite side is 28 ft. Find the area.

9. The lengths of the parallel sides of a trapezoid are 12 ft. and 17 ft., and the perpendicular distance between these sides is 8 ft. Find its area.

10. The parallel sides of a garden are 84 yd. and 92 yd. respectively, and their perpendicular distance $27\frac{1}{2}$ yd.; what did it cost at $1200 an acre?

244. Let ABC be a triangle, and let the rectangle $BCDE$ be drawn. Then it is evident that the triangles a and a' and c and c' are equal. Hence the triangle ABC is one-half of the rectangle $BCDE$. *Hence, to find the area of a triangle, multiply one-half the measure of the base by that of the altitude.*

$$a = \tfrac{1}{2}\,bh.$$

245. To find the area of a triangle when the lengths of the sides are given:

Find one-half of the sum of the measures of the sides; subtract from this the measure of each side separately. The square root of the product of these four results will give the measure of the area of the triangle.

Find the area of a triangle whose sides are 9 in., 7 in., and 6 in.

$$2 s = 9 + 7 + 6 = 22.$$

$$s = 11, \quad s - 9 = 2, \quad s - 7 = 4, \quad s - 6 = 5.$$

$$\sqrt{11 \cdot 2 \cdot 4 \cdot 5} = \sqrt{440} = 20.9.$$

$$\therefore \text{ the area} = 20.9 \text{ sq. in.}$$

Draw this triangle and see if its area seems to be nearly equal to that of a rectangle 7 in. by 3 in. (*i.e.* 21 sq. in.).

Exercise 183

1. Find the area of each of these triangles:

Base	Altitude	Base	Altitude
12 in.	8 in.	80 rd.	30 rd.
9 ft.	6 ft.	15 ch.	8 ch.
16 yd.	12 yd.	13 ch.	5 ch.

2. Find the area of a triangular piece of park whose base is 8 rd. and altitude 5 rd. What is its value at $12,000 an acre?

3. Find the value of a triangular field, whose base is 75 rd. and altitude 48 rd., at $72 an acre.

4. Find the altitude of a triangle whose base is 12 in. and area 48 sq. in. ($6 x = 48$.)

5. Find the altitude of each of these triangles:

Area	Base	Area	Base
108 sq. in.	18 in.	4 A.	16 ch.
196 sq. in.	28 in.	$2\frac{1}{2}$ A.	10 ch.
2 A.	32 rd.	1 A.	110 yd.

6. Find the base of each of these triangles:

Area	Altitude	Area	Altitude
36 sq. in.	6 in.	$4\frac{1}{2}$ A.	10 ch.
84 sq. ft.	14 in.	$7\frac{1}{2}$ A.	40 rd.
1 A.	16 rd.	$1\frac{1}{2}$ A.	120 yd.

7. Find the area of a triangle whose sides are 10 in., 8 in., and 6 in. Draw this triangle and test your result.

8. Find the areas of the triangles the lengths of whose sides are respectively:

(1) 13 yd., 10 yd., and 13 yd.   (4) 8 in., 7 in., and 5 in.

(2) 13 yd., 24 yd., and 13 yd.   (5) 9 in., 12 in., and 7 in.
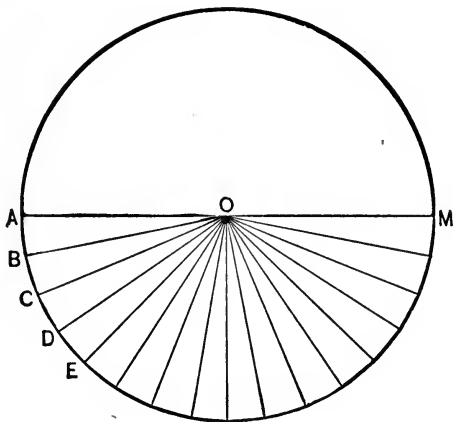
(3) 13 ft., 4 ft., and 15 ft.   (6) 3 in., 5 in., and 7 in.

9. The sides of a triangular piece of park are 17 rd., 15 rd., and 8 rd. Find its value at $2400 per acre.

10. The sides of a triangular field are 41 rd., 40 rd., and 9 rd. Find its value at $96 per acre.

11. Find the number of acres in a triangular field whose sides are 12 ch., 16 ch., and 8 ch.

246. To find the area of a circle:

Draw a circle on cardboard and cut it out — the larger the better. Divide each half of the circle as the semicircle in the figure is divided, the arcs A, B, C, D, etc., being as nearly equal as possible. Cut the circle into two equal parts along the line AOM.

x

Cut along *OB*, *OC*, etc., cutting nearly to the points *B*, *C*, *D*, but not separating the parts entirely at these points. Spread the resulting figure out as in the darker part of the figure below.

Then cut up the other semicircle in the same way; spread open the parts and fit the two semicircles together, as in the figure. The resulting figures will be nearly a rectangle. The smaller the arcs *AB*, *BC*, etc., the more nearly the area will be to a rectangle whose base is equal to one-half the circumference and whose altitude is equal to the radius of the circle.

Hence the measure of the area of a circle is one-half the product of the measures of the circumference and the radius. It may also be expressed thus:

$$\text{The measure of the area} = \tfrac{1}{2}\,cr.$$
$$\text{Again, since } c = 3.1416 \times 2\,r,$$
$$\text{the measure of the area} = 3.1416\,r^2.$$

Both formulas are useful.

The last rule may be read: *The measure of the area of a circle is found by multiplying the square of the measure of the radius by* 3.1416.

On referring to Exercise 52, example 16, it will be seen that *the circumference of a circle is equal to the diameter multiplied by* 3.1416.

Note. — $\tfrac{22}{7}$ may be substituted in the above formula for 3.1416 when desired. Why?

247. (1) Find the circumference of a circle whose diameter is 6 in.

The circumference $= 3.1416 \times 6$ in. $= 18.8496$ in.

(2) Find the diameter of a circle whose circumference is 25 in.

Let $x =$ the number of inches in the diameter.

Then $\qquad 25 = 3.1416\,x$, or $3.1416\,x = 25$.

$\qquad x = 25 \div 3.1416 = 7.9$.

$\qquad \therefore$ the diameter $= 7.9$ in.

(3) Find the area of a circle whose diameter is 12 in.

The area $= 3.1416 \times 6^2$ sq. in.

$= 3.1416 \times 36$ sq. in.

$= 113.09$ sq. in.

Exercise 184

1. Find the circumference of each of the following circles:

Diameter $= 4$ in., 12 in., 25 in., 7 in., 15 in.

Radius $\quad = 5$ in., 7 in., $2\frac{1}{2}$ in., $4\frac{1}{4}$ in., 9 in.

2. Find the diameter of a circle whose circumference is 36 in.

3. Find the diameter of each of the following circles:

Circumference $= 16$ in., 45 in., 24 in., 29 in.

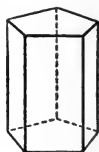4. Find the circumference of a pond whose radius is 12 ft. 6 in.

5. Ladysmith, South Africa, when besieged, was defended by cannon that commanded a radius of 4 mi. Show that the investing line must have been at least 25 mi. in circumference.

6. Find the area of each of the following circles:

Radius $= 2$ in., 5 in., 10 in., 7 in., $2\frac{1}{2}$ in.

7. A cow is tied to a stake with a chain 24 ft. in length. How many square yards can she graze over?

8. A circular pond is 12 yd. in diameter.   Find its area.

9. A circular lake 32 rd. in diameter has a road around it which is 4 yd. wide.   Find the number of square yards in the road.

10. Find the total pressure on a plate 14 in. in diameter, the pressure per square inch being 25 lb.   (Use $\frac{22}{7}$.)

11. Out of a circle of radius 3 ft. is cut a circle of radius 2 ft. Find the area of the remainder.

12. There is a circular fish-pond of 90 ft. radius, surrounded by a walk 5 ft. wide.   Find the area of the walk.

13. Show that the area of a circle whose radius is 35 in. is equal to the sum of the areas of four circles of 10 in., 15 in., 18 in., and 24 in. radius respectively.

14. The diameter of a semicircle is 10 in.   Find its perimeter.

248. A **Prism** is a solid whose bases are equal polygons and lateral faces parallelograms.

Fold a sheet of paper into a prism whose base is (*a*) a triangle ; (*b*) a square ; (*c*) a pentagon.   Unfold the prisms into rectangles.

249. If the lateral surface of a right prism is placed in one plane, it will form a rectangle whose length is the perimeter (*p*) of prism and whose width is the altitude (*h*) of the prism.

To find the area of the lateral surface of a prism multiply the measure of the perimeter of its base by that of its altitude.

$$a = ph.$$

To find the volume of a prism multiply the measure of the area of its base by that of its altitude.

$$v = bh.$$

Exercise 185

1. Find the lateral area of a right prism the perimeter of whose base is 9 in. and altitude 6 in. Make this prism by folding a sheet of paper 9 in. by 6 in. parallel to the 6 in. side.

2. Find the lateral area of each of these right prisms:

Perimeter of base = 24 in., 36 in., 2 ft. 10 in.

Altitude = 8 in., 15 in., 1 ft. 6 in.

3. Find the lateral area of a square prism whose altitude is 16 ft. and the side of whose base is 9 ft. Find its total area.

4. Find the lateral area of a triangular prism whose altitude is 20 in. and the sides of whose base are 13 in., 17 in., and 11 in.

5. Find the total area of a right triangular prism whose altitude is 75 in. and the sides of whose base are 51 sq. in., 45 in., and 24 in. long.

6. Find the total area of a right triangular prism whose altitude is 6 in. and whose base is an equilateral triangle of side 4 in.

7. Find the volume of a prism whose base contains 25 sq. in. and whose altitude is 7 in.

8. Find the volume of each of these prisms:

Area of base = 9 sq. in., 28 sq. in., 3 sq. ft.

Altitude = 4 in., 10 in., 1 ft. 9 in.

9. Find the volume of each of these prisms:

(1) Base a square of side 5 in.; altitude $7\frac{1}{2}$ in.

(2) Base a triangle of base 6 in. and altitude 3 in.; altitude 4 in.

(3) Base a triangle, sides 51 in., 45 in., 24 in.; altitude 2 ft. 2 in.

(4) Base an equilateral triangle, side 10 in.; altitude 7 in.

(5) Base a rectangle, sides 9 in., 6 in.; altitude $5\frac{1}{2}$ in.

(6) Base a parallelogram, length 11 in., width 6 in.; altitude 8 in.

250. If we wrap a rectangular sheet of paper about a right circular cylinder, we find that the area of the curved surface of the cylinder is a rectangle whose base is the circumference of the cylinder and altitude the height of the cylinder.

Hence the measure of the area of the curved surface of a right circular cylinder is equal to the product of the measure of the perimeter of the base and that of the altitude.
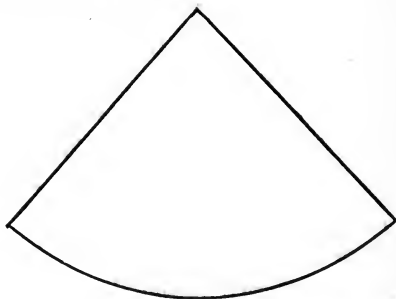
$$a = ph.$$

What must we add to this to find the *total* area of the cylinder?

To find the measure of the volume of a cylinder, take the product of the measures of the area of the base and the altitude.

$$v = bh.$$

251. The curved surface of a right circular cone can be unwrapped into a portion of a circle. Similarly the lateral surface of a regular pyramid can be unwrapped into series of equal triangles whose common altitude is equal to the slant height of the pyramid.

Hence the measure of the lateral surface of a right circular cone or regular pyramid is one-half the product of the measure of the perimeter of its base by that of its slant height.

$$a = \tfrac{1}{2} ps.$$

252. Make a cylinder out of paper and also a right circular cone having the same altitude and base. Fill the cone with some dry material and empty it into the cylinder. Do this three times and the cylinder will be just filled. Hence the volume of a right circular cone is one-third that of a cylinder of equal base and altitude. *Hence, to find the volume of a right circular cone, multiply one-third the measure of the area of the base by the measure of the altitude.*

$$v = \tfrac{1}{3} bh.$$

Use the same rule to find the volume of a pyramid.

253. (1) The length of the radius of the base of a right circular cylinder is 5 in. and its altitude is 8 in. Find its volume and the area of its curved surface.

The measure of the area of the base = $3.1416 \times 25 = 78.54$.

The measure of the volume of the cylinder = $8 \times 78.54 = 628.32$.

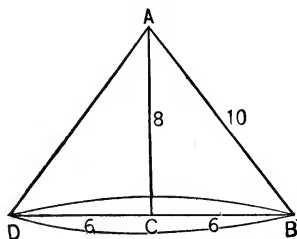$\therefore$ the volume = 628.32 cu. in.

Again,

The measure of the circumference of the base = $3.1416 \times 10 = 31.416$.

The measure of the area of the curved surface = $8 \times 31.416 = 251.328$.

$\therefore$ the area = 251.328 sq. in.

What must we add to this to find the *total* area of the cylinder?

(2) Find the area of the curved surface and also the volume of a cone whose altitude is 8 in. and whose base is 12 in. in diameter.

Since the altitude AC is perpendicular to the diameter BCD, the triangle ACB is a right triangle whose sides are 8 in., 6 in., and 10 in.

Hence AB is 10 in.

Also the circumference of the base = 3.1416 × 12 in. = 37.6992 in.

The measure of the area = $\frac{1}{2}$ × $\frac{10}{1}$ × 37.6992 = 188.496.

$$\therefore \text{ the area} = 188.496 \text{ sq. in.}$$

What must we add to this to find the *total* area of the cone?

Again,

The altitude of the cone = 8 in.

The area of the base = 3.1416 × 6² or 113.0976 sq. in.

The measure of the volume = $\frac{1}{3}$ × 8 × 113.0976 = 301.5936.

$$\therefore \text{ the volume} = 301.59 \text{ cu. in.}$$

Exercise 186

1. Find the area of the curved surface of a right circular cylinder, the circumference of whose base is 12 in. and altitude 8 in.

2. Find the area of the curved surface of each of these right circular cylinders:

(*a*) Circumference of base = 10 in., 15 in., 18 in.

Altitude = 9 in., 8 in., 12 in.

(*b*) Diameter of base = 6 in., 16 in., 7 in.

Altitude = 4 in., 14 in., 5 in.

3. Find the area of: (1) The curved surface of a right circular cylinder of altitude 12 in. the diameter of whose base is 14 in. (2) Both ends. (3) The total area. (Use $\frac{22}{7}$.)

4. Find the total area of a right circular cylinder the radius of whose base is 5 in. and altitude 8 in.

5. Find the volume of a cylinder the area of whose base is 18 sq. in. and altitude 8 in.

6. Find the volume of each of these right circular cylinders:

(*a*) Area of base = 16 sq. in., 27 sq. in., 135 sq. in.

Altitude = 5 in., 9 in., 15 in.

(b) Radius of base = 10 in., 7 in., 9 in., 8 in.

Altitude = 10 in., 4 in., 6 in., 8 in.

7. Find the area of the lateral surface of a right circular cone the circumference of whose base is 14 in. and slant height 11 in.

8. Find the area of the lateral surface of each of these right circular cones:

(a) Circumference of base = 26 in., 17 in., 1 ft. 6 in.

Slant height = 20 in., 8 in., 9 in.

(b) Radius of base = 5 in., 8 in., $\frac{1}{6}$ yd.

Slant height = 6 in., 11 in., $\frac{1}{4}$ yd.

9. Find the area of: (1) The lateral surface of a right circular cone the radius of whose base is 6 in. and slant height 8 in. (2) The area of its base. (3) Its total area.

10. Find the total area of the surface of a right circular cone the radius of whose base is 5 in. and slant height 12 in.

11. Find the area of the lateral surface of a regular pyramid whose slant height is 12 in. and base is an equilateral triangle, one of whose sides is 8 in. long.

12. Find the area of the convex surface of a regular pyramid whose slant height is 8 in. and whose base is a square, its side being 4 in.

13. Find the total area of the pyramid in the preceding example.

14. Find the volume of a right circular cone the area of whose base is 16 sq. in. and altitude 6 in.

15. Find the volume of each of these right circular cones:

(a) Area of base = 24 sq. in., 62 sq. in., 79 sq. in.

Altitude = 7 in., 15 in., 14 in.

(b) Radius of base = 6 in., 9 in., 10 in., 15 in.

Altitude = 8 in., 7 in., 14 in., 25 in.

16. Find the volume of a square pyramid whose altitude is 18 in. and the edge of whose base is 6 in.

17. Find the volume of each of these square pyramids:

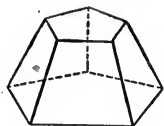Altitude = 16 in., 12 in., 25 in., 2 ft. 6 in.

Side of square = 9 in., 8 in., 16 in., 1 ft. 2 in.

18. Find the volume of a pyramid whose altitude is 15 in. and base an equilateral triangle of side 10 in.

19. The radius of the base of a right circular cone is 6 in. and its altitude 8 in. Find the area of its cone surface.

20. The radius of the base of a right circular cone is 8 in. and its slant height 17 in. Find its volume.

21. Find the volume of a pyramid whose altitude is 15 in. and base (1) a rectangle whose sides are $7\frac{1}{2}$ in. and 6 in.; (2) a triangle whose sides are 10 in., 24 in., and 26 in. long.

254. That portion of a cone included between its base and a plane parallel to its base is called a *frustum* of the cone.

That portion of a pyramid between its base and a plane parallel to its base is called a *frustum* of the pyramid.

255. *To find the lateral area of the frustum of a regular pyramid or of a right circular cone, multiply one-half the measure of its slant height by that of the sum of the perimeters of its bases.*

$$a = \tfrac{1}{2}(p + p')s.$$

To find the volume of a frustum of a regular pyramid or of a right circular cone, multiply one-third the measure of its alti-

*tude by the sum of the areas of the two bases and the square
root of their product.*

$$v = \frac{h}{3}(b + b' + \sqrt{bb'}).$$

Exercise 187

1. Find the area of the lateral surface of the frustum of a
right circular cone, the perimeters of whose bases are 27 and 23 in.
and slant height 15 in.

2. Find the area of the lateral surfaces of each of these frus-
tums of pyramids:

$$p = 12 \text{ in.}, \quad 25 \text{ in.}, \quad 1 \text{ ft. } 10 \text{ in.}, \quad 2 \text{ ft. } 3 \text{ in.}$$
$$p' = 18 \text{ in.}, \quad 16 \text{ in.}, \quad 1 \text{ ft. } 3 \text{ in.}, \quad 1 \text{ ft. } 7 \text{ in.}$$
$$s = 8 \text{ in.}, \quad 10 \text{ in.}, \quad 1 \text{ ft. } 2 \text{ in.}, \quad 2 \text{ ft. } 1 \text{ in.}$$

3. Find the lateral area of a frustum of a right circular cone
whose slant height is 12 in. and whose bases are circles whose
radii are 6 in. and 4 in.

4. Find the entire area of the frustum in example 3.

5. Find the lateral area of the frustum of a regular pyramid
whose slant height is 10 in. and bases equilateral triangles of side
8 in. and 6 in.

6. Find the entire area of the frustum in example 5.

7. Find the volume of the frustum of a pyramid whose bases
contain 2 sq. ft. and 8 sq. ft. and whose altitude is 6 ft.

8. Find the volume of the frustum of a square pyramid, the
sides of whose bases are 4 ft. and 5 ft. and altitude 12 ft.

9. Find the volume of a frustum of a cone whose bases con-
tain 27 sq. in. and 12 sq. in. and whose altitude is 15 in.

10. Find the volume of the frustum of a right circular cone,
the radii of whose bases are 10 ft. and 20 ft. and whose altitude
is 24 ft.

11. Find the volume of the frustum of a triangular pyramid, the sides of whose bases are 3 ft., 4 ft., 5 ft., and 6 ft., 8 ft., 10 ft., and whose altitude is 7 ft.

12. Find the number of cubic feet in a log, the radii of whose ends are 8 in. and 1 ft. and length 30 ft.

256. The measure of the area of the surface of a sphere is equal to four times the square of the radius multiplied by 3.1416.

$$a = 4 \times 3.1416 \ r^2.$$

257. If we imagine a sphere to be divided into a large number of small cones, as in § 246 we divided the circle into triangles, the centre of the sphere being the vertex of each cone, and a small portion of the circumference being its base, we can think of the volume of the sphere as being equal to the sum of the volumes of the cones. The altitude of each cone is equal to the radius of the sphere, and the total area of their bases is equal to the area of its surface. Hence the volume of the sphere is given by the formula:

$$V = \tfrac{1}{3} r (4 \times 3.1416 \times r^2)$$
$$= \tfrac{4}{3} \times 3.1416 \ r^3.$$

Hence the measure of the volume of the sphere is $\tfrac{4}{3}$ of 3.1416 times the cube of the measure of the radius.

258. (1) Find the surface of a sphere whose radius is 6 in.

The measure of the area $= 4 \times 3.1416 \times 6^2 = 452.3904$.

∴ the area $= 452.39$ sq. in.

(2) Find the volume of a sphere whose diameter is 8 in.

The measure of the volume $= \tfrac{4}{3} \times 3.1416 \times 4^3 = 268.0832$.

∴ the volume $= 268.08$ cu. in.

Exercise 188

1. Find the surface of a sphere whose radius is 3 in.

2. Find the surface of a sphere 12 in. in diameter.

3. Find the volumes of the spheres given in examples 1 and 2.

4. Find the surface of a sphere 5 ft. in diameter.

5. Find the volume of a sphere whose diameter is 16 ft.

6. Place a croquet or base ball between two chalk boxes. Place a foot measure in line with one edge of each box. What is the diameter of the ball? What is the area of its surface? What is its volume?

7. With a pair of compasses draw a circle with the diameter found in example 6. Cut out this circle and pass the ball through the hole.

8. If the pressure of the air is equal to 15 lb. a square inch, what is the pressure on the surface of a sphere 6 in. in diameter?

Exercise 189

1. Find the cost of a field 25 rd. long and 20 rd. wide at $96 an acre.

2. What is the area of a parallelogram 7 ft. 6 in. long and 3 ft. 4 in. wide?

3. The base of a triangle is 15 ft. 9 in. and the altitude 12 ft. 4 in. Find its area.

4. Find the number of square yards in a triangle whose sides are 13 ft., 14 ft., 15 ft.

5. The perimeter of a flower bed in the form of an equilateral triangle is 27 ft. What is its area?

6. How many acres are there in a square field each side of which is 330 yd.?

7. What is the area of a trapezoid whose parallel sides are 4 ft. 6 in. and 8 ft. 3 in., and whose altitude is 5 ft. 3 in.?

8. What is the value of a field in the form of a trapezoid whose parallel sides are 8.6 ch. and 4.4 ch., and whose altitude is 5.4 ch., at $75 an acre?

9. A cow is fastened to a stake by a chain 40 yd. long. How many square yards of grass more than an acre can she feed on?

10. What is the side of a square equal in area to a circle whose radius is 100 ft.?

11. The sides of a triangular wheat field are 275 yd., 220 yd., 165 yd. Find the value of wheat grown on it, the crop averaging 25 bu. to the acre and worth 80 ¢ per bushel.

12. The slant height of a triangular pyramid is 10 ft. and each side of the base 2 ft. Find the total area of its surface.

13. What is the total area of the surface of a right cone whose slant height is 20 in. and whose base is a circle of radius 10 in.?

14. What is the area of the lateral area of a right cone whose altitude is 15 in. and the diameter of whose base is 16 in.?

15. Find the total area of the surface of the frustum of a square pyramid, each side of the bases being, respectively, 10 in. and 6 in., and the slant height 15 in.

16. Find the total area of the surface of a frustum of a cone, whose greater diameter is 18 in. and less diameter 8 in., and slant height 24 in.

17. Find the area of the surface of a sphere whose diameter is 22 in.

18. Find the area of the surface of the earth, supposing it a sphere of diameter 7960 mi.

19. Find the entire surface of a cylinder whose height is 10 ft. and base a circle of 5 ft. diameter.

20. Find the volume of a square prism whose length is 5 ft. 6 in., each side of its base being 1 ft. 4 in.

21. How many gallons of water will a prismatic vessel contain, its base being a rectangle 14 in. by 6 in., and its altitude 22 in.?

22. The length of a cylindrical piece of timber is 18 ft. and the diameter of its base is 1 ft. Find its volume.

23. Find the volume of a square pyramid, each side of whose base is 4 ft. and height 12 ft.

24. A conical church spire is 100 ft. high and the diameter of its base is 18 ft. Find its volume.

25. What is the volume of a frustum of a square pyramid whose height is 6 ft., the sides of the greater end being 7 in. and of the smaller 5 in. ?

26. Find the volume of a squared piece of timber, its length being 18 ft., each side of the greater end being 18 in. and of the small end 12 in.

27. Find the volume of a tapering round piece of timber whose length is 10 ft. and the diameters of the ends 8 in. and 4 in. respectively.

28. Find the volume of a sphere 2 ft. in diameter.

29. A vessel in the form of a right circular cone is 4 ft. deep, and the diameter of its base is 3 ft. Find how many gallons of water it will contain.

30. A cylindrical cistern, 8 ft. in diameter and 6 ft. deep, is $\frac{2}{3}$ full of water. Find the number of gallons of water in the cistern.

31. An excavation 2 yd. deep, in the form of the frustum of a square pyramid, has its upper base 8 yd. long, and its lower base 4 yd. long. Find the number of wagon loads of earth required to fill it.

CHAPTER XIX

THE METRIC SYSTEM OF WEIGHTS AND MEASURES

259. The French or **Metric System** of Weights and Measures is based upon the decimal system. It is used in scientific treatises, and has been adopted by most of the nations of Europe and South America. It is also in partial use in the United States and Canada.

260. The fundamental unit of the metric system is the **Meter**, which is 39.37 in. long. The original standard meter is a platinum rod, called the French Standard Meter, which is deposited in the Archives at Paris.

FOUR INCHES IN SIXTEENTHS OF AN INCH
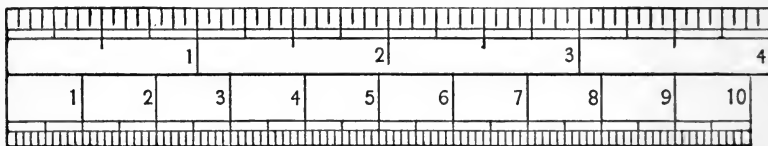
The length of the measure in the diagram is $\frac{1}{10}$ of a meter, and is called a decimeter. It is divided into 10 equal parts, each of which is called a centimeter. Each of these is divided into 10 equal parts, each of which is called a millimeter.

261. In order that pupils may study this system of weights and measures to the best advantage, the school should be provided with a system of metric weights and measures, and each pupil with a foot rule on which the decimeter, centimeter, and millimeter are marked.

An intelligent study of the system can, however, be made if the teacher has a metric stick and a liter for reference.

262. The names of the higher or lower units in the metric system are formed by attaching certain prefixes to the names of the standard units, thus :

> *Deca* signifies   10 times the unit.
> *Hecto* signifies   100 times the unit.
> *Kilo*   signifies 1000 times the unit.

Deci signifies the   10th part or   .1 of the unit.
Centi signifies the   100th part or   .01 of the unit.
Milli signifies the 1000th part or .001 of the unit.

Units of Length

1 *millimeter* (mm.) = .001 meter
1 *centimeter* (cm.) = .01 meter
1 decimeter (dm.)   = .1 meter
1 *meter* (m.)   = *standard unit*
1 decameter (Dm.) = 10 meters
1 hectometer (Hm.) = 100 meters
1 *kilometer* (Km.)   = 1000 meters
1 myriameter (Mm.) = 10,000 meters

The units in common use are the *millimeter*, *centimeter*, *meter*, and *kilometer*.

263. Write out the table of the units of length, thus :

> (*a*) 10 millimeters = 1   centimeter, etc.
>
> (*b*)   1 millimeter = $\frac{1}{10}$ centimeter, etc.

Exercise 190

1. Measure with a yardstick in the school yard a distance of 11 yd. Measure along the same distance 10 times with the meter. What is the difference in inches ?

Y

2. Reduce 11 yd. and also 10 m. to inches, and verify the result obtained in the first question.

3. Measure the length of the schoolroom in meters and decimals of a meter.

4. Find the hypotenuse of a right triangle whose sides are (1) 6 m. and 8 m., (2) 24 cm. and 45 cm.

5. Librarians frequently use the centimeter as the unit in registering the heights of books. Express in centimeters the height of your (*a*) Arithmetic, (*b*) History, (*c*) Geography, (*d*) and also of several other books.

6. Measure and express in terms of the centimeter the distances between the lines on ruled paper.

7. Measure and express in terms of the centimeter the height of the schoolroom thermometer.

8. How many centimeters are there in a full line of this book ?

9. Press tightly together the leaves of this book. Make a layer just 1 cm. thick. Count the leaves and find the thickness of one leaf as a decimal of a millimeter.

10. Cut a slit 2 mm. wide, by 3 cm. long, in a sheet of paper.

11. How many millimeters are there in the width of your pencil ?

12. Find the number of inches in 1 Km., reduce the result to a decimal of a mile, and show that 1 Km. is nearly equal to $\frac{5}{8}$ of a mile.

13. If a train travels at the rate of 20 m. a second, what is the rate in kilometers per hour ?

14. Show that 5 in. are very nearly equal to 127 mm.

15. For what do we use the inch, the yard, and the mile ? For what are the millimeter, the centimeter, the meter, and the kilometer respectively used ?

UNITS OF AREA

264. The principal units for measuring land are the square meter, called the **Centare** (ca.), the square decameter called the **Are** (a.), and the square hectometer called the **Hectare** (ha.).

---

1 sq.
cm.
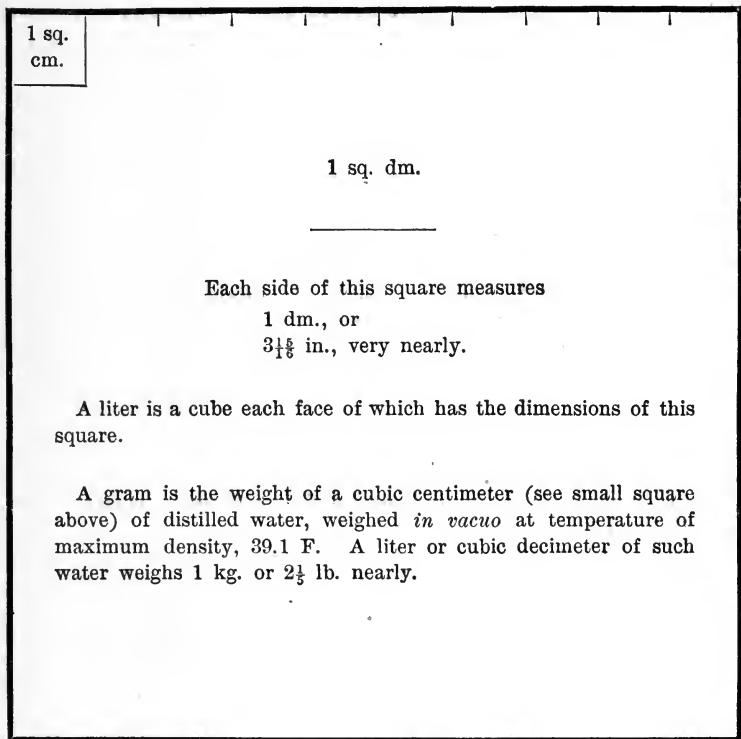
1 sq. dm.

Each side of this square measures
    1 dm., or
    $3\frac{15}{16}$ in., very nearly.

A liter is a cube each face of which has the dimensions of this square.

A gram is the weight of a cubic centimeter (see small square above) of distilled water, weighed *in vacuo* at temperature of maximum density, 39.1 F. A liter or cubic decimeter of such water weighs 1 kg. or $2\frac{1}{5}$ lb. nearly.

---

265. How many units of length are there in the side of a square meter, the decimeter being the unit?

How many units of area are there in a square meter, the square decimeter being the unit?

How many units of area are there in a square decimeter, the square centimeter being the unit of area?

What part of a square meter is a square decimeter?

Write a square decimeter as a decimal of a square meter.

Write a square centimeter as a decimal of a square decimeter. As a decimal of a square meter. What is the ratio of each unit of area in the metric system to the next smaller, and also to the next higher?

Units of Area

1 square millimeter (q. mm.)	= .000001 of a square meter
1 *square centimeter* (q. cm.)	= .0001 of a square meter
1 square decimeter	= .01 of a square meter
1 *square meter* (q. m.)	= *standard unit*
1 square decameter	= 100 square meters
1 square hektometer	= 10,000 square meters
1 *square kilometer* (q. Km.)	= 1,000,000 square meters
1 *centare* (ca.)	= .01 *are* (a.)
1 *hectare* (ha.)	= 100 *ares*

Note that the square meter is 1⅕ times as large as the square yard.

Exercise 191

1. Cut out of paper 1 q. dm. and also 1 q. cm. What is the ratio of the two areas?

2. Draw 1 sq. ft. and measure it with 1 q. dm. of paper as the unit of area.

3. A square decimeter contains .10764 sq. ft. Find the number of square decimeters contained in a square foot, correct to two decimal places, and compare the result with that obtained in example 2.

4. Make a drawing of 1 q. m., and draw 1 sq. yd. within it.

5. Mark out an *are* on the school ground.

6. State the table of area, expressing each unit of area as equal to 100 times the next lower.

7. Find the area of a page of this book in square centimeters.

UNITS OF VOLUME

266. The principal units of volume are the cubic meter, also called the **Stere**, and the cubic decimeter, called the **Liter**.

267. How many units of length are there in a side of a meter, the decimeter being the unit?

How many units of volume are there in the cubic meter or stere, the cubic decimeter or liter being the unit?

What is the ratio of each unit of volume in the metric system to the next smaller unit? To the next larger?

<div align="center">UNITS OF VOLUME</div>

1 cubic millimeter (c. mm.)	= .000000001 of a cubic meter
1 *cubic centimeter* (c. cm.)	= .000001 of a cubic meter
1 cubic decimeter	= .001 of a cubic meter
1 *cubic meter* (c. m.)	= *standard unit*

Exercise 192

1. Make a liter out of paper.

2. Fill a quart, liquid measure, with sand and empty it into your liter. Which of the two measures is larger?

3. Fill a liter with sand and empty it into a quart, dry measure. Which of the two is larger?

4. Fill a gallon measure, using the liter as a dipper, and note how many liters are equivalent to the gallon.

5. 1 liter is equal to .264 gal.; find, correct to two decimal places, the number of liters in a gallon. Compare this result with that obtained in example 4.

6. State some purposes for which the liter is used.

7. Make a cubic centimeter out of paper.

8. Make the necessary measurements and compute the volume of the room in cubic centimeters.

9. Make the necessary measurements and compute the volume of a box in liters.

10. Express as a decimal part of a cubic meter the volume of a beam 3 m. long, 10 cm. wide, and 5 cm. thick.

11. A cylindrical vessel having a base of a square meter is filled with water to the depth of 2 m. How many liters of water does it contain?

12. How many liters of water may be held by a vessel measuring $25 \times 35 \times 75$ cm.?

13. What will it cost to build a wall 1 Hm. long, $\frac{1}{2}$ dm. thick, and 1 m. high, at $5 a cubic meter?

UNITS OF WEIGHT

268. The principal units of weight are the **Gram** and the **Kilogram**.

The **Gram** is the weight of a cubic centimeter of distilled water at 40°, at which temperature water is at its maximum density.

A nickel weighs 5 g.

A liter of distilled water at 40° weighs 1 kg. The kilogram is nearly equal to $2\frac{1}{5}$ lb. Avoirdupois.

A cubic meter of water at 40° weighs a metric ton (1000 kg.).

Exercise 193

1. One gram is equal to 15.432 gr. Show that 1 kg. is approximately equal to $2\frac{1}{5}$ lb. Avoirdupois.

2. A cubic meter of distilled water at 40° weighs how many kilograms? If 1 kg. is equal to $2\frac{1}{5}$ lb. Avoirdupois, how many pounds does 1 c. m. of water weigh?

3. Show that a metric ton weighs about 10% more than our short ton.

4. If sulphuric acid is 1.8 times as heavy as water, what weight of the acid will a two-liter bottle contain?

5. What part of a liter is 750 g. of water?

6. What is the weight of 1 deciliter of water?

7. If alcohol is 80% as heavy as water, what will 375 c. cm. of alcohol weigh?

8. If 20 c. cm. of lead weighs 227 g., what is the ratio of the weight of lead to that of an equal volume of water?

9. If a quantity of iron weighs 7.8 times as much as an equal quantity of water, what is the weight of an iron bar $75 \times 4 \times 3$ cm.?

10. A body weighing 512 g. in air weighs 428 g. in water. What per cent of its weight is lost?

11. A liter flask was two-fifths filled with water; the remaining space being filled with sand, the weight was found to be 2050 g. Required the weight of a liter of sand.

12. If the pressure of the air on the surface of a body is 1 kg. to the square centimeter, what is the pressure of the air on the surface of a sphere whose radius is 10 cm.?

13. A cubical block of ice measures 3 dm. along its edge. What will be its weight if ice weighs 94% as much as an equal volume of water?

14. What is the weight of air in a room 5 m. long, 3 m. wide, 4 m. high, if 1 c. dm. of air weighs .0018 kg.?

15. Emperor William of Germany was struck in the face with a fishplate weighing 550 g. Find its weight in pounds.

CHAPTER XX

Miscellaneous Exercise 194

1. Write the following in figures:

(*a*) Fifty thousand nine hundred nine.

(*b*) Nine hundred thousand ninety.

(*c*) Six hundred fifty thousand seven hundred.

(*d*) Eight hundred seven thousand eight.

(*e*) Seven hundred seventy thousand sixty-seven.

(*f*) Nine million ninety thousand ninety-nine.

(*g*) Eighty million nine hundred thousand thirty.

(*h*) Nine hundred seventy million eight hundred eighty-seven thousand.

(*i*) Six hundred seventeen million ninety-three.

(*j*) Nine hundred nineteen thousand four hundred eleven.

(*k*) Six hundred four thousand twenty-five.

2. Write in figures:

Twenty-five thousand four hundred ninety; ninety-nine thousand nine hundred seventeen; nine hundred seven thousand six hundred six; one million; MDCCCXCV. And in words: 9009; 16,060; 7018; 207,509; 75,115.

3. (*a*) Define and give examples of quantity, unit, and number.

(*b*) Explain the basis of our system of numeration.

4. Write in figures (placed for addition): Nine hundred nineteen; three hundred eleven; seven hundred seventy; eight hundred ninety-seven; six hundred eight; three hundred nine; XCVII; LXVII; CXIX; CDL; and DCXL.

5. Add:

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4567890123
5678901234
6789012345
7890123456
8901234567
0912345678
6598695326
8396876549
7788995566
3453453456
```

6. Write down neatly the following statement of six weeks' cash receipts; add the amounts vertically and horizontally, and prove the correctness of the work by adding your results:

	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Total
1st	$28.79	$34.71	$35.33	$30.10	$27.97	$47.81	
2d	23.87	30.03	29.38	33.84	26.77	48.77	
3d	16.99	27.09	28.77	30.16	24.95	43.07	
4th	29.13	33.72	30.81	39.17	28.47	50.05	
5th	18.47	32.29	26.73	34.45	28.88	54.39	
6th	19.02	27.06	29.04	29.89	29.51	61.93	
Total							

7. Solve, as in example 6:

	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Total
1st	$65.95	$24.89	$79.79	$40.78	$37.59	$89.61	
2d	58.71	41.65	24.67	94.26	70.26	42.51	
3d	47.58	99.57	50.60	80.71	91.82	89.76	
4th	29.69	70.80	87.91	74.93	36.63	21.90	
5th	81.45	56.93	54.82	96.57	12.72	96.67	
6th	42.63	68.77	81.79	60.86	31.87	75.82	
Total							

8. Mr. Jones bought one house for $865 and another for $984, and sold them both for $1900. How much did he gain?

9. Juliette has $149, Florence has $87 more than Juliette, and Elizabeth has $115 more than both. How many dollars has Elizabeth?

10. Thanksgiving Day, 1901, the Michigan University football team increased its record of 451 points by 50. Find its record for the season of 1901.

11. Two men together receive $97.75, but one receives $18.25 more than the other. How much does each receive?

12. A and B start together and walk in the same direction, A at the rate of 4 mi. an hour, and B at the rate of 3 mi. an hour. At the end of 7 hr. A turns and goes back. How many miles will B have gone when he meets A?

13. In a factory, 12 men, 16 women, and 30 boys are employed. At the end of a week they receive $330. A man is paid as much as 2 women, and a woman as much as 3 boys. What is the share of each?

14. A man bought a number of cows for $1080; he sold half of them for $810, thereby gaining $15 on each one sold. What did each cow cost?

15. A clerk received a salary of $650 a year. He spent 50 ¢ a day the first year, $4 a week the second year, and $22 a month the third year. How much did he save in three years?

16. The subtrahend is 9564, the remainder is 1965. What is the minuend? The multiplier is 96 and the product is 82,848. What is the multiplicand?

17. The dividend is 1800, the quotient is 17, and the remainder 66. What is the divisor?

18. How many times can 506 be subtracted from the product of 6072 and 13,986?

19. The quotient of a division is 834. What quotient would have been obtained if both dividend and divisor had been first multiplied by 13? Why?

20. Subtract $847\frac{19}{24}$ from $1003\frac{5}{12}$, explaining fully each step.

21. Simplify $\frac{1}{2} - \frac{2}{3}$ of $\frac{5}{8} + \frac{7}{78}$, and find how many times the result is contained in $\frac{3}{8} \div (\frac{7}{9}$ of $\frac{3}{14} - \frac{1}{8})$.

22. Divide the sum of $\frac{2}{5}$ of $8\frac{1}{3}$ and $2\frac{1}{7}$ of $5\frac{5}{6}$ by the difference between $\frac{3}{7}$ of $3\frac{1}{2}$ and $\frac{1}{2}$ of $\frac{1}{3}$ of $2\frac{2}{3}$.

23. Prove, (1) $\frac{2}{3}$ of $\frac{2}{5} = \frac{4}{15}$; (2) $\frac{2}{5}$ of $\frac{2}{3} = \frac{2}{3}$ of $\frac{2}{5}$.

24. Simplify $3\frac{1}{2} + \dfrac{2\frac{1}{2} - \frac{1}{8}}{2\frac{1}{2} + \frac{1}{8}} - \dfrac{7}{10}$ of $3\frac{1}{3} - \dfrac{5}{6}$.

25. A boy's age now is $\frac{1}{5}$ of his father's. In 6 yr. it will be $\frac{1}{3}$ his father's present age. How old is he?

26. A house and lot are together worth $\$2100$; $\frac{1}{4}$ of the value of the house is equal to $\frac{1}{3}$ of the value of the lot. Find the value of each.

27. The circumference of a wheel is $\frac{22}{7}$ of its diameter. Find the diameter of a wagon wheel which makes 360 revolutions in going a mile.

28. A man owned a $\frac{3}{5}$-interest in a mill, and sold $\frac{3}{7}$ of his interest to one man, and $\frac{1}{9}$ of his interest to another. What part of the mill did each of the three men then own?

29. If to a certain number its $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$ be added, the sum will be 122; required the number.

30. Find the number which is 207 more than the sum of $\frac{1}{5}$ and $\frac{1}{7}$ of itself.

31. A man spent $\frac{5}{19}$ of his money for a house, $\frac{3}{7}$ of the remainder for cattle, and the rest for a farm. If the farm cost him $\$357$ less than the house and cattle together, what did he pay for all?

32. A legacy of $\$9500$ is to be divided among A, B, and C, so that A will get $\frac{5}{19}$ of the whole, and B will get $\frac{3}{4}$ as much as C. Find the shares of each.

33. A man spent $\frac{2}{7}$ of his money for provisions, $\frac{5}{8}$ of the remainder for clothing, $\frac{2}{15}$ of the remainder for charity, and had $\$9.10$ left. How much did he have at first?

34. John Smith sells a merchant 752 lb. of cheese at $11\frac{3}{4} ¢$ per pound, and receives the following goods in exchange:

<table>
<tr><td>11 yd. silk @ $2.25;</td><td>96 lb. nails @ $3\frac{5}{8} ¢$;</td></tr>
<tr><td>400 lb. sugar @ $4\frac{7}{8} ¢$;</td><td>56 yd. gray cotton @ $9\frac{3}{8} ¢$;</td></tr>
<tr><td>12 lb. raisins @ $11 ¢$;</td><td>11 yd. white cotton @ $10 ¢$;</td></tr>
</table>

3 pr. gloves @ $75 ¢$.

Find the balance due John Smith.

35. A man owns a horse and saddle; $\frac{1}{4}$ of the value of the horse is equal to 4 times the value of the saddle; the horse and saddle together are worth $170. Find the value of each.

36. A man bought a horse and carriage for $280, and $\frac{2}{5}$ of the cost of the carriage was equal to $\frac{2}{3}$ of the cost of the horse. What was the cost of each?

37. Divide the product of .037 and .0025 by the sum of .9, .02, and .005.

38. Divide 6 by .000725, correct to four decimal places.

39. Add together 1.302, 3.2589, and 40.93. Multiply the sum by .00297 and divide the product by 90.09.

40. Multiply 350.4 by .0105 and divide the product by .0000219.

41. What decimal must be taken from the sum of $69\frac{1}{5}$, 8.2, 5.445, .065, and $20\frac{1}{12}$, so that it will contain 6.05 an exact number of times?

42. A drover lost .065 of his flock by wolves, .105 by disease, and .27 by theft. He then sold .75 of what remained, and had 280 sheep left. Find the number in his original flock.

43. Find the amount of the following bill:

> 1328 ft. siding, at 1.62\frac{1}{2}$ per C.;
> 48,480 cu. ft. timber, at 59.37\frac{1}{2}$ per M.;
> 7400 fence rails, at $7.75 per C.;
> 8400 fence pickets, at $15.00 per M.;
> 5680 lb. hay, at $12.50 per T.

44. A cooper paid $78.32 for 16,488 bbl. staves. Required the price per M.

45. A rectangular field is 7 ch. 75 l. long and 4 ch. 87½ l. wide. How many rods of fencing are required to enclose it?

46. How many miles of road, 3 rd. wide, will contain 8 A. of land?

47. Make a drawing that will show the number of square yards in a square rod. (Scale 1 yd. to 1 in.)

48. Find the value of a piece of land 20 ft. × 40 rd., at $1000 per acre.

49. A certain map is drawn on a scale of 8 mi. to an inch. On this map the township of Scott measures $1\frac{5}{16}$ in. in length and $1\frac{1}{8}$ in. in width. How many acres does it contain?

50. Find the expense of sodding a plot of ground which is 40 yd. long and 100 ft. wide, with sods each 1 yd. in length and 1 ft. in breadth, the sods, when laid, costing 75¢ per hundred.

51. A floor 16 ft. 8 in. by 14 ft. 2 in. is to be laid with square tiles. Find the dimensions of the largest tiles that can be used without cutting or fitting.

52. Find the cost of papering a room 24 ft. long, 21 ft. wide, 12 ft. high, at 25¢ a roll, 12 yd. long and 21 in. wide.

53. How much will it cost to plaster the walls and ceiling of a room 15 ft. long, 12 ft. wide, and 11 ft. high, at $32\frac{1}{2}$¢ per square yard?

54. A room 18 ft. by 16 ft. is carpeted with carpet $\frac{3}{4}$ yd. wide, and the smallest possible number of yards of the carpet is used. Find (*a*) the number of breadths, (*b*) the number of yards.

55. How many thousand shingles, 18 in. long and 4 in. wide, lying $\frac{1}{3}$ to the weather, are required to shingle the roof of a building 54 ft. long, with rafters 22 ft. long, the first row of shingles being double?

56. A schoolroom is 30 ft. long, 24 ft. wide, and 10 ft. high above the wainscoting. The trustees pay $ 20 per thousand for a new floor, $ 15 per thousand for a new board ceiling, 10 ¢ per square yard for painting the ceiling, 4 ¢ per square yard for tinting the walls, and $ 2 per day for 6 da. labor. Find the total cost.

57. A cubical cistern is 5 ft. deep. How many gallons of water will it hold when it is $\frac{4}{5}$ full?

58. How many cubical blocks, each edge of which is $\frac{1}{3}$ ft., are equivalent to a block of wood 8 ft. long, 4 ft. wide, and 2 ft. thick?

59. If the ceiling of a square room is 15 ft. high, how many square feet of floor must it have in order that 50 pupils and the teacher may each have 300 cu. ft. of air?

60. Four-foot wood piled $5\frac{1}{2}$ ft. high requires how many feet in length of the pile for $2\frac{1}{2}$ cd. ?

61. What is the value of a pile of wood 360 ft. long, 12 ft. wide, 6 ft. high, at $ 3.20 per cord?

62. A square plot of ground that contains $\frac{9}{40}$ A. is covered with cordwood (4 ft. long) to an average height of 12 ft. What is the wood worth at $ 4.12 per cord?

63. Required the cost of 35 pieces of scantling 18 ft. long, 4 in. wide, and 2 in. thick, at $ 14 per thousand, board measure.

64. How many board feet are there in 12 scantlings 16 ft. by 4 in. by 2 in. ?

65. It is required to build a sidewalk $\frac{1}{4}$ mi. in length, 8 ft. wide, and 2 in. thick, supported by three continuous lines of scantlings 4 in. square. What will the lumber cost at $ 17 per thousand?

66. Find the value of the following lumber at $ 15 per thousand: —

20 pieces 2 × 4, 18 ft. long;

20 pieces 4 × 4, 12 ft. long;

20 pieces 3 × 10, 16 ft. long.

67. A farmer sold a lot of barley, weighing 2712 lb., when barley was 40 ¢ per bushel. In weighing the grain, the dealer made a mistake and took it as rye, and paid for it at 49 ¢ per bushel. How much did the farmer gain or lose by the mistake?

68. The weight of a cubic foot of water is $62\frac{1}{2}$ lb., and 1 gal. contains 231 cu. in. Find the weight in ounces of 1 pt. of water.

69. A lake whose area is 45 A. is covered with ice 3 in. thick. Find the weight of the ice in tons, if 1 cu. ft. weighs 920 oz. Avoir.

70. In what time would a field, 80 by 60 rd. pay for under-draining lengthwise, at 2 ¢ per foot, if the field yields 2 bu., at 66 ¢, per acre more than before draining? The drains are 4 rd. apart, and the first drain runs down the centre of the field.

71. Find the amount of the following bill:

June 1, 1896, G. Murray & Co. sold to John Scott, 4886 bu. 36 lb. wheat @ 58 ¢ per bushel, 4532 lb. peas @ 52 ¢ per bushel, 38 bu. 3 pk. barley @ 54 ¢ per bushel, 465 lb. flour @ $ 1.50 per hundredweight, 4685 lb. bran @ $ 15 per ton. Write out a receipt in full for payment of account, June 26.

72. Find the length of the shortest line that can be exactly measured by a yard measure, a ten-foot pole, or a two-rod chain.

73. Required the cost of 1 doz. silver spoons, each weighing 18 pwt. 18 gr. at $1.15 per ounce.

74. Reduce 7 gal. 3 qt. 1 pt. to the fraction of a barrel.

75. I sow 11 bu. 2 pk. 4 qt. of wheat, and raise therefrom 215 bu. 2 qt. How much is the average yield per bushel of seed?

76. The running time of the Empire State Express from New York to Buffalo is 8 hr. 30 min., and the distance is 440 mi. If stops of 5 min. each are made at Albany, Utica, Syracuse, and Rochester, what is its average speed per hour?

77. Find (*a*) the exact number of days from Jan. 17, 1899, to April 5, 1899; (*b*) the difference in time by subtraction of dates.

78. A railroad train moves 1 mi. in 65 sec. What is its speed per hour?

79. A note given August 15, 1901, for 90 days, will mature when?

80. A can walk $3\frac{1}{3}$ mi. in 50 min., and B can walk $2\frac{1}{4}$ mi. in 36 min. How many yards will A be ahead of B when A has gone 6 mi., if they start together?

81. A farmer delivered at a warehouse four loads of wheat weighing respectively 2113 lb., 2310 lb., 2270 lb., and 2091 lb. How much should he have received at 72¢ per bushel?

82. The difference in longitude between two places being 9° 34′ 25″, what is the difference in time?

83. A man has a salary of $400 a year, and has $500 in the bank. If he spends $500 a year, in what time will his money be all gone? (Allow no interest.)

84. What is the shortest stick that can be cut into pieces, 9 in., 12 in., or 15 in. in length, with nothing remaining?

85. (a) What is meant by a Common Multiple of two or more numbers?

(b) Find the L. C. M. of 36, 54, 105.

86. (a) What is meant by the prime factors of a number?

(b) Find the prime factors of 13,230, 22,050, and 23,625.

(c) By means of the prime factors find their G. C. M. and L. C. M.

87. Resolve 16,335 and 18,018 into their prime factors, and from inspection of these write the prime factors of their (a) L. C. M. and (b) G. C. M.

88. A farmer bought a number of horses and cows for $2000. There were 3 times as many cows as horses, and a horse costs twice as much as a cow. If each horse costs $80, how many cows did he buy?

89. The difference in weight of two chests of tea is 25 lb.; the value of both at 65¢ per pound is $113.75. How many pounds of tea are in each chest?

90. What is the smallest sum of money with which you can buy chickens at 25 ¢, or geese at 50 ¢, or turkeys at 75 ¢, or lambs at $3, or sheep at $5, or pigs at $7, or cows at $35, or horses at $140, and have exactly $15 left for expenses?

91. Ten cents will buy 3 oranges, 4 lemons, or 5 apples. How many apples are worth as much as 5 doz. oranges and 7 doz. lemons?

92. One workman charges $3 for a day's work of 8 hr., and another $3.50 for a day's work of 9 hr. Which had I better employ, and how much shall I have to pay him for work that he can do in a fortnight, working 6 hr. a day?

93. A can do a piece of work in $\frac{2}{3}$ of a day, and B in $\frac{1}{2}$ of a day. If $1.40 is paid for the work, how much should A receive?

94. How many oranges must a boy buy and sell to make a profit of $9.30, if he buys at the rate of 5 for 3 ¢, and sells at the rate of 4 for 3 ¢?

95. A and B dig a ditch in 50 hr. With C's help they could have done it in $18\frac{3}{4}$ hr. In what time could C do $\frac{3}{5}$ of the work alone?

96. Three men can dig a certain drain in 8 da. They work at it for 5 da., when one of them falls ill, and the other two finish the work in 5 da. more. How much of the work did the first man do before he fell ill?

97. A boy can run 6 times around a circular plot of ground in 52 sec.; another boy can run 9 times around the same plot in 80 sec. If they start from the same place at the same time, and run in the same direction, how many rounds will each make before the faster boy overtakes the slower?

98. Express in the form of a vulgar fraction the average of $\frac{3}{8}$, $\frac{\frac{3}{4}}{1\frac{4}{5}}$, .7, .4$\frac{4}{9}$, and .486$\frac{1}{9}$.

99. In a granary there are 4 bins, each 10 ft. long and 5 ft. wide. How high must they be boarded in front to be capable of holding 860 bushels?

z

100. The outfit of a livery stable is worth $3000; $\frac{1}{7}$ the value of the horses is equal to $\frac{1}{5}$ the value of vehicles, harness, etc. Find the value of the horses.

101. A farmer agreed to pay his hired man 10 sheep and $160 for 1 yr. labor. The man quit work at the end of 7 mo., receiving the sheep and $60 as a fair settlement. Find the value of each sheep.

102. Divide $1200 among A, B, and C, so that A may have $70 more than B, and twice as much as C.

103. A train going 25 mi. an hour starts at 1 o'clock P.M. on a trip of 280 mi.; another going 37 mi. an hour starts for the same place at 12 min. past 4 o'clock P.M. When and where will the former be overtaken?

104. In the number, 28,672, the value expressed by the first two digits from the left is how many times the value expressed by the fourth digit from the left?

105. A town whose population was 10,000 increased 10% every year for 3 yr. What was the population at the end of that period?

106. A house and lot was sold for $7030, at a loss of $16\frac{2}{3}\%$ of its cost. Find the cost.

107. Five men in a factory accomplish as much as 8 boys. What per cent of a man's work does a boy do? What per cent of a boy's work does a man do?

108. Forty-five per cent of a carload of melons were sold to one dealer, and $33\frac{1}{3}\%$ of those left to another. How many were there in the car before any were sold, if after the second sale there remained 110?

109. In a certain school 48% of the pupils are boys, and there are 39 girls. Find the number of boys.

110. How many pounds of flour will be required to make 1000 lb. of bread, if the bread weigh 30% more than the flour used?

111. 93 lb. 6 oz. is what per cent of 43 lb. 12 oz. ?

112. Water in freezing expands 10%. If 1 cu. ft. of water weighs 1000 oz., find the weight of 1 cu. ft. of ice.

113. Give answers to the following:

(a) $15\frac{3}{4}\%$ of 660 = ?   (d) $.2\%$ of 40 = ?

(b) 660 is $15\frac{3}{4}\%$ of what number?   (e) 40 is $.2\%$ of what number?

(c) $\frac{3}{7}$ is what per cent of $\frac{3}{5}$?

(f) What per cent of itself must be added to a number so that the sum diminished by 10% of itself may be 17% more than the original number?

114. Brooms are bought wholesale at $20 a gross. What per cent profit will be made by selling them at 20 ¢ each?

115. A merchant purchases sugar at $4.50 per hundredweight. At what price per pound must he sell it in order to gain $5\frac{5}{9}\%$?

116. I bought a house for $4000 and spent 40% of the cost in repairs. What must I rent it for a month in order to make a clear gain of 5% of the total cost, taxes and repairs amounting to $80 yearly?

117. By selling a piano for $260, a dealer loses 20%. How much should he have sold it for to gain 5%?

118. A man having lost 20% of his capital is worth exactly as much as another who has just gained 15% on his capital. The second man's capital was originally $9000. What was the first man's capital?

119. A dealer sold an article for $8.10 and lost 10%. At what selling price would he have gained 10%?

120. A bookseller deducts 10% from the market price of his books, and after this has a gain of 25%. He sells a book for $7.20. Find the cost price of the book, and what per cent the marked price is in advance of the cost price.

121. A merchant bought 1000 yd. of carpet at 60 ¢ a yard, and sold $\frac{2}{5}$ of it at a profit of 30%, $\frac{1}{2}$ at a profit of 20%, and the rest at a loss of 20%. How much did he receive for the carpet?

122. A sells goods to B at a gain of 12%, and B sells the same goods to C at a gain of $7\frac{1}{2}$%. C paid $3762.50 for the goods. How much did A pay for them?

123. A machinist sold two seed-drills for equal sums of money. He gained 25% on the one and lost 25% on the other. His total loss was $9.60. Find the cost of each drill.

124. R purchased a house and lot for $3300, paid $1325 for repairs, and now rents the premises for $30 a month. If he expends annually for taxes $47.50, and for incidental repairs $35, what is his per cent of annual income on his investment?

125. A merchant closed out a stock of cloaks for $311.04, at a loss of 28%. Required the loss by the transaction.

126. By selling my cloth for $1.26 per yard I gain 11¢ more than I lose by selling it at $1.05 per yard. What would I gain by selling 800 yd. at $1.40 per yard?

127. A merchant marks his goods at 40% in advance of cost, and in selling uses a pound weight $\frac{1}{8}$ oz. too light. If he throws off 10% of his marked price, find his gain per cent.

128. State the relation between 1 lb. Troy and 1 lb. Avoirdupois. What is the gain per cent when the selling price per ounce Avoirdupois is the same as the cost per ounce Troy?

129. A man bought a bankrupt stock at 60¢ on the dollar of the invoice price, which was $4840. He sold half of it at 10% advance on invoice price, half the remainder at 20% below invoice price, and the balance at 50% of invoice price. His expenses were 10% of his investment. Find his loss or gain (a) in money, and (b) in rate per cent.

130. The list price of an article is $150. If trade discounts of 25% and $16\frac{2}{3}$% are allowed, what is the net price?

131. A dealer buys stoves at a discount of 22% from list price, and sells them at list price; what is his per cent of gross profit on the investment?

132. Required the net price of an article listed at $400, 30%, 10%, and 5% off.

133. From the list price of a line of goods a purchaser is allowed a trade discount of 20%; a further discount of 10% off the trade price for taking a quantity, and a still further discount of 5% off his bill for cash. Find his gain per cent by selling at 10% less than the list price.

134. The net price of a reaper is $158.40, and the trade discounts allowed are 20% and 10%. Find the list price.

135. A commission merchant sold coffee for me and remitted $1960, after deducting his commission of 2%. What is the value of the coffee?

136. If an agent receives $1092 to buy pork, how many pounds, at 6½¢ per pound, can he buy and retain his commission of 5% for buying?

137. A commission merchant sold 1014 bu. of oats, at 41¢ per bushel, paid $33.74 freight charges, and retained 3½% commission. How much should he remit to the consignor?

138. A lad earned $21.16 collecting accounts for a physician. He was allowed 5¾%. What amount did he collect?

139. Find the premium paid to insure a house worth $7500 for ⅔ of its value for 3 yr., the rate for each year being ⅜% of the policy.

140. What premium must be paid to insure a cargo of 4880 bu. of wheat, valued at 78¢ per bushel, at 1½%, the policy being for only ⅝ of its value?

141. A building is insured for $400 more than ⅚ of its cost at 4%. If destroyed, the loss will be $216. Find the cost of the building.

142. A dealer shipped 200 bbl. of apples to Liverpool; the average cost of the apples was $3.75 per barrel. For what sum must he have the apples insured at ¾% premium to guard against all loss, in case of shipwreck, his other expenses being $75?

143. If in a certain town $3093.75 was raised from a $\frac{3}{4}\%$ tax, what was the assessed valuation of the property in the town ?

144. A tax of $24,750 is levied on a town, the assessed valuation being 15 mills on a dollar. What tax does a man pay on an income of $1100, of which $400 is exempted ?

145. A farmer whose property is assessed at $9600 pays on the dollar $1\frac{3}{4}$ mills for township rates, $1\frac{1}{4}$ for county rates, $1\frac{1}{2}$ for railway bonus, and $2\frac{1}{2}$ for school rate. How much does he pay in all ?

146. B's tax was $86.2755 when the rate was 7.635 mills on a dollar. What was the assessed valuation of his property ?

147. A certain school section is assessed for $150,000. The trustees have built a schoolhouse costing $1800.

(*a*) What will the schoolhouse cost a ratepayer whose property is assessed for $4500 ?

(*b*) What would be the rate of taxation per annum on the whole section if the house were paid for in six equal annual payments, without interest ?

148. A clerk pays $7.50 taxes on his salary. What is his total salary if $400 of it is exempt from taxation and a $2\frac{1}{2}\%$ rate is levied on the remainder ?

149. What per cent must be assessed on $1,500,000 to produce $29,400 after paying 2% for collecting ?

150. An importer receives an invoice of kid gloves billed at $680, pays a duty of 50% ad valorem, and sells them at an advance of $33\frac{1}{3}\%$ on their gross cost to him. How does the price paid by the purchaser compare with the exporter's price ?

151. A merchant imports 75 cases of indigo, gross weight 196 lb. each, allowing 15% for tare. What was the duty at $5\cancel{c}$ per pound ?

152. What will $1 amount to in 216 da., at $7\frac{1}{2}\%$ per annum, simple interest ?

153. Find the simple interest on $597.50 for 5 mo. 12 da. at 8% per annum.

154. How long will it take $450, at 8%, to yield $21.30 interest?

155. What amount will be due July 1, 1902, on a note of $80, drawn Feb. 6, 1901, and bearing interest at $5\frac{1}{4}$% per annum, exact interest?

156. Find the sum due Sept. 2, 1899, on a note for $147.33, given Jan. 13, 1899, and bearing interest at 4% per annum.

157. Find the exact interest on $225 from July 13, 1899, to Sept. 3, 1899, at 6%.

158. Find the interest on $1, at $7\frac{1}{2}$% per annum, from Jan. 1, 1899, to June 3, 1899. (Complete answer required.)

159. What sum will amount to $354.09 in 7 mo., at 3% per annum?

160. In what time will $1350 earn $31.88 at 5% per annum?

161. Find the face of a draft that cost $434.70, at $\frac{5}{8}$% premium.

162. If the interest is $12.57, the time 8 mo. 2 da., and the rate per annum $5\frac{1}{2}$%, what is the principal?

163. Find the exact interest on $150 from July 16 to Dec. 9, at 5% per annum.

164. A person borrows money for 6 mo. at 4%, simple interest, and repays at the end of the time, as principal and interest, $816. How much did he borrow?

165. Find the simple interest on $912.50, at 8%, from Feb. 13, 1901, to Dec. 19, 1902.

166. A note of $360, drawn April 20, 1900, was paid July 2, 1901, with interest at $7\frac{1}{2}$% per annum. Find the amount paid, simple interest.

167. Oct. 15, 1899, a young man deposited in the savings bank the sum of $860.75. May 20, 1900, he withdrew the principal and simple interest at 4% per annum. What amount did he withdraw?

168. Bought a horse for $160, and gave in payment my note dated Aug. 15, 1899, with interest at $7\frac{1}{2}$% per annum until paid. Jan. 9, 1900, I sold the horse for $200 cash, and paid my note. What was my net gain?

169. If for $7 I can have the use of $35 for 3 yr. 4 mo., how much a month shall I have to pay for the use of $8750?

170. Jan. 1, 1899, a person borrowed $2417.50 at $6\frac{3}{4}$%, simple interest, promising to return it as soon as it amounted to $2582.50. On what day did the loan expire? (365 da. = 1 yr.)

171. March 1, 1899, a storekeeper bought goods amounting, at catalogue prices, to $840, on which he was allowed successive discounts of $33\frac{1}{3}$% and 5%. The account is payable in 60 da., after which time interest is to be charged at 7% per annum. June 1, 1899, he paid $100. How much is due July 1, 1899?

172. Find the proceeds of a note for $200 given at Albany, N.Y., for 3 mo., and discounted at the bank the day it was made at 6%.

173. Find the proceeds of a note for $168 due Oct. 20, 1899, and discounted Sept. 25, 1899, at a Brooklyn, N.Y., bank, at 6% per annum.

174. $ $1234\frac{50}{100}$.   St. Louis, Jan. 15, 1899.

Ninety days after date, I promise to pay A. Bee, or order, the sum of one thousand two hundred and thirty-four $\frac{50}{100}$ dollars, at the Bank of Commerce here. Value received.   C. Dee.

This note was discounted Feb. 10, 1899, at 6% per annum. Find the proceeds.

175. A note for $230, drawn Jan. 2, 1899, at 3 mo., and bearing interest at 8% per annum, was discounted Feb. 1 at 7%. Find the proceeds.

176. Find the proceeds of the following note:

$ 2400.   HAMILTON, OHIO, Feb. 3, 1899.

Five months after date, value received, I promise to pay Thomas Cowan, or order, the sum of two thousand four hundred dollars, at the Bank of Hamilton, with interest at 6% per annum.

VANCE ALLEN.

Discounted May 22, 1899, at 7%.

177. The discount on a note for $ 3600, which matured April 21, 1899, and was discounted Feb. 24, 1899, was $ 45.60. Find the rate of discount.

178. A buys 600 yd. of silk at 95¢ per yard, and sells it at once, receiving in payment a 90-day note for $ 700, which he at once discounts at a bank at 6% per annum. Find the gain.

179. For what sum must a note be drawn June 1, 1899, payable in 90 da., so that when discounted June 14, at 8%, the proceeds will be $ 717.20 ?

180. Jan. 1, A owes a bank $ 15,000. He offers for discount certain notes: $ 2500 due Feb. 15, $ 3700 due March 13, and $ 7500 due April 1. If these are discounted at 8% per annum, how much cash must he pay ?

181. Find the proceeds of a note for $ 292.73, discounted at a bank, for 35 da., at 6% per annum, exact interest method.

182. Find the value of $(1.03)^4$.

183. A man has the choice of loaning his money at $7\frac{1}{2}\%$, compound interest, or at 8%, simple interest, money and interest to be paid at end of 3 yr. Show which is the better investment.

184. An annual deposit of $ 250 is made with a loan company which pays 4% per annum on deposits, compounded half-yearly. Find the amount of all these deposits when the fourth has been made.

185. June 30, 1899, I borrowed $ 16.50, to be returned April 30, 1901. With compound interest at $6\frac{1}{2}\%$, what amount did I then pay ?

186. A man puts $350 in a savings bank each year, making his first deposit Dec. 31, 1899. How much will there be to his credit Jan. 1, 1903, the bank adding 4% per annum?

187. $1200 is to be divided between two persons, A and B, so that A's share is to B's share as 2 to 7. Find the share of each.

188. What is the ratio of $3\frac{7}{8}$ to $\frac{5}{6}$? Answer in per cent.

189. Divide 1026 into four parts that shall be in the ratio of 3, 11, 17, and 23.

190. An upright pole 16 ft. long casts a shadow 5 ft. 4 in. long, and at the same hour the shadow of a tree is found to be 26 ft. 9 in. Required the height of the tree.

191. The sum of three numbers is 940. The first number equals $\frac{5}{9}$ of the second, and the second equals $\frac{7}{10}$ of the third. Find the numbers.

192. One-sixth of the square of a certain number is 384. Find the number.

193. Find the square root of .6 correct to three decimal places.

194. Find, within one inch, the side of a square whose area is 5 A.

195. A rectangular field whose length is $\frac{4}{3}$ of its width contains 2 A. 112 sq. rd. Find the length of a diagonal.

196. Required the base of a right-angled triangle whose hypotenuse is $16\frac{1}{4}$ ft., and perpendicular $9\frac{3}{4}$ ft.

197. A ladder 78 ft. long stands perpendicularly against a building. How far must it be pulled out at the foot that the top may be lowered 6 ft.?

198. A road runs round a circular pond; the outer circumference is 440 yd., and the width of the road is 20 yd. Find the area of the pond.

199. In order to drain a swamp a ditch was dug 1 mi. long, 3 ft. deep, 6 ft. wide, at the surface, and 4 ft. wide at the bottom. Find the total cost at 9¢ per cubic yard.

200. How many gallons of water will a circular cistern 6 ft. in diameter and 7 ft. deep contain? (1 cu. ft. = 7.48 gal.)

201. The surface of a cube is 432 sq. ft. What is its volume?

202. (*a*) A circular cistern, 8 ft. in diameter and 9 ft. in depth, is filled with water to the height of 6 ft. How many gallons of water in the cistern?

(*b*) If a sphere whose diameter is 4 ft. is submerged in the water in the cistern, how high will it cause the water to rise?

203. How many cords are there in a cylindrical log 20 ft. long and 3 ft. 6 in. in diameter?

204. Find the diameter of a circle whose area is equal to the sum of the areas of two circles whose diameters are 12 in. and 16 in. respectively.

205. Find the area of the curved surface of a right circular cone the radius of whose base is 3.5 in. and whose altitude is 7 in.

206. A chord of a circle, whose radius is 12 in., subtends a right angle at the centre of the circle. Find the area of the smaller segment cut off by this chord.

207. A spherical shell, internal diameter 14 in., is filled with water. Its contents are poured into a cylindrical vessel whose internal radius is 14 in. Find the depth of the water in the cylinder.

208. The sides of a triangle are 40, 45, and 50 ft. respectively. Find its area.

209. The diameter of a circular plate of lead is 13 in. From this is cut out a circular plate of radius 6 in., and the remainder of the lead is moulded into the form of a circular plate, with $\frac{1}{4}$ of the former thickness. Find the diameter of this plate.

210. The sides of a triangle are 13, 14, and 15 ft. Find its area.

211. The external dimensions of a rectangular covered box, made of inch stuff, are 7, 8, and 9 ft. Find the capacity of the box and the quantity of lumber in it.

212. A ball of yarn 3 in. in diameter makes one mitten. How many similar mittens will a ball 6 in. in diameter make?

213. Find the volume of a cylinder the radius of whose base is 10 in., the altitude being 18 in.

214. Find the volume of a cone the radius of whose base is 10 in., the altitude being 18 in.

215. How often can the cone in example 214 be filled and emptied into the cylinder in example 213?

216. The length of the radius of the base of a right circular cylinder is 9 in. and its altitude is 16 in. Find the volume.

217. Find the area of the curved surface of the cylinder in example 216. Find the area of its entire surface.

218. Find the volume of a cone whose altitude is 15 in. and whose base is a circle 10 in. in diameter.

219. Find the volume of a cone whose altitude is 12 in. and the diameter of whose base is 5 in.

220. Find the area of the curved surface of a cone whose altitude is 20 in. and the radius of whose base is 15 in. Find also its total area.

221. If the diameter of a cylindrical well be 5 ft., and its depth 27 ft., how many cubic yards of earth were removed in order to form it?

222. A farmer employs a number of men and 8 boys; he pays the boys $.65 and the men $1.10 per day. The amount that he paid to all was as much as if each had received $.92 per day. How many men were employed? (*x* men.)

223. Two men start from the same point at the same time to walk in the same direction around a block of land 1¼ mi. on each side. A goes at the rate of 4 mi. and B 3 mi. an hour. How far will A walk before he overtakes B?

224. Find the cost of the material required to fence 2½ mi. of railway (both sides), posts placed 8 ft. apart, an 8-in. base 1 in. thick, a 2 × 4 in. rail at top, and 6 strands of wire. The posts cost 12½ ¢ each, the lumber $14 per thousand, and the wire 4 ¢ per pound. (A pound of wire stretches 1 rd.)

CHAPTER XXI

APPENDIX

STOCKS AND BONDS

269. The capital of a bank or other public company is called **Stock**.

It is usually divided into a definite number of equal parts or **Shares**.

The original value of a share, generally $100, $50, or $25, is called its **Par Value**.

270. The **Market Value** of a share is the sum for which it can be sold.

Stock is said to be *above par*, or at a premium, when the market value is greater than its par value ; it is said to be *below par*, or at a discount, when the market value of the share is less than its par value.

Thus if $100 stock sells for $112 money, the stock is at 12% premium, and it is said to sell at 112.

If $100 stock sells for $96 money, the stock is at 4% discount, and is quoted at 96.

271. A **Stock Broker** is a person who buys or sells stocks, bonds, or similar securities. His commission, called **Brokerage**, is reckoned at a certain *rate per cent*, which varies, the most common rate being $\frac{1}{8}$ of 1% or $\frac{1}{8}$%.

272. A **Bond** is a note bearing interest issued by a government or corporation. There are two kinds of bonds — *registered* and *coupon* bonds.

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A **Registered Stock** or **Bond** is one which is registered on the books of the company or government issuing it, and which cannot be sold or transferred except in writing at the office of the treasurer.

An **Interest Coupon** is an interest certificate payable to the bearer, which is attached to the bond, and which is detached when the interest becomes due.

One coupon is attached to the bond for each instalment of interest to be paid on it.

273. The following is the quotation of United States bonds in the market of Oct. 11, 1901:

	BID	ASKED		BID	ASKED
New 2s	109	109½	Coupon 4s	112	112¾
Coupons	109	109½	Registered 4s new .	139	140
New 3s	107¼	108¼	Coupon 4s new . .	139	140
New 3s coupon . .	108	109	Registered 5s . .	107½	108½
New 3s small. . .	108	109	Coupon 5s	107½	108½
Registered 4s . . .	112	112¾			

274. The following is the quotation of stock in the market of Oct. 11, 1901:

STOCKS	HIGHEST	LOWEST	CLOSING	
			Oct. 11	Oct. 10
Am. Sugar	119¾	117½	119	118½
Amal. Copper	90¼	88½	89⅞	89⅜
Atchison	79	77⅜	78⅛	77½
C. R. I. & P.	144	140¼	142½	146
Del. & Hudson	166½	166	166	165¾
Manhattan	121½	119⅝	121	120½
So. Pacific	59¾	57⅜	59⅜	57

Exercise 195

1. At what different prices is Amalgamated Copper stock quoted, Oct. 11, 1901 ?

2. What will a seller receive from his broker for 1 share of Atchison stock, Oct. 11, 1901, at each of the quoted prices, brokerage being $\frac{1}{8}\%$? What from 1 share of Am. Sugar ?

3. What will a buyer have to pay for 1 share of Manhattan stock at each quotation, Oct. 11, 1901, brokerage $\frac{1}{8}\%$? What for C. R. I. & P. ?

4. At what per cent premium are the different quotations for Am. Sugar, C. R. I. & P., and Del. & Hudson stock, Oct. 11, 1901 ?

5. At what per cent discount are the different quotations for Amal. Copper, Atchison, and So. Pacific stock, Oct. 11, 1901 ?

6. What would I receive for 1 share of Del. & Hudson, Oct. 11, 1901, sold at the highest price, brokerage $\frac{1}{8}\%$? What for 10 shares ? What for 100 shares ?

7. What would I have to pay for 1 share of Atchison stock, Oct. 11, 1901, bought at the lowest price, brokerage $\frac{1}{8}\%$? What for 10 shares ? What for 100 shares ?

8. What would I have to pay for 1 share of Am. Sugar stock, Oct. 11, 1901, at the lowest quoted price, brokerage $\frac{1}{8}\%$? What for 10 shares ? What for 100 shares ?

9. What would I receive for 1 share of So. Pacific stock, Oct. 11, 1901, sold at the lowest quotation, brokerage $\frac{1}{8}\%$? What for 10 shares ? What for 100 shares ?

10. What will 1 share of Amal. Copper stock cost, Oct. 11, 1901, at the closing price, brokerage $\frac{1}{8}\%$? How many shares can I buy for $180 ? For $450 ? For $360 ?

11. What will 1 share of Atchison stock cost, Oct. 11, 1901, at the lowest quotation, brokerage $\frac{1}{8}\%$? How many shares can be bought for $155 ? For $775 ?

12. What is the difference between the highest and lowest quotations of Manhattan stock, Oct. 11, 1901 ?

13. What is the difference between the closing prices of Del. & Hudson stock, Oct. 10 and Oct. 11, 1901 ?

14. What is the difference between the highest and lowest prices of Atchison stock, Oct. 11, 1901 ?

15. At what per cent discount is stock which is quoted at 88 ? $98\frac{1}{4}$? $72\frac{3}{4}$?

275. (1) How much will be realized by selling out 66 shares of Missouri Pacific stock at $95\frac{1}{2}$, brokerage $\frac{1}{8}\%$?

> 1 share of stock sells for $\$95\frac{1}{2} - \$\frac{1}{8}$, or $\$95\frac{3}{8}$ money.
>
> ∴ 66 shares of stock sell for $66 \times \$95\frac{3}{8}$ or $\$6294.75$ money.
>
> Note. — The brokerage = $66 \times \$\frac{1}{8} = \$8.25$.

(2) How many shares of Manhattan stock at $124\frac{1}{8}$, brokerage $\frac{1}{8}\%$, can I buy for $\$5591.25$?

> 1 share costs $\$124\frac{1}{8} + \$\frac{1}{8}$, or $\$124.25$.
>
> ∴ the number of shares = $\$5591.21 \div \$124.25 = 45$.
>
> Note. — The brokerage = $45 \times \$\frac{1}{8} = \$5.62\frac{1}{2}$.

(3) A broker realizes $\$7.25$ from a sale of stock, brokerage $\frac{1}{8}\%$. What was the par value of the stock sold ?

> $\frac{1}{8}\%$ of the par value = $\$7.25$.
>
> 1% of the par value = $\$58$.
>
> 100% of the par value = $\$5800$.
>
> ∴ the par value = $\$5800$.

(4) I sold through my broker 95 shares of Twin City Rapid Transit stock, receiving for it $\$9476.25$, brokerage $\frac{1}{8}\%$. Find at what price the stock was quoted.

> 95 shares sell for $\$9476.25$.
>
> 1 share sells for $\$9476.25 \div 95 = \$99.75$;
>
> *i.e.* excluding brokerage, the selling price = $99\frac{3}{4}$.
>
> ∴ stock is quoted at $99\frac{3}{4} + \frac{1}{8}$ or $99\frac{7}{8}$.

(5) What annual income will be realized from $ 3828.12½, invested in the U. S. 3's at 109¼, brokerage ⅛% ?

1 share costs $ 109¼ + $ ⅛ = $ 109⅜ = $ 109.375.

The number of shares = $ 3828.125 ÷ $ 109.375 = 35.

∴ the income = 35 × $ 3 = $ 105.

(6) What amount of money must be invested in 6% stock at 119¾, brokerage ⅛%, to realize an income of $ 978 ?

1 share yields an income of $ 6.

The number of shares = $ 978 ÷ $ 6 = 163.

1 share costs $ 119¾ + $ ⅛ = $ 119⅞.

163 shares cost 163 × $ 119⅞ = $ 19,539.62½.

∴ $ 19,539.62½ must be invested.

(7) If 6% stock is bought at 109⅞, what per cent does it pay on the investment, brokerage ⅛% ?

1 share costs $ 109⅞ + $ ⅛ = $ 110.

$ 110 yields an income of $ 6.

∴ the rate per cent = $\frac{6}{110}$ or $5\frac{5}{11}$% of the investment.

(8) What must I pay for 6% stock to realize an income of 8% on the investment, brokerage ⅛% ?

8 % of the cost of 1 share = $ 6.

1 % of the cost of 1 share = $ ¾.

100 % of the cost of 1 share = $ 75 ;

i.e. including brokerage, the cost price is $ 75.

∴ stock is quoted at 74⅞.

Exercise 196

1. What will 25 shares of Ill. Central stock cost at 148, brokerage ⅛% ?

2. What is realized from the sale of 208 shares of Chi. & Alton pfd. R. R. stock at 71½, brokerage ⅛% ?

2 A

3. What did I pay for 39 shares Union Pacific, Oct. 11, 1901, stock selling at $99\frac{1}{4}$ and brokerage being $\frac{1}{8}\%$?

4. Find what I received from the sale of 84 shares of Western Union stock at $92\frac{1}{2}$, brokerage $\frac{1}{8}\%$.

5. What is the cost of $20,000 U. S. 4's at $112\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

6. Find the cost of $24,000 U. S. 4's at $116\frac{3}{8}$, brokerage $\frac{1}{8}\%$.

7. October 11, 1901, 96 shares of Pullman stock at $214\frac{1}{2}$ were sold on the New York stock exchange, brokerage $\frac{1}{8}\%$. Find the amount received by the owners of the stock.

8. How many shares of Atchison pfd. stock at $97\frac{1}{4}$ can I buy for $3505.50, brokerage $\frac{1}{8}\%$?

9. October 11, 1901, Bal. and Ohio stock was quoted at $101\frac{1}{4}$. How many shares were bought for $4257.75, brokerage $\frac{1}{8}\%$?

10. A stockholder sold D. L. and W. R. R. stock at $157\frac{1}{4}$, receiving all together $3771. How many shares did he sell, brokerage being $\frac{1}{8}\%$?

11. How many shares of U. S. Steel pfd. stock must I sell at 95, brokerage $\frac{1}{8}\%$, to receive $9677.25 ?

12. If from my sales of Western Union stock at $95\frac{1}{4}$, I receive $6278.25, how much stock did I sell, brokerage being $\frac{1}{8}\%$?

13. How many shares of N. Y. Central stock at $158\frac{1}{2}$ can be bought for $2855.25, brokerage $\frac{1}{8}\%$?

14. A broker sells 24 shares of stock on a commission of $\frac{1}{8}\%$. How much does he realize ?

15. How many shares of stock does a broker sell to realize a commission of $16.25, brokerage $\frac{1}{8}\%$?

16. A broker realizes $12.50 from the sale of stock, brokerage $\frac{1}{8}\%$. What was the par value of the stock sold and what did it sell for at $70\frac{3}{8}$?

17. A broker received $46.50 for buying stock on a commission of $\frac{5}{8}\%$. How much stock did he buy ?

18. I sold through my broker 40 shares of stock, receiving for it $4860, brokerage $\frac{1}{8}\%$. At what price was the stock quoted?

19. A person received 6053.12\frac{1}{2}$ for $6500 stock after paying his broker $\frac{1}{8}\%$. Find at what per cent discount the stock was sold.

20. October 10, 1901, $1654.25 was paid for 26 shares of Pacific Coast stock, brokerage $\frac{1}{8}\%$. At what was Pacific Coast stock quoted, Oct. 10?

21. What annual income will be obtained from $6071, invested in U. S. 4's coup. of 1925 at 116$\frac{5}{8}$, brokerage $\frac{1}{8}\%$?

22. A person paid $8578.50 for U. S. 4's at 112$\frac{3}{4}$, brokerage $\frac{1}{8}\%$. What was his income from the bonds?

23. If I invest $8583.75 in stock at 95$\frac{1}{4}$, paying 5% dividend, what will be my income, brokerage $\frac{1}{8}\%$?

24. What income will be realized from $9229.50 invested in stock at 109$\frac{3}{4}$, brokerage $\frac{1}{8}\%$, paying a dividend of 5$\frac{1}{2}\%$?

25. What amount of money must be invested in 8% stock at 158$\frac{1}{2}$, brokerage $\frac{1}{8}\%$, to realize an income of $1096?

26. What sum must I invest in 4$\frac{1}{2}\%$ stock at 99$\frac{3}{4}$, to produce an annual income of $1638, brokerage $\frac{1}{8}\%$?

27. How much must I invest in U. S. 5's at 112$\frac{3}{4}$ to realize an annual income of $450, brokerage $\frac{1}{8}\%$?

28. If street railway stock bought at 232 yields a half-yearly dividend of 6$\frac{1}{2}\%$, how much must I invest to obtain a semiannual income of $325, brokerage $\frac{1}{8}\%$?

29. If I buy stock through a broker who charges $\frac{1}{8}\%$, how much must I invest in stock at 153, paying 9% dividends, to secure an income of $1350?

30. If 4$\frac{1}{2}\%$ stock is bought at 74$\frac{7}{8}$, brokerage $\frac{1}{8}\%$, what per cent does it pay on the investment?

31. If 8% stock is bought at 159$\frac{7}{8}$, what per cent does it pay on the investment, brokerage $\frac{1}{8}\%$?

32. What must I pay for 4% stock to pay 5% on the investment, brokerage $\frac{1}{8}\%$?

33. What rate of interest do I realize on an investment in 6% stock at $107\frac{7}{8}$, brokerage $\frac{1}{8}\%$?

34. What must I pay for 5% stock to yield an income of 6% on my investment?

35. A person receives \$600 from an 8% bank dividend. How much stock does he own?

36. A person having \$5000 bank stock sells out when it is at 40% premium. What amount of money does he receive, brokerage being $\frac{1}{8}\%$?

37. Bought through a broker 1600 shares (\$100) R. R. stock at $69\frac{1}{4}$, brokerage $\frac{1}{8}\%$. What was the cost of the stock?

38. A speculator bought 36,500 shares (\$100) R. R. stock at $39\frac{3}{4}$, and sold them at $40\frac{3}{8}$. What was his gain, brokerage $\frac{1}{8}\%$, on both transactions?

39. A bank declared a dividend of $3\frac{1}{2}\%$. How much should a stockholder owning 120 shares (\$50) receive?

40. One company guarantees to pay 6% on shares of \$100 each; another guarantees at the rate of $5\frac{4}{5}\%$ on shares of \$30 each; the price of the former is \$124.50, and of the latter \$34. Find the rates of interest which they return to the purchaser.

41. A broker receives \$42,100 to invest in U. S. 5-20 bonds, after reserving $\frac{1}{4}\%$ on the par value of the amount purchased. What was his commission, the bonds being at a premium of 5%?

42. A man bought through a broker 1900 shares (\$100) R. R. stock at $54\frac{3}{4}$ and sold them at $55\frac{3}{8}$. What was his net profit on the transaction, brokerage each way $\frac{1}{8}\%$?

43. An insurance company declared a dividend of 9%. What rate is that on the market value of the shares which are at 185?

44. Compare the rates on the cash values of 6% on stock at 216 and $3\frac{1}{2}\%$ on stock at 125.

45. Sold 37 shares ($ 25) B. and L. Association stock, receiving therefor $ 1019.81. At what rate was the stock sold?

46. Bought through a broker 750 shares ($ 50) in the Farmers' Loan and Savings Society, paying therefor $43,968.75. At what quotation were they bought, brokerage $\frac{1}{8}\%$?

47. Bought stock at $197\frac{5}{8}$ and sold it at $194\frac{7}{8}$, having meanwhile received a dividend of 6% on it. My net gain on the transaction after paying $\frac{1}{8}\%$ brokerage each way is $ 336. How many shares ($ 40) did I buy?

48. How many railway shares ($ 100) at 40% discount must be sold in order that the proceeds invested in bank stock, which is 4% below par, and pays a dividend of 7%, may yield an income of $ 1680, brokerage included?

49. Explain the terms: Stocks, Shares, Dividends. When is stock at par? At a premium? At a discount?

50. When the $3\frac{1}{2}$ per cents are at 98, what must be the price of another stock yielding $4\frac{1}{2}\%$, so that the latter may be as profitable as the former, brokerage included?

COMPOUND PROPORTION

276. (1) If 20 men can dig 60 yd. of earth in 4 da., how many yards can 30 men dig in 9 da.?

Men	Yd.	Da.
20	60	4
30	x	9

Multiply 60 yd. by $\frac{30}{20}$. ∵ 30 men can dig $\frac{30}{20}$ as many yards as 20 men.

Multiply the result by $\frac{9}{4}$. ∵ in 9 da. 30 men can dig $\frac{9}{4}$ as much as in 4 da.

$$\therefore \ x \text{ yd.} = \frac{60}{1} \text{ yd.} \times \frac{30}{20} \times \frac{9}{4} = 202\frac{1}{2} \text{ yd.}$$

(2) If 120 bu. of oats last 14 horses 56 da., in how many days will 6 horses consume 90 bu.?

Bu.	Horses	Da.
120	14	56
90	6	x

Multiply 56 da. by $\frac{90}{120}$. ∵ 90 bu. will last $\frac{90}{120}$ as many days as 120 bu.

Multiply the result by $\frac{14}{6}$. ∵ 90 bu. will last 6 horses $\frac{14}{6}$ as long as 14 horses.

$$\therefore x \text{ da.} = \frac{56}{1} \text{ da.} \times \frac{90}{120} \times \frac{14}{6} = 98 \text{ da.}$$

277. In each of the above two solutions, in order that the ratio may easily be seen, the items in the question have been written in horizontal lines. In number (1) we are required to find the number of yards, and the problem is to determine the ratio resulting from each comparison, and how it affects the number of *yards*.

In number (2) we are required to find the number of days, and the problem is to determine the ratios, and how they affect the number of *days*.

278. To prove the answer correct, substitute the answer in place of x in the horizontal line and omit one of the quantities, frame the question, and then solve.

Bu.	Horses	Da.
120	14	56
90	x	98

If 120 bu. of oats last 14 horses for 56 da., how many horses will 90 bu. last 98 da. ?

On solving, x will be found equal to 6, which proves the former solution correct. How many questions can be made from the numbers in the two lines, including the original one?

Solve the following questions. State one or more questions in proof for each problem, and prove your answers correct.

Exercise 197

1. If 7 horses are kept 20 da. for $14, how many will be kept 7 da. for $28 ?

2. If 3 men earn $75 in 20 da., how many men will earn $78.75 in 9 da. at the same rate ?

3. If 16 horses eat 96 bu. of corn in 42 da., in how many days will 7 horses eat 66 bu. ?

4. If 16 horses can plough 1280 A. in 8 da., how many acres will 12 horses plough in 5 da. ?

5. If 20 men can perform a piece of work in 12 da., find the number of men who could perform another piece of work 3 times as great in $\frac{1}{5}$ of the time.

6. If 252 men can dig a trench 210 yd. long, 3 wide, and 2 deep in 5 da. of 11 hr. each, in how many days of 9 hr. each will 22 men dig a trench of 420 yd. long, 5 wide, and 3 deep ?

7. If 10 men can reap a field of $7\frac{1}{2}$ A. in 3 da. of 12 hr. each, how long will it take 8 men to reap 9 A., working 16 hr. a day ?

8. If 25 men can do a piece of work in 24 da., working 8 hr. a day, how many hours a day would 30 men have to work in order to do the same piece of work in 16 da. ?

9. A town which is defended by 1200 men, with provisions enough to sustain them 42 da., supposing each man to receive 18 oz. a day, obtains an increase of 200 men to its garrison. What must now be the allowance to each man, in order that the provisions may serve the whole garrison for 54 da. ?

10. If 560 flagstones, each $1\frac{1}{2}$ ft. square, will pave a courtyard, how many will be required for a yard twice the size, each flagstone being 14 in. by 9 in. ?

11. If 20 men in 3 wk. earn \$ 900, in what time will 12 men earn \$ 1500 ?

12. If $\frac{9}{14}$ of a meadow be mown by 12 men in 6 da., find in what time the remainder could be mown by 10 men.

13. If 36 men, working 16 da., can dig a trench 72 yd. long, 18 yd. wide, and 12 yd. deep, how many men can dig a trench 64 yd. long, 27 yd. wide, and 18 yd. deep in 24 da. ?

14. If 25 men build a wall 15 ft. high, 2 ft. thick, and 50 ft. long, in 12 da. of 9 hr. each, how many hours per day must 40 men work to build a wall 60 ft. long, 3 ft. thick, and 20 ft. high in 27 da. ?

15. A miller has a bin 8 ft. long, $4\frac{1}{5}$ ft. wide, and $2\frac{1}{2}$ ft. deep, holding 68 bu. How deep must he make another bin which is to be 18 ft. long and $3\frac{1}{2}$ ft. wide, so that its capacity may be 408 bu.?

16. What is the weight of a block of stone 12 ft. 6 in. long, 6 ft. 6 in. broad, and 8 ft. 3 in. deep, when a block of the same stone 5 ft. long, 3 ft. 9 in. broad, and 2 ft. 6 in. deep, weighs 7500 lb.?

COMPOUND PARTNERSHIP

279. In **Compound Partnership** the time is taken into account as well as the capital in determining the gain or loss of each partner.

280. (1) A, B, and C enter into partnership. A puts in $700 for 12 mo., B $500 for 9 mo., and C $600 for 8 mo. Divide a profit of $2065 equitably among them.

> The gain on $700 for 12 mo. = the gain on $8400 for 1 mo.
>
> The gain on $500 for 9 mo. = the gain on $4500 for 1 mo.
>
> The gain on $600 for 8 mo. = the gain on $4800 for 1 mo.

∴ the proportional parts representing the gains are 84, 45, and 48, or 28, 15, and 16. $\qquad 28 + 15 + 16 = 59.$

∴ the respective gains are $\frac{28}{59}$, $\frac{15}{59}$, and $\frac{16}{59}$ of $2065 = $980, $525, and $560.

(2) A commenced business with $4000 stock; 3 mo. after, he took in B with a capital of $2000; and 4 mo. after B became a partner, he took in C with a capital of $600; at the end of the year the firm had gained $3450. Find the share of each.

> A's capital = $4000 for 12 mo. = $48,000 for 1 mo.
>
> B's capital = $2000 for 9 mo. = $18,000 for 1 mo.
>
> C's capital = $600 for 5 mo. = $3000 for 1 mo.

∴ the respective gains are proportional to 48, 18, and 3; *i.e.* 16, 6, and 1.

$$16 + 6 + 1 = 23.$$

∴ the respective shares are $\frac{16}{23}$, $\frac{6}{23}$, and $\frac{1}{23}$ of $3450 = $2400, $900, and $150.

Exercise 198

1. D and E enter into partnership; D puts in $480 for 3 mo., and E $900 for 4 mo. They gain $840. What is each man's share in the gain?

2. A, B, and C entered into partnership; A put in $1200 for 8 mo., B $800 for 10 mo., and C $400 for 12 mo. They gained $3920. What was each man's share of the gain?

3. A, B, and C are partners; A puts in $5000 for 6 mo., B $6000 for 8 mo., and C $900 for 11 mo. The profit is $5575.50. What is the share of each?

4. Three graziers hire a pasture for their common use, for which they pay $318. One puts in 20 oxen for 6 mo., another 24 oxen for 8 mo., and the third 28 oxen for 4 mo. How much of the rent should each pay?

5. A and B enter into partnership; A contributes $15,000 for 9 mo., and B $12,000 for 6 mo. They gain $5750. Find each man's share of the gain.

6. A, B, and C rent a field for $56.50; A puts in 70 cattle for 6 mo., B 40 for 9 mo., and C 50 for 7 mo. What ought each to pay?

7. Three merchants enter into partnership; the first invests $1855 for 7 mo., the second invests $887.50 for 10 mo., and the third invests $770 for 11 mo., and they gain $434. What should be each partner's share of the gain?

8. L, M, and N entered into partnership and invested respectively $19,200, $22,500, and $28,300. At the end of 5 mo. L invested $3800 additional, M $2500, and N $3700. At the end of a year the net gain of the firm was found to be $7850. What was each partner's share of this?

9. A and B enter into partnership; A puts in $400 at first, and $500 at the end of 2 mo.; B puts in $300 at first, and $600 at the end of 3 mo. The profit at the end of the year is $470. How should this be divided?

10. A and B engage in trade: A invests $6000, and at the end of 5 mo. withdraws $2000; B puts into the business $4000, and at the end of 7 mo. $6000 more. Divide a gain of $6800 at the end of the year.

11. A, B, and C form a partnership with a joint stock of $15,600; A's stock continues in trade 6 mo., B's 8 mo., and C's 12 mo. A's gain is $1200, B's $2400, and C's $1800. What stock did each put in?

12. Two men complete in a fortnight a piece of work for which they are paid $46.75; one of them works alternately 9 hr. and 8 hr. a day, the other works $8\frac{1}{2}$ hr. for 5 da. in the week, and does nothing on the remaining day. What sum should each receive?

13. A and B are partners; A's capital is to B's as 4 to 9. At the end of 4 mo. A withdraws $\frac{1}{2}$ of his capital, and B $\frac{2}{3}$ of his. At the end of the year their whole gain is $4600. How much belongs to each?

CUBE ROOT

281. The product of $3 \times 3 \times 3$ is 27; of $5 \times 5 \times 5$ is 125. The cubes whose sides measure 3 and 5 units of length contain 27 and 125 units of volume. We say that 27 is the cube of 3 and that 125 is the cube of 5; that 3 is the cube root of 27, and that 5 is the cube root of 125. The cube of 5 is written 5^3, and the cube root of 5 is indicated thus: $\sqrt[3]{5}$.

5^3 is also called the third power of 5.

282. The cubes of

1,	2,	3,	4,	5,	6,	7,	8,	9,	10,

are 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000.

283. The cube roots of

1, 8, 27, 64, 125, 216, 343, 512, 729, 1000,

are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

284. These two paragraphs should be mastered by the pupils as the corresponding paragraphs in square root.

285. The product $4 \times 4 \times 4$ is written 4^3.

The product $4.6 \times 4.6 \times 4.6$ is written $(4.6)^3$.

The product $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$ is written $(\frac{3}{4})^3$.

The cube root of 4 is written $\sqrt[3]{4}$.

The cube root of .4 is written $\sqrt[3]{.4}$.

The cube root of $\frac{3}{4}$ is written $\sqrt[3]{\frac{3}{4}}$.

Exercise 199

Write the following products as powers:

1. $2 \times 2 \times 2$.
2. 3×3.
3. $5 \times 5 \times 5$.
4. $5 \times 5 \times 5 \times 5$.
5. $5 \times 5 \times 5 \times 5 \times 5$.
6. $5 \times 5 \times 5 \times 5 \times 5 \times 5$.
7. $\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$.
8. $\frac{7}{8} \times \frac{7}{8} \times \frac{7}{8}$.
9. $2.3 \times 2.3 \times 2.3$.
10. $3.12 \times 3.12 \times 3.12$.
11. $.12 \times .12 \times .12$.
12. $.1 \times .1 \times .1$.
13. $.02 \times .02 \times .02$.

Write the following powers as products and find their values:

14. 4^3.　　15. 12^3.　　16. 2.5^3.　　17. $(\frac{2}{3})^3$.　　18. $(\frac{1}{5})^3$.

19. $.02^3$.　　20. $.1^3$

Prove the following statements by multiplication:

21. $\sqrt[3]{216} = 6$.
22. $\sqrt[3]{15625} = 25$.
23. $\sqrt[3]{15.625} = 2.5$.
24. $\sqrt[3]{1.728} = 1.2$.
25. $\sqrt[3]{.008} = .2$.
26. $\sqrt[3]{.064} = .4$.
27. $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$.
28. $\sqrt[3]{\frac{125}{343}} = \frac{5}{7}$.
29. $\sqrt[3]{\frac{1728}{3875}} = \frac{4}{5}$.
30. $\sqrt[3]{\frac{512}{.1728}} = \frac{2}{3}$.

Exercise 200

1. Find the length of one edge of a cube containing 512 cu. in. Find the length of all its edges. Find the area of one of its faces. Of all its faces.

2. Find the area of one face of a cube containing 729 cu. in.

3. Find the number of units of length in a cube containing 343 units of volume. Find the number of units of area in one face.

4. Find the edge of a cube one of whose faces contains 144 sq. in. Find its volume.

5. Find the volume of a cube one of whose faces contains 225 sq. in.

6. What is the edge of a cube whose volume is 8 units of volume? 27 units of volume?

7. The ratio of the volumes of two cubes is $\frac{8}{27}$. What is the ratio of their edges?

8. The ratio of the volumes of two cubes is $64 : 125$. What is the ratio of their edges?

9. The ratio of the edges of two cubes is $\frac{5}{7}$. What is the ratio of their volumes?

10. The edges of two cubes are as $7 : 9$. What is the ratio of their volumes?

Exercise 201

1. Find the cubes of 14, 25, 36, 54, 75, and 99.

2. From the results in § 282, state how many digits there are in the cube of a number of 1 digit.

3. From the results in example 1, state how many digits there are in the cube of a number of 2 digits.

4. In long division how many figures form a group? In square root? In cube root?

5. How many digits are there in the cube root of 512? 64? 8? What are they?

6. Divide 389,017 into groups of figures; 29,791; 3375. How many figures are there in the cube root of each of these numbers?

7. Cube 73, 31, and 15.

8. Judging from the results given in § 283, state the number of digits in the cube root of a number containing 1, 2, or 3 digits.

9. Write the cubes of 10, 20, 30, 40, 50, 60, 70, 80, and 90.

10. State how many digits there are in the cube root of a number containing 4, 5, or 6 digits.

11. What is the first digit in the cube root of 2744? 39,304?

286. To find the cube root of a number, we shall first see how the cube of a number is found.

Since $54 = 50 + 4$, we can cube 54 thus:

$$
\begin{array}{r}
50 + 4 \\
50 + 4 \\
\hline
(4 \times 50) + 4^2 \\
50^2 + (4 \times 50) \\
\hline
50^2 + 2(4 \times 50) + 4^2 \\
50 + 4 \\
\hline
(4 \times 50^2) + 2(4^2 \times 50) + 4^3 \\
50^3 + 2(4 \times 50^2) + (4^2 \times 50) \\
\hline
50^3 + 3(4 \times 50^2) + 3(4^2 \times 50) + 4^3
\end{array}
$$

Since 4 divides each of the last three terms, we can put this result $= 50^3 + 4\{3 \times 50^2 + 3 \times 4 \times 50 + 4^2\}$.

We now wish to recover from such a number as 157,464 its cube root. Plainly the tens' digit of the root is 5, *i.e.* the first part of the root is 50.

$$
\begin{array}{r|l}
50 & 157'464\,\underline{|50 + 4} \\
& 125000 \\
\hline
3 \times 50^2 = 7500 & 32464 \\
3 \times 4 \times 50 = 600 & \\
4^2 = 16 & \\
\hline
8116 & 32464
\end{array}
$$

To find the second term, note that in the expression

$$4\{3 \times 50^2 + 3 \times 4 \times 50 + 4^2\}$$

the number **4** is the second digit in the number 54 that was cubed; hence in the work of taking the cube root, the other factor $\{3 \times 50^2 + 3 \times 4 \times 50 + 4^2\}$ will be the real divisor and 3×50^2 the trial divisor.

Therefore, squaring 50 and multiplying by 3, we have 7500. Dividing 7500 into 32,464, we find the quotient to be 4; completing the divisor by adding $3 \times 4 \times 50$ or 600, and 4^2 or 16, we find the divisor to be $7500 + 600 + 16$ or 8116. Multiplying this by 4, we have 32,464. Hence we conclude that the cube root of 157,464 is 54. To prove the result correct, cube 54.

287. The work of extracting the cube root may be shortened thus:

$$
\begin{array}{r|l}
 & 157'464\,\underline{|54} \\
 & 125 \\
\hline
300 \times 5^2 = 7500 & 32464 \\
30 \times 5 \times 4 = 600 & \\
4^2 = 16 & \\
\hline
8116 & 32464
\end{array}
$$

Find the cube root of 926,859,375.

$$
\begin{array}{r|l}
 & 926'859'375\,\underline{|975} \\
 & 729 \\
\hline
300 \times 9^2 = 24300 & 197859 \\
30 \times 9 \times 7 = 1890 & \\
7^2 = 49 & \\
\hline
26239 & 183673 \\
300 \times 97^2 = 2822700 & 14186375 \\
30 \times 97 \times 5 = 14550 & \\
5^2 = 25 & \\
\hline
2837275 & 14186375
\end{array}
$$

288. The cube of 3.19 is equal to 32.461759. From this it is evident that corresponding to the two figures in the decimal part of the number, viz. 19, we have two groups of three figures, viz. 461 and 759, in the decimal part of the cube. Hence in pointing off, begin at the decimal point and mark the number off into periods of three figures each to the right of the decimal and then again to the left. Mark off into periods the number 95.443993.

(1) Find the cube root of 95.443993.

```
                          95.443'993 | 4.57
                          64
    300 × 4² = 4800       31443
   30 × 4 × 5 =  600
        5² =     25
              5425        27125
   300 × 45² = 607500     4318993
  30 × 45 × 7 =  9450
        7² =     49
             616999       4318993
```

(2) Extract the cube root of 16.

```
                           16 | 2.519
                            8
    300 × 2² = 1200        8000
   30 × 2 × 5 =  300
        5² =     25
              1525         7625
   300 × 25² = 187500      375000
  30 × 25 × 1 =   750
        1² =      1
              188251       188251
  300 × 251² = 18900300    186749000
 30 × 251 × 9 =  667700
        9² =     81
           19568081        176112729
```

(3) To extract the cube root of such a number as 843.7295, add ciphers thus, 843.729'500, and extract the cube root.

(4) Extract the cube root of $\frac{64}{343}$.

$$\sqrt[3]{\frac{64}{343}} = \frac{\sqrt[3]{64}}{\sqrt[3]{343}} = \frac{4}{7}.$$

(5) Extract the cube root of $\frac{16}{343}$.

$$\sqrt[3]{\frac{16}{343}} = \frac{\sqrt[3]{16}}{\sqrt[3]{343}} = \frac{2.519}{7} = .359.$$

(6) **Extract the cube root of $\frac{16}{25}$.**

$$\frac{16}{25} = .64 \text{ or } .640.$$

$$\sqrt[3]{.640} = .861.$$

$$\therefore \sqrt[3]{\frac{16}{25}} = .861.$$

Which of the three denominators is not a perfect cube? When should a fraction be reduced to a decimal before extracting its cube root? Why?

Exercise 202

Find the cube root of:

1. 29,791.   2. 54,872.   3. 110,592.

4. 804,357.   5. 941,192.

6. Compare the processes of long division, of extracting the square root, and the cube root of a number. Note in what respect they are similar and in what respect they are different.

7. 2,406,104.   13. .001906624.

8. 69,426,531.   14. 3, .3, .03, .003, .0003.

9. 8,365,427.   15. $\frac{8}{27}$, $\frac{125}{729}$, $\frac{5832}{12167}$.

10. 389.017.   16. $\frac{250}{686}$, $\frac{135}{320}$.

11. 32.461759.   17. $\frac{2}{3}$, $\frac{49}{216}$, $\frac{5}{7}$.

12. .000912673.   18. $3\frac{3}{8}$, $405\frac{28}{125}$, $7\frac{1}{5}$.

19. A cubical block of stone contains 50,653 cu. ft. What is the area of its side?

20. A cube contains 56 cu. ft. 568 cu. in. Find its edge.

21. One gallon contains 231 cu. in. Find the edge of a cube equal to it.

22. Find the length of the inside edge of a cubical vessel which will just hold 10 gal.

23. Three cubes of lead, measuring respectively $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$ of an inch on the edge, were melted together and cast into a single cube. Find the length of the edge of the cube thus formed, neglecting loss of lead in melting and casting.

24. Four cubes of lead, measuring respectively 6, 7, 8, and 9 in. on the edge, were melted together and cast into a single cube. Find the length of the edge of the cube thus formed, if 4 % of the lead was lost in melting and casting.

25. Find the volume of a cube, the area of whose surface is 100.86 sq. in.

26. A cube measures 5 in. on the edge. A second cube is 3 times the volume of the first. By how much does the length of an edge of the second cube exceed that of an edge of the first cube ?

27. By raising the temperature of a cube of iron the length of each of its edges was increased by 5 %. Find correct to four decimals the ratio of increase in the volume of the cube.

28. Each edge of a cube is diminished by $\frac{1}{10}$ of its length. By what fraction of itself is the volume diminished ? By what fraction of itself is the area of the surface diminished ?

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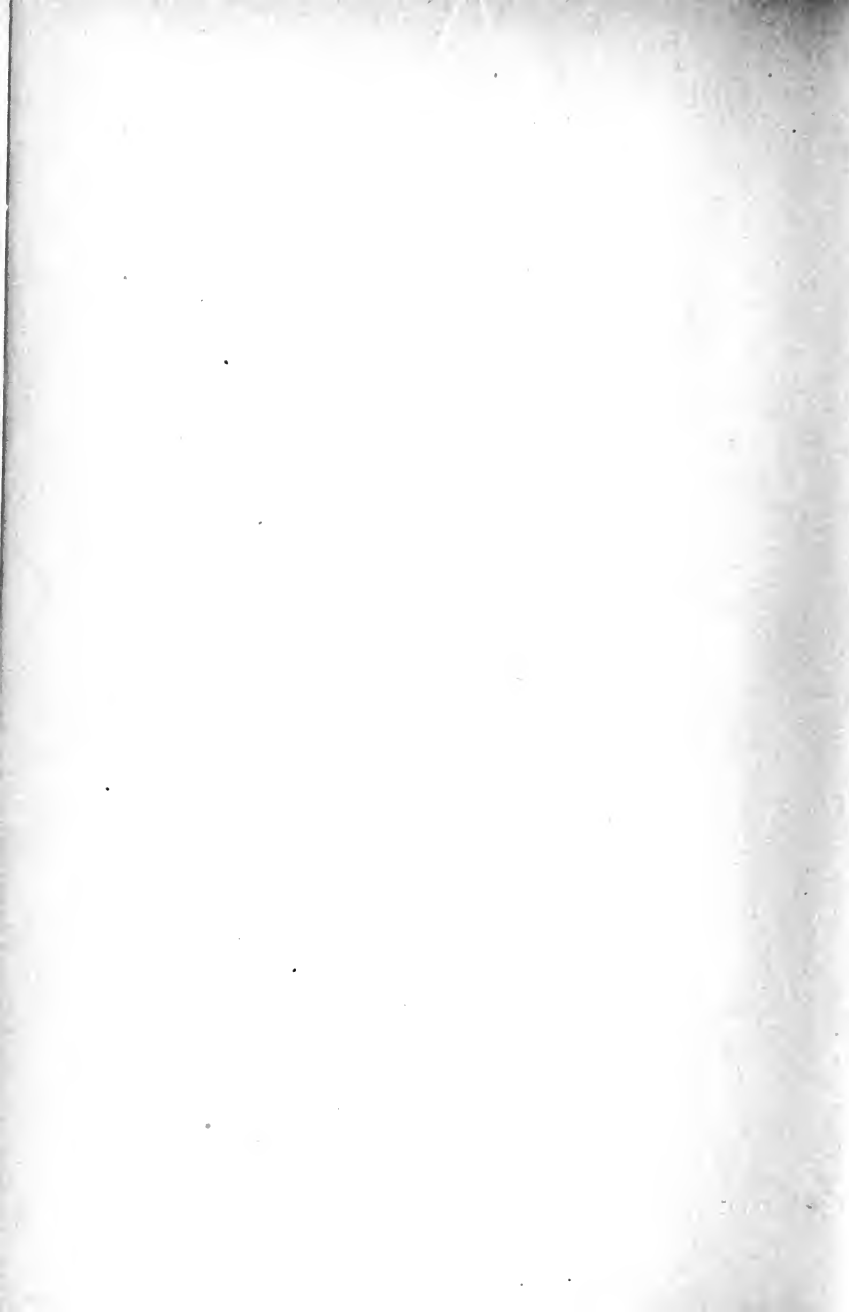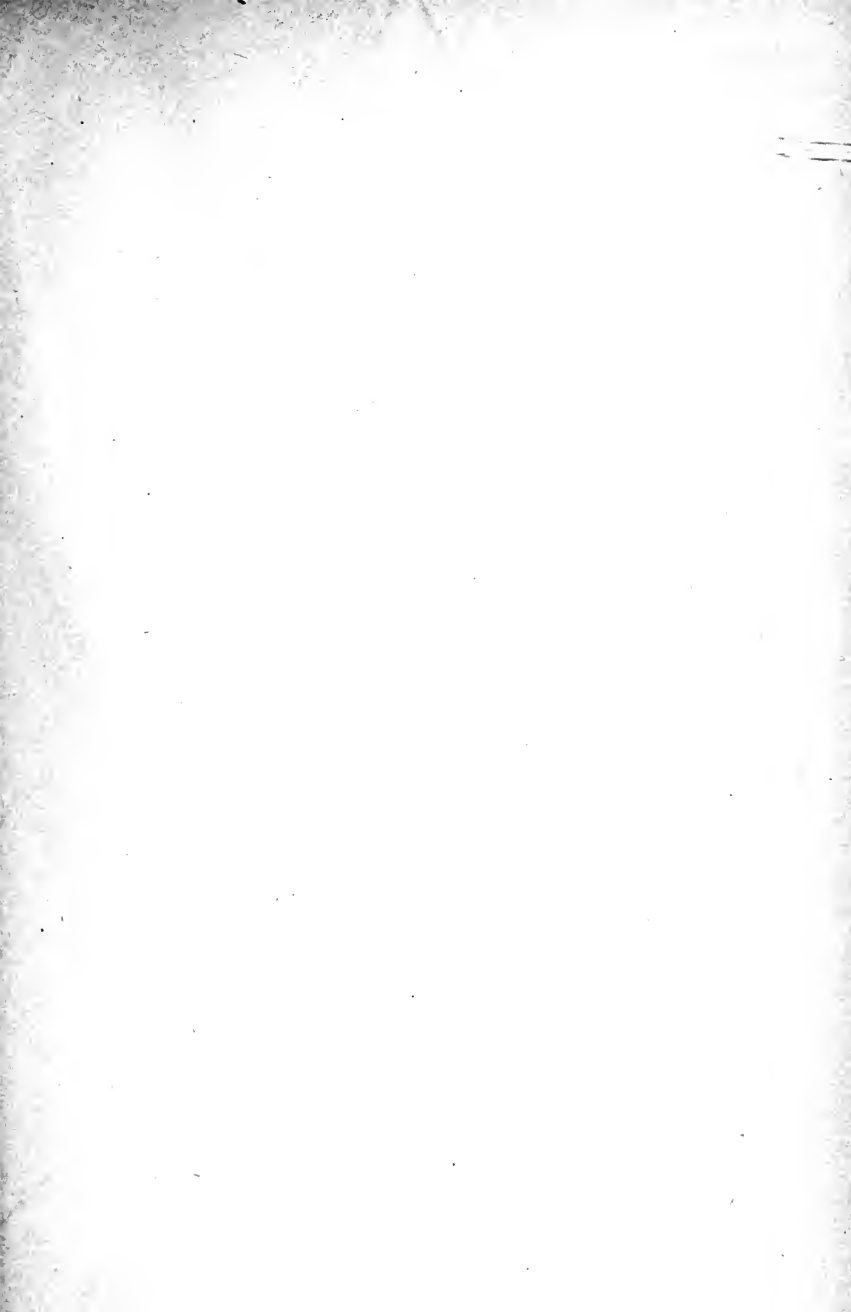

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